Capital Deaccumulation and the Large Persistent Effects of Financial Crises

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Abstract

In a panel of OECD and emerging economies, I find that recessions are associated with larger initial drops in investment and more persistent drops in output if they occur simultaneously with banking crises. Furthermore, the banking crises that are followed by more persistent output slumps are associated with particularly large initial drops in investment. I present a model to account for these patterns, in which a financial shock temporarily increases the costs of external finance for investing entrepreneurs, leading to a drop in investment and a persistent slump in output. Critical to the model is the distinction between different types of capital with different depreciation rates. Intangible capital and equipment have high depreciation rates, leading these stocks to drop substantially when investment falls after a financial shock. This can induce a slump in output and employment that persists for close to a decade, through the contribution of equipment and intangibles to production and labor demand. I show that the consequences of such a financial shock correspond to several features of the US Great Recession (2007-2014), including the large drop in equipment and intangible capital. In the model, TFP and government spending shocks do not lead to large declines in investment or as persistent output decreases, so the model is also consistent with the more transitory output drops seen after non-financial recessions, where such shocks may have been more important.

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1 Introduction

This paper proposes a model to account for the long and protracted slumps in output that often follow financial crises. The length of the slump that followed the financial crisis of 2008 in many countries has been seen as puzzling in the literature, with many real and monetary models with financial shocks predicting much sharper recoveries (see, for instance, Kydland and Zarazaga (2013); Galí, Smets and Wouters (2012)). However, the long slump after 2008 does not appear to have been particularly exceptional. In a panel of 40 OECD and emerging economies from 1970 onwards, I find that output typically shows no sign of returning to trend 6 years after a recession featuring a banking crisis. This finding is consistent with a large empirical literature. By contrast, the majority of banking crises in the dataset last fewer than 4 years, suggesting that the slump in output persists substantially beyond the period of financial stress. Research on the behavior of credit spreads also supports this interpretation. For instance, Krishnamurthy and Muir (2015) find that increases in the spread predict output losses during banking crises, but output does not immediately return to trend once spreads have returned to normal levels.

I hypothesize that drops in investment during financial crises may be a major force contributing to long subsequent slumps. I find in my panel that recessions marked by banking crises not only feature longer-lived output drops than other recessions, but are also characterized by larger drops in investment at the onset of the recession. I also find that banking crises that are followed by more persistent output drops are associated with larger drops in the investment-output ratio around the time of the crisis. Neither of these two findings can be explained by the fact that recessions which are initially more severe are also more persistent. This is because recessions marked by financial crises do not feature much larger initial output drops than other recessions, even in cases where the output drop is ultimately very persistent. Furthermore, these two findings are not predicted by a standard RBC model in which persistent decreases of output are caused by persistent TFP shocks. In these models, persistent TFP shocks tend to induce major decreases in consumption on impact, rather than particularly large drops in investment.

Motivated by these findings, I present a model in which a financial shock leads to a large drop in investment. This in turn leads to a persistent decrease in the capital stock, causing a persistent slump in output. The quantitative importance I find for this mechanism contrasts with the existing literature, which has found movements in the aggregate capital stock, as

\footnote{Gali, Smets and Wouters (2012) argue that their New Keynesian model can account for the slowness of the recovery from the US Great Recession, but only if the US economy continued to be hit by repeated negative shocks in the years after 2008. No shock individually has a persistent negative effect on output.}

\footnote{Empirical studies have found financial crises to be followed by very persistent decreases in output relative to the pre-crisis trend in the US, in the world and in the OECD. Such empirical studies include Cerra and Saxena (2008), Furceri and Mourougane (2012), Jalil (2015), Krishnamurthy and Muir (2015) and Abiad et al. (2009). The latter also find financial crises to be followed by persistent decreases in employment.}

\footnote{That financial crises are followed by particularly large investment slumps has also been documented by Ottonello (2014).}
conventionally measured, to be of less importance in economic fluctuations. This is because the
depreciation rate is sufficiently low that the capital stock does not fluctuate much except at
very low frequencies, both in models and in the data.\footnote{See Schwartzman (2012), and Bigio (2012) for more discussion of this.} The key novel feature of my model that
overturns this conclusion and generates larger and persistent output slumps is the important
role of equipment and intangible capital in production. Unlike standard models, I disaggregate
the capital stock into structures, on the one hand, and equipment and intangibles on the other.
These are treated as separate inputs into a Cobb-Douglas production function. Equipment and
structures represent the two types of business capital traditionally measured in the national
accounts. Intangible capital refers to the assets firms have built up through past investment in
product design, marketing and customer support, research and development, human capital and
organizational development. Many of these assets are still not included in US national accounts.
Crucially, I calibrate a relatively large stock of intangible capital, based on the estimates of
\textcite{Corrado and Hulten (2010)}\footnote{These estimates are not atypical of the recent intangible capital literature. This is discussed in more depth in footnote \ref{foot:corrado} on page \pageref{foot:corrado}.} The calibration implies a correspondingly large role for equipment
and intangibles in production.\footnote{For the model to match long-run average investment rates in structures, equipment and intangibles in the
steady state, I find that the elasticity of output with respect to equipment and intangibles needs to be 0.37,
versus an elasticity of 0.15 for structures.}

The propagation mechanism in the model results from movements in the stock of equipment
and intangible capital, rather than from movements in structures. This is because equipment and
intangible capital differ from structures in their depreciation rates. Equipment and intangible
capital face estimated depreciation rates of around 20\% per year, whereas the depreciation rate
of structures is around 4\% per year. I model a financial shock as a transitory increase in the
riskiness of investment, which reduces the ability of entrepreneurs to borrow externally. This
leads to a short-lived rise in the credit spread and a large drop in investment. The decrease in
investment causes a substantial decrease in the stock of equipment and intangibles, due to the
high depreciation rate of these types of capital. By contrast, the stock of structures does not
decrease as much, due to the low depreciation rate of structures. Nevertheless, since equipment
and intangibles contribute more to production than do structures, the net effect is a large decrease
in output which persists until the stock of equipment and intangibles can be replenished. This
takes many years. I find that the model produces a decrease in output after a financial shock
that is around 50\% larger than in an almost identical setting with only one type of capital and
a depreciation rate typical of the literature.

The US Great Recession and other financial crises have been marked not only by persistent
decreases in output, but also persistent decreases in employment. I find that the model can
replicate this feature as well, provided that wages do not decrease too much in the slump. In order
to explore the effects of varying wage rigidity, I model the labor market using search frictions
and consider two distinct wage bargaining protocols: Nash bargaining, and an alternating offer bargaining protocol based on [Hall and Milgrom (2008)]. The latter nests Nash bargaining as a special case, but is also consistent with greater wage rigidity, depending upon the parameters. In the model, the drop in equipment and intangible capital after a financial shock acts to reduce labor demand, due to the contribution of equipment and intangibles to the production function. If wages are sufficiently rigid, employment falls. I find that the model with Nash bargaining produces only a modest drop of employment in the years after the financial crisis. However, I find that the model with alternating offer bargaining produces a large decrease in employment after a financial shock that persists for close to a decade, at parameter values consistent with the empirical behavior of wages. Indeed, I estimate the parameters of this bargaining process using a Bayesian approach, based on NIPA and BLS data on wages, output and employment. I find that the data favors a parametrization with much more rigid wages than Nash bargaining. In addition to its effect on aggregate employment dynamics, this parametrization produces a decrease in output after a financial shock that is more than twice as large as in the model with Nash bargaining.

I find that the model with alternating offer bargaining can account for approximately one third of the persistent decrease in output and employment observed in the US Great Recession. I analyze how the model economy responds to a series of financial shocks that lead to the same spikes in the credit spread as in occurred in the US over the 2007-2014 period. In the model economy, these shocks produce a slump in output and employment equal to around a third of the drop in these variables relative to trend in the Great Recession. I find that the behavior of key aggregate variables in the model shows striking similarities to actual economic developments between 2007 and 2014 in three key respects. First, output and employment show no tendency to return to their pre-recession trend in the seven years after the financial crisis. Second, the model economy experiences a significant decrease in the stock of equipment and intangible capital after the shock. Third, it experiences a smaller decrease in the stock of structures. In sum, these similarities suggest that the declining stock of equipment and intangibles may be the cause of around a third of the persistently low output and employment in the Great Recession.

I show that the effects of non-financial shocks in the model are consistent with the more transitory decreases in output that appear to occur in non-financial recessions. Specifically, I augment the model to include TFP shocks as well as a government sector, allowing shocks to the level of public employment, as in [Michaillat (2014)]. Due to the existence of financial and search frictions, private employment does not change rapidly in response to changes in public employment in the short run. Consequently, the public employment multiplier is close to 1. Moreover, the responses of output to TFP and public employment shocks is sizable and shows some persistence, but rather less persistence than for financial shocks. In addition, TFP shocks
and government employment shocks have less effect on investment than financial shocks. As such, these shocks may account for the more transitory decreases in output and smaller decreases in investment seen in non-financial recessions.

The remainder of the paper is organized as follows. Section 2 discusses how the paper relates to the existing literature. Section 3 considers the empirical behavior of output and investment across a large sample of financial and non-financial recessions. Section 4 presents the model. Section 5 discusses model calibration and quantitative results and compares model predictions to the US Great Recession. Section 6 concludes.

2 Relevant Literature

This paper contributes a mechanism through which financial crises have persistent effects on aggregate output through movements in the stock of equipment and intangible capital. As far as I am aware, this is the first paper in the literature to emphasize that the relatively high depreciation rates of these types of capital imply that their stocks can decrease substantially after a financial shock, contributing to persistently low output. Furthermore, I show that if wages are set by an alternating offer bargaining protocol similar to Hall and Milgrom (2008) then this mechanism can contribute to persistently high unemployment.

The paper relates most closely to the emerging literature on modeling the persistent economic effects of financial crises but the mechanism is distinct from those discussed in this literature. Much of the literature on the persistent effects of financial crises draws on endogenous growth models along the lines of Romer (1990). In an endogenous growth setting, a financial crisis can reduce innovation and so have a permanent negative effect on total factor productivity and output. Prominent examples of this approach include Queralto (2013) and Anzoategui et al. (2015). Critical to the endogenous growth approach is the idea that large scale spillovers arise from technological innovation, leading to persistent negative effects on other firms when a financially constrained firm stops innovating. Unlike in these models, in my setting, the persistent effects of financial crises occur entirely within an exogenous growth context, without any spillover effects of intangible investment. The mechanisms I discuss in this paper are therefore distinct and complementary to this literature.

Other models of the persistent economic effects of financial crises include those of Ottonello (2014), Khan and Thomas (2011) and Schmitt-Grohé and Uribe (2012). As in this paper,

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7 Other examples related to this approach include Bianchi and Kung (2014), Garcia-Macia (2013), Gornemann (2015) and Guerrón-Quintana and Jinmai (2014).

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It is also worth noting that investment in equipment and marketing appear to be substantially more volatile in US data than investment in R&D. Since these two forms of investment presumably are not associated with strong spillover effects, this motivates the focus of this paper on the persistent effects of a collapse in investment on output through an alternative channel.
Ottonello explores the relationship between what he calls ‘investment slumps’ and slow recoveries after financial crises. However, the channel he focuses on is different, relying on capital becoming unemployed after a financial crisis rather than a deaccumulation of capital after the crisis. Khan and Thomas likewise focus on the misallocation of capital that follows a financial crisis. My approach differs from both these papers in drawing a connection between low investment in intangibles and the persistence of financial crises. Finally, Schmitt-Grohè and Uribe (2012) find very persistent effects of financial crises in the context of a liquidity trap. Although this is likely to be a very important mechanism in the Great Recession, it presumably does not apply in the case of financial crises that occurred when interest rates were substantially above the zero-lower bound. This paper, by contrast, considers the effects of financial shocks in a context where money is neutral and so liquidity traps do not play a role.

This paper also closely relates to the literature on the measurement of intangible investment and its behavior over the business cycle and during the Great Recession. I rely heavily on the work estimating the stock of intangible capital and intangible investment in the United States by Corrado and Hulten (2010). Consistent with the importance accorded to the effect of financial crises on investment in intangible capital in this paper, Campello, Graham and Harvey (2010) find that the tightening of firms’ financing constraints in the Great Recession significantly decreased their investment in intangible capital, especially in marketing. Additionally, Hall (2015) and Reifschneider, Wascher and Wilcox (2015) argue that low investment and the deaccumulation of capital following the financial crisis had a sizeable and persistent negative effect on the economy’s aggregate capacity, although these papers do not construct general equilibrium models that give rise to this mechanism.

A small literature has discussed the implications of disaggregating the aggregate capital stock into different sub-categories, such as equipment and structures. However, this literature has not, to my knowledge, simultaneously considered the implications of including intangibles. Furthermore, it has not attempted to account for the persistent effects of financial shocks. Most similar to this paper is Lopez and Olivella (2014), who consider the effect of financial shocks on skilled and unskilled unemployment in a model with two types of capital. They find that allowing for two types of capital amplifies the financial shock when only one type of capital

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9Since it appears difficult to fully account for the severe persistent slumps following financial crises through one channel alone, it is likely that multiple channels are responsible. Thus, the mechanisms discussed by Ottonello (2014) and Khan and Thomas (2011) likely also play an important role explaining in the persistent effects of financial crises, in addition to the channel I discuss.

10Other models that find persistent effects of financial crises with liquidity traps include Christiano, Eichenbaum and Trabandt (2014) and Eggertsson and Mehrotra (2014).

11Other work attempting to measure intangible capital stocks includes that of Atkeson and Kehoe (2005) and Eisfeldt and Papanikolaou (2014). McGrattan and Prescott (2014) also consider the role played by intangible capital in the Great Recession, and find that the importance of technology shocks in business cycles and the Great Recession is increased when allowance is made for intangible capital.

12See Archibugi, Filippetti and Frenz (2013) and the references cited therein for other work discussing the effect of the financial crisis on intangible investment.
can be pledged as collateral. However, their model does not discuss intangible capital and does not produce persistent decreases in employment following financial shocks. Also is the work of Benhabib, Perli and Sakellaris (2006). They find that disaggregating the capital stock into two types can contribute to the persistence of economic fluctuations, although they do not discuss financial shocks or intangible capital. Chang (1995) observes that the stock of equipment is much more volatile than the stock of structures over the business cycle and considers how distortionary taxes may contribute to this. Slavik and Yazici (2014) analyzes optimal dynamic taxation in a model with two types of capital.

This paper also relates to the literature on the interaction between financial frictions and search frictions in labor markets. Prominent examples include Garin (2015) and Petrosky-Nadeau (2014). Both papers emphasize that the interaction of financial frictions and labor search frictions increases the persistence of movements in employment following economic shocks. Although the mechanism they discuss also operates in this paper to some degree, I find that the role of movements in equipment and intangible capital is quantitatively much more important.

3 Motivating Evidence

A large literature has found that banking crises are associated with particularly persistent declines in output. In this section I present evidence from a sample of developed and emerging economies that the persistent declines in output after financial crises might be due to low levels of investment as a result of the crisis. I consider ‘financial’ and ‘non-financial’ recessions in a sample consisting of OECD countries and countries tracked by JP Morgan’s Emerging Markets Outlook over the period 1970-2014. I define a recession as ‘financial’ if it occurs within a year of a banking crisis, as measured by Reinhart and Rogoff (2009), and ‘non-financial’ otherwise. I use World Bank annual data and drop countries for which GDP or investment data is not available for the whole sample period, leaving a sample of 21 OECD and 19 emerging economies.

Figure 1 shows the average decrease in GDP relative to the 8-year linear pre-recession trend, in the years during and after a financial and non-financial recession. The year of the recession is marked as ‘1’ on the X axis. It is apparent from the figure that both financial and non-financial...
Figure 1: GDP Decline After Financial Vs. Non-Financial Recessions

Figure 2: Investment/Output Decline After Financial Vs. Non-Financial Recessions
recessions are associated with decreases in GDP that are surprisingly persistent. There is no evidence of a return of GDP to trend 6 years after the last pre-recession year. Furthermore, Figure [1] is consistent with other findings in the literature that financial recessions are especially associated with persistently poor output performance. Indeed, while the output drop is slightly larger for financial recessions than others in the year of the recession, the gap between the two lines increases noticeably over the next 5 years.

However, the difference between financial and non-financial recessions is starker for the behavior of investment. Figure [2] shows the behavior of the investment-output ratio \( \frac{I}{Y} \) relative to the pre-recession trend for financial and non-financial recessions. The drop in \( \frac{I}{Y} \) in Figure [2] is more than twice as large for financial recessions in the first year of the recession and the following year. By contrast, the drop in GDP in Figure [1] was only moderately larger for financial recessions in these years. Thus, these two figures suggest that financial recessions may be associated with somewhat larger GDP drops than non-financial recessions in the years immediately after the recession. However, they suggest, that financial recessions may see substantially larger drops in \( \frac{I}{Y} \) in these years. These two patterns survive if the same figures are plotted in a sample of only OECD countries; a sample of only emerging economies, or in a sample ending in 2006. It also applies if variables are detrended using a 2-30 year or 2-40 year band pass filter instead of a linear trend, or if the measure of financial crisis used is that of Laeven and Valencia (2012) instead of Reinhart and Rogoff.

This begs the question of whether the persistently poor performance of GDP after financial recessions may be related to the drop in investment during the recession. Some evidence suggesting that this may be the case is provided by Figures [3] and [4]. These figures show the behavior of GDP and investment in ‘more persistent’ as compared to ‘less persistent’ financial recessions. I define a financial recession as ‘more persistent’ if the output loss relative to trend satisfies the following condition:

\[
\min[Y_{Loss_5}; Y_{Loss_6}] > \max[Y_{Loss_1}; Y_{Loss_2}; Y_{Loss_3}]
\]

where \( Y_{Loss_i} \) denotes the loss in GDP, relative to the linear pre-recession trend, \( i \) years after the last pre-recession year. This rough measure of persistence is designed to reduce the risk that recessions are marked as more persistent simply because another recession rapidly follows, or because the recession starts late in the year.\(^{17}\) In the sample, approximately half of financial recessions are labelled as more persistent by this measure.

It is apparent from Figure [3] that the output loss from more persistent financial recessions is no worse than for less persistent financial recessions in the year of the recession. However, output continues to decline relative to trend for the more persistent financial recessions, whereas the

\(^{17}\)The patterns are substantively very similar if recessions are instead measured as persistent when \( \text{mean}[Y_{Loss_5}; Y_{Loss_6}] > \text{mean}[Y_{Loss_1}; Y_{Loss_2}; Y_{Loss_3}] \).
Figure 3: GDP Decline After More Vs. Less Persistent Financial Recessions

Figure 4: Investment/Output Decline After More Vs. Less Persistent Financial Recessions
less persistent ones show a gradual recovery. This is hardly surprising – more persistent financial recessions were defined as such because they show larger output losses relative to trend in later years. The pattern in Figure 4, however, does not follow automatically from the way persistence is defined. This figure shows the decrease in the investment-output ratio in the more persistent and less persistent financial recessions. It appears that, even in the first recession year and the subsequent year, more persistent financial recessions see bigger drops in the investment-output ratio than less persistent financial recessions. The difference between the two lines in Figure 4 is especially marked in the year after the first recession year.

This cross national evidence is suggestive that there may be a connection between the behavior of investment during financial recessions and the persistent output drops that follow these recessions. Unfortunately, without data on the types of investment and capital that see big decreases after a financial recession, it is impossible to infer whether there are particularly large decreases in the stocks of equipment and intangible capital in these recessions, a key part of the mechanism for persistent slumps in the model in this paper. Few countries have investment data disaggregated into different types (structures, equipment and intangibles) for a long time span, and many countries have very poor or non-existent measures of intangible capital even today. Fortunately, there is sufficient data to draw conclusions about the dynamics of different types of capital in the US, as structures and equipment have been differentiated in the US national accounts for decades. In Section 5.6, I turn to this data in order to demonstrate the explanatory power of the model developed in this paper, comparing the behavior of types of investment and capital in the US Great Recession with the movements predicted by the model.

4 The Model

The model is built around a DSGE framework with two types of capital: structures and intangibles, which differ only in their depreciation rates. ‘Intangibles’ is intended to proxy for equipment and intangible capital in the data, both of which have relatively high depreciation rates in contrast to structures. There are two kinds of friction in the model: financial frictions and labor market frictions. The financial friction is that investment and hiring are carried out by entrepreneurs who are subject to borrowing constraints. Borrowing constraints are due to a costly state verification problem based on the canonical framework of Bernanke, Gertler and Gilchrist (1999). The labor market friction is that entrepreneurs wishing to employ workers must
pay hiring costs and post vacancies. Vacancies match with unemployed workers, as in the now standard framework of Mortensen and Pissarides (1994). Entrepreneurs and workers bargain over wages. The next section, 4.1, summarizes the agents in the model and their interactions. In the following sections I discuss the more standard parts of the model first, before considering the financial frictions and wage bargaining.

4.1 Agents

Each household consists of a large family of workers and entrepreneurs who share consumption. In addition to workers and entrepreneurs, there are a large number of competitive banks and final goods firms owned by households. Entrepreneurs own structures and intangibles, and hire workers. Entrepreneurs use their workers and capital to produce identical intermediate goods which they sell to final goods producing firms. The final goods firms then transform intermediates into consumption and capital goods. Consumption goods are sold to households and capital goods are sold to entrepreneurs. Banks exist in order to transfer funds from workers to entrepreneurs. Workers can hold risk-free deposits in banks and entrepreneurs can borrow from banks, in order to fund investment and hiring. In addition to workers, entrepreneurs, final goods producers and banks, there is a government, which taxes households and hires workers in order to produce public goods. In the baseline model, I assume that the government sets taxes, public employment and public goods production equal to zero each period. I subsequently consider an extension in which these three quantities vary over time.

4.2 Preferences

All agents act to maximize the expected present discounted utility of their household. The preferences of the representative household are as follows.

$$\sum_{t=0}^{\infty} \beta^t \left( \log(C_t) - \nu N_t - \nu N_{G,t} + Y_{G,t} \right)$$  (1)

Here $C_t$ is aggregate consumption, $N_t$ is the measure of workers who are employed in the private sector, $N_{G,t}$ workers are employed in the public sector and $Y_{G,t}$ is total production of public goods. It is assumed that the total measure of workers in the economy is equal to 1. Hence, at any time $t$, there are $1 - N_t - N_{G,t}$ unemployed workers.

4.3 Shocks

I consider three distinct shocks in the model: a shock to the riskiness of investment by entrepreneurs, denoted by $\sigma_t$, a government employment shock, affecting $N_{G,t}$ and a shock to total
factor productivity, denoted by \( Z_t \). I assume that \( \sigma_t = \bar{\sigma}e_{\sigma,t} \), \( N_{G,t} = \bar{N}_G e_{g,t} \) and \( Z_t = \bar{Z}e_{z,t} \), where \( e_{\sigma,t}, e_{g,t} \) and \( e_{z,t} \) follow AR(1) processes in logs:

\[
\begin{align*}
\log(e_{\sigma,t}) &= \rho_{\sigma} \log(e_{\sigma,t-1}) + \epsilon_{\sigma,t} \\
\log(e_{g,t}) &= \rho_{g} \log(e_{g,t-1}) + \epsilon_{g,t} \\
\log(e_{z,t}) &= \rho_{z} \log(e_{z,t-1}) + \epsilon_{z,t}
\end{align*}
\]

The values of all aggregate shocks are revealed at the start of the period. I assume \( \sigma_t+1 \) is revealed one period in advance, at time \( t \).

4.4 Final Goods Producers

Entrepreneurs produce intermediate goods which they sell to final goods producers. The final goods producers then use these intermediate goods to produce consumption, structures and intangibles according to a constant returns to scale technology. The production function of the representative final goods producer is as follows:

\[
C_t + I_{S,t} + I_{I,t} + \kappa(I_{S,t}, I_{S,t-1}, I_{I,t}, I_{I,t-1}) = Y_t
\]

where \( Y_t \) denotes the quantity of the intermediate good purchased from the entrepreneurs, and \( C_t, I_{S,t} \) and \( I_{I,t} \) denote the quantity of consumption goods, structures and intangibles produced. These are sold to entrepreneurs and households.

Here \( \kappa(\cdot) \) is a convex adjustment cost function, given by:

\[
\begin{align*}
\kappa(I_{S,t}, I_{S,t-1}, I_{I,t}, I_{I,t-1}) &= \kappa_0 \left( \frac{I_{S,t} - I_{S,t-1}}{I_{S,t-1}} \right)^2 I_{S,t-1} + \kappa_0 \left( \frac{I_{I,t} - I_{I,t-1}}{I_{I,t-1}} \right)^2 I_{I,t-1} \\
&\quad + \kappa_1 \left( \frac{I_{S,t} - I^*_S}{I^*_S} \right)^2 I^*_S + \kappa_1 \left( \frac{I_{I,t} - I^*_I}{I^*_I} \right)^2 I^*_I
\end{align*}
\]

Here \( I^*_S \) and \( I^*_I \) denote the steady state levels of structures and intangible investment. This adjustment cost function implies that the level of investment adjustment costs is given by \( \kappa_0 \) and the level of capital adjustment costs is given by \( \kappa_1 \).

Normalize the price of consumption goods to 1. Profits of the representative final goods producer are equal to \( C_t + p_{S,t}I_{S,t} + p_{I,t}I_{I,t} - p_{Y,t}Y_t \), where \( p_{S,t}, p_{I,t} \) and \( p_{Y,t} \) denote the prices of structures, intangibles and intermediate goods respectively. In the competitive equilibrium, final goods producers make zero profits and the price of intermediate goods satisfies \( p_{Y,t} = 1 \).

The quantity of capital goods produced by the final goods firms determines the stock of capital goods.
capital goods next period:

\[ K_{S,t+1} = (1 - \delta_S)K_{S,t} + I_{S,t} \]  
\[ K_{I,t+1} = (1 - \delta_I)K_{I,t} + I_{I,t} \]  

(8) (9)

4.5 Government

The public sector produces public goods according to the production function:

\[ Y_{G,t} = Z_G N_{G,t} \]  

(10)

Since \( Y_{G,t} \) enters into the household’s utility an additively separable way in (1), the value of \( Z_G \) does not affect the decisions of any private agent.

The government pays public sector employees the wage \( w_{G,t} \). It is assumed to set this equal to the average private sector wage \( w_t \). The government funds wage payments to its employees using lump-sum taxes on households. Ricardian equivalence holds in the model, so it is without loss of generality to assume that the government balances its budget every period. That is, the lump sum tax \( \tau_t \) is equal to total government spending:

\[ \tau_t = w_t N_{G,t} \]  

(11)

4.6 Labor Markets

Recall that workers are either employed or unemployed. Workers who are employed in the private sector are matched with entrepreneurs and separate with probability \( \delta_N \) at the end of each period. Workers employed by the government also separate from jobs with probability \( \delta_N \). Unemployed workers search for jobs randomly and at the end of each period they find a job in the private sector with probability \( f_t \) and find a job in the public sector with probability \( f_{G,t} \).

The probability of finding a job in the private sector \( f_t \) is given by the ratio of aggregate private hiring to the number of unemployed workers, that is:

\[ f_t = \frac{N_{t+1} - (1 - \delta_N)N_t}{1 - N_t - N_{G,t}} \]  

(12)

The probability of finding a job in the public sector is determined in a similar way:

\[ f_{G,t} = \frac{N_{G,t+1} - (1 - \delta_N)N_{G,t}}{1 - N_t - N_{G,t}} \]  

(13)

Entrepreneurs that wish to hire workers must pay hiring costs equal to \( \frac{h_t}{p_{V,t}} \) units of the inter-
mediate good. Therefore, hiring a worker costs the same amount as \( h_t \) units of the consumption good. As in Pissarides (2009) hiring costs consist of two parts. First of all, entrepreneurs that wish to hire have to pay a cost of \( h_1 \) units of the intermediate good in order to post a vacancy. Once a vacancy has matched with a worker, the entrepreneur must pay an additional cost of \( h_0 \) units of the intermediate good in order to hire the worker. These represent training costs.

Let the total number of matches in the labor market in period \( t \) be given by the Cobb-Douglas function

\[
M_t = A_{M}^{1-\psi} (v_t + v_{G,t})^{1-\psi} (1 - N_t - N_{G,t})^{\psi}.
\]

This depends on the number of private and public sector vacancies, \( v_t + v_{G,t} \), and the number of unemployed workers, \( 1 - N_t - N_{G,t} \). Then the probability that a vacancy matches with a worker is given by

\[
\left( \frac{A_{M}(v_t + v_{G,t})}{1 - N_t - N_{G,t}} \right)^{-\psi}.
\]

It is assumed that a law of large numbers holds such that an entrepreneur that wishes to hire a worker therefore posts exactly \( \left( \frac{A_{M}(v_t + v_{G,t})}{1 - N_t} \right)^{\psi} \) vacancies and hires the worker with probability 1.

Therefore the costs of hiring a worker are given by:

\[
h_t = p_{Y,t} h_0 + p_{Y,t} h_1 \left( \frac{A_{M}(v_t + v_{G,t})}{1 - N_t - N_{G,t}} \right)^{\psi} \tag{14}
\]

Equally, the total quantity of private sector vacancies posted is equal to:

\[
v_t = [N_{t+1} - (1 - \delta N)N_t] \left( \frac{A_{M}(v_t + v_{G,t})}{1 - N_t - N_{G,t}} \right)^{\psi} \tag{15}
\]

Public sector vacancies \( v_{G,t} \) find workers at the same rate as private sector vacancies. Therefore, the total number of public sector vacancies is given by:

\[
v_{G,t} = [N_{G,t+1} - (1 - \delta N)N_{G,t}] \left( \frac{A_{M}(v_t + v_{G,t})}{1 - N_t - N_{G,t}} \right)^{\psi} \tag{16}
\]

The government in turn pays a hiring cost of \( h_t \) units of public goods per worker it hires.

4.7 Entrepreneurs

An entrepreneur holds three types of physical asset: structures \( k_S \), intangibles \( k_I \), and workers \( n \), which she uses to produce intermediate goods. At the end of each period \( t \) an entrepreneur chooses her stock of capital and workers in order to produce in the next period. Denote her choices by \( \tilde{k}_{S,t}, \tilde{k}_{I,t}, n_{t+1} \).

Entrepreneurs can buy structures and intangibles from final goods firms and each other at prices \( q_{S,t} \) and \( q_{I,t} \) and hire workers at cost \( h_t \). Entrepreneurs can also sell workers to one

\[20\] Provided that public sector hiring costs are paid in units of public goods, the precise level of these hiring costs will only affect the stock of public goods in equilibrium and will not affect any other aggregate variables, since public goods consumption and private consumption are additively separable in the household’s utility function.
Entrepreneurs can fund purchases of capital and workers by issuing debt to banks as well as by selling existing assets. After acquiring capital and workers at time $t$, an entrepreneur draws a shock $\xi$ at the start of period $t+1$ which affects the productive value of these assets. Specifically, if an entrepreneur bought $\tilde{k}_s$ and $\tilde{k}_I$ units of structures and intangibles at the end of period $t$ and draws shock $\xi_{t+1}$ at the start of $t+1$, then the entrepreneur starts period $t+1$ with structures and intangibles given by:

$$k_{S,t+1} = \xi_{t+1}\tilde{k}_{S,t}$$
$$k_{I,t+1} = \xi_{t+1}\tilde{k}_{I,t}$$

As in [Bernanke, Gertler and Gilchrist (1999)](#), I assume that $\xi$ has a unit mean lognormal distribution that is independently drawn across time and across entrepreneurs. Let $\sigma_{t+1}$ denote the standard deviation of $\log(\xi)$ at time $t+1$, which varies over time in response to aggregate shocks as discussed in section 4.3 above.

After observing the shock $\xi$, entrepreneurs use their workers and capital to produce intermediate goods, which they sell to final goods producers. In addition to affecting the entrepreneur’s stock of capital, the shock $\xi$ affects the entrepreneur’s total factor productivity for one period. Indeed, an entrepreneur with assets $k_{S,t+1}, k_{I,t+1}$ and $n_{t+1}$ that draws shock $\xi_{t+1}$ produces output $y_{t+1}$ according to:

$$y_{t+1} = Z_{t+1}\tilde{k}_{S,t}^\alpha_{S}k_{I,t+1}^\alpha_{I}(\xi_{t+1}n_{t+1})^{1-\alpha_{I}-\alpha_{S}} \quad \equiv \xi_{t+1}Z_{t+1}\tilde{k}_{S,t}^\alpha_{S}k_{I,t+1}^\alpha_{I}n_{t+1}^{1-\alpha_{I}-\alpha_{S}}$$

where the second line uses that $k_{S,t+1}$ and $k_{I,t+1}$ themselves depend on $\xi_{t+1}$. Here $Z_{t+1}$ is the aggregate level of total factor productivity, which varies in response to shocks as discussed in section 4.3. Define $\tilde{y}_{t+1} = Z_{t+1}\tilde{k}_{S,t}^\alpha_{S}k_{I,t+1}^\alpha_{I}n_{t+1}^{1-\alpha_{I}-\alpha_{S}}$, so that we may write $y_{t+1} = \xi_{t+1}\tilde{y}_{t+1}$.

Simultaneously with production, the entrepreneur pays wages $w_{t+1}(\xi_{t+1})$, which in general depend on $\xi_{t+1}$. These wages are set by bargaining, as discussed in section 4.13 below. After period $t+1$ production, fraction $\delta_S$ and $\delta_I$ of the entrepreneur’s structures and intangibles respectively are lost to depreciation, and fraction $\delta_N$ of her workers exogenously separate from their jobs.

The net effect of this is that the entrepreneur who bought assets $\tilde{k}_{S,t}, \tilde{k}_{I,t}$ and $n_{t+1}$ at time $t$
earns a return on these assets $R_{t+1}^K$ equal to:

$$R_{t+1}^K = \left( \frac{\xi_{t+1}(\bar{p}_{t+1} y_{t+1} - w_{t+1}(\xi_{t+1}) n_{t+1})}{q_{S,t} \hat{k}_{S,t} + q_{I,t} \hat{k}_{I,t} + h_{t} n_{t}} \right)$$

$$+ \left( \frac{\xi_{t+1}(q_{S,t+1}(1 - \delta_S) \hat{k}_{S,t} + q_{I,t+1}(1 - \delta_I) \hat{k}_{I,t}) + h_{t+1}(1 - \delta_N)n_{t+1}}{q_{S,t} \hat{k}_{S,t} + q_{I,t} \hat{k}_{I,t} + h_{t} n_{t}} \right)$$

It is convenient to assume now, and verify later on, that the wage paid by the entrepreneur takes the following form in equilibrium:

$$w_{t+1}(\xi_{t+1})n_{t+1} = w_{0,t+1}n_{t+1} + \vartheta(1 - \alpha_I - \alpha_S)\xi_{t+1} p_{Y,t+1} \tilde{y}_{t+1}$$

where $w_{0,t+1}$ is an aggregate variable unaffected by $\xi_{t+1}$ or the entrepreneur’s decisions, and $\vartheta$ is a parameter representing the bargaining weight of labor. Then, the entrepreneur’s return can be written as:

$$R_{t+1}^K = R_{0,t+1}^K + \xi_{t+1} R_{1,t+1}^K$$

where

$$R_{0,t+1}^K = \frac{-w_{0,t+1}n_{t+1} + h_{t+1}(1 - \delta_N)n_{t+1}}{q_{S,t} \hat{k}_{S,t} + q_{I,t} \hat{k}_{I,t} + h_{t} n_{t}}$$

$$R_{1,t+1}^K = \frac{[1 - \vartheta(1 - \alpha_I - \alpha_S)] p_{Y,t+1} \tilde{y}_{t+1} + q_{S,t+1}(1 - \delta_S) \hat{k}_{S,t} + q_{I,t+1}(1 - \delta_I) \hat{k}_{I,t}}{q_{S,t} \hat{k}_{S,t} + q_{I,t} \hat{k}_{I,t} + h_{t} n_{t}}$$

Therefore, the entrepreneur’s rate of return is homogeneous of degree zero with respect to the variables $(\hat{k}_{S,t}, \hat{k}_{I,t}, n_{t+1})$. As will become evident, the linearity of the entrepreneur’s problem ensures that, in equilibrium, all entrepreneurs choose levels of capital and labor proportional to their net worth. Thus, the values of $R_{0,t+1}^K$ and $E_t[R_{1,t+1}^K]$ are invariant across entrepreneurs.

### 4.8 Financial Markets

Entrepreneurs may finance their purchases of capital and labor at the end of a period $t$ by taking loans from the banks. The banks in turn raise funds by allowing workers to save in risk-free deposits. Thus, the banks transfer funds from the workers to the entrepreneurs.

The loan obtained by an entrepreneur from a bank is assumed to take the form a standard debt contract. That is, the entrepreneur who borrows $b_t$ at time $t$, must repay $b_{t+1} = (1 + i_{t+1})b_t$ after producing in period $t + 1$, if she has the resources to repay, and will default on her debt otherwise. The interest rate on the debt $i_{t+1}$ is assumed to depend on the aggregate state at
time $t+1$ as discussed below.\footnote{The details of the debt contract are imposed exogenously here. However, that the contract between the entrepreneur and the bank should take the form of a debt contract can be justified on grounds of costly state verification. See footnote \ref{footnote:debt}.}

Entrepreneurs who draw very low $\xi_{t+1}$ may find themselves unable to afford to repay their debt. Let $\xi_{t+1}$ denote the cutoff value, such that the entrepreneur will be able to repay her debt if and only if she draws $\xi_{t+1} \geq \xi_{t+1}$. Then, in particular it follows that:

\begin{equation}
(R^K_{0,t+1} + R^K_{1,t+1}(\xi_{t+1}))(q_{s,t}\tilde{k}_{s,t} + q_{l,t}\tilde{k}_{l,t} + h_{t}n_{t+1}) = b_t(1 + i_{t+1})
\end{equation}

Given $\xi_{t+1}$ this equation determines $i_{t+1}$ in each state $t+1$.

If the entrepreneur draws $\xi_{t+1} < \xi_{t+1}$, she declares bankruptcy, in which case the bank confiscates all her assets and sells them to other entrepreneurs. However, the bank must also pay a monitoring cost in this case, in order to verify that the entrepreneur truly cannot repay her debt. The monitoring cost is assumed to equal fraction $\mu$ of the value of the entrepreneur’s assets and output, that is $\mu[R^K_{0,t+1} + \xi_{t+1}R^K_{1,t+1}](q_{s,t}\tilde{k}_{s,t} + q_{l,t}\tilde{k}_{l,t} + h_{t}n_{t+1})$\footnote{More precisely, the equation follows only if $\xi_{t+1} \in (0, \infty)$. However, this will always be the case in equilibrium.} Monitoring costs are paid in units of the intermediate good, so a bank that pays a monitoring cost of, for instance, $\mu$, must purchase $\frac{\mu}{p_{Y,t}}$ units of the intermediate good.

It is assumed that each bank must pay a risk-free return on all deposits it issues and cannot borrow through any other means. Consequently, each bank must make non-negative profits in every period, in every state of the world. Furthermore, free entry and perfect competition among banks imply that banks cannot make positive expected profits on any loan. Then, an equilibrium will exist in which banks make strictly zero profits on every loan in expectation in every aggregate state.\footnote{As discussed in \cite{Bernanke:1999}, the standard debt contract assumed here can be viewed as an incentive compatible solution to a costly state verification problem, in which the entrepreneur privately observes her realization of $\xi$, and the bank must pay the monitoring cost in order to observe it. The debt contract is optimal given the constraint that the bank make non-negative expected profits on every loan in every aggregate state $t+1$.} That is, the following break-even condition for the bank will hold on each loan in each aggregate state $t+1$:

\begin{equation}
b_t(1 + r_t) = [1 - F_t(\xi_{t+1})](1 + i_{t+1})b_t + (1 - \mu)(q_{s,t}\tilde{k}_{s,t} + q_{l,t}\tilde{k}_{l,t} + h_{t}n_{t+1}) \int_0^{\xi_{t+1}} R^K_{0,t+1} + \xi R^K_{1,t+1}dF_t(\xi)
\end{equation}

Here $r_t$ is the risk free rate paid by the bank to its depositors, which is predetermined at time $t$.

\footnote{I will assume here that the equilibrium takes this form. Given the restrictions imposed on banks due the need to issue risk-free deposits and free entry, the only potential deviation from this equilibrium to consider would be a bank that makes positive expected profits on some loans in some aggregate states, and negative expected profits on other loans in those states, with zero expected profits on each loan overall. However, such a deviation would increase the proportion of entrepreneurs that default, by making the required repayment from entrepreneurs more stochastic, and so would increase monitoring costs with no benefits, which cannot be profitable.}
and is identical across banks in a competitive equilibrium.

4.9 Evolution of Net Worth

Let $x_t$ denote the value of the entrepreneur’s net worth at time $t$, just before investment is undertaken. Then her expected period $t + 1$ net worth will be given by:

\[
E_t[x_{t+1}] = E_t \left[ \int_{\xi_{t+1}}^{\infty} (R_{0,t+1}^K + \xi R_{1,t+1}^K)(q_s \tilde{k}_{S,t} + q_I \tilde{k}_{I,t} + h t n_{t+1}) - b_t(1 + i_t)dF(\xi, \sigma_t) \right]
\]

(24)

\[
= (q_s \tilde{k}_{S,t} + q_I \tilde{k}_{I,t} + h t n_{t+1})E_t \left[ R_{1,t+1}^K \int_{\xi_{t+1}}^{\infty} (\xi - \xi_{t+1})dF(\xi, \sigma_t) \right]
\]

(25)

\[
= x_t L_t E_t \left[ R_{1,t+1}^K (1 - \Gamma_t(\xi_{t+1})) \right]
\]

(26)

where $L_t$ and $\Gamma_t$ are defined by:

\[
L_t = \frac{q_s \tilde{k}_{S,t} + q_I \tilde{k}_{I,t} + h t n_{t+1}}{x_t}
\]

(27)

\[
\Gamma_t(\xi_{t+1}) = [1 - F_t(\xi_{t+1})] \xi_{t+1} + G_t(\xi_{t+1})
\]

(28)

and

\[
G_t(\xi_{t+1}) = \int_{0}^{\xi_{t+1}} \xi dF_t(\xi)
\]

(29)

That is, $L_t$ denotes the leverage ratio and $[1 - \Gamma_t(\xi_{t+1})] R_{1,t+1}^K$ denotes the fraction of the total return $R_{t+1}^K$ retained by the entrepreneur in expectation. Note that (22) was used in the derivation of (25).

After entrepreneurs have produced, they sell assets to repay their debts, as well as taking on new debts in order to produce once more. Furthermore, after production and default have taken place, fraction $\chi$ of entrepreneurs’ net worth is exogenously transferred to workers. In addition, fraction $1 - \chi$ of workers’ wealth is exogenously transferred to entrepreneurs’. This is a modeling device to prevent entrepreneurs from saving enough to outgrow their financial constraints, a common concern in the literature, as discussed by Quadrini (2011). It can be interpreted variously as representing entry and exit, or that firms are not free to vary their payments of dividends.

One interpretation of this transfer is that the workers’ wealth is held by inactive entrepreneurs. Each period fraction $1 - \chi$ of active entrepreneurs remain active, with the rest becoming inactive. In turn, fraction $1 - \chi$ inactive entrepreneurs become active, with the rest remaining inactive. The scales of transfers to and from active entrepreneurs that this implies are particularly large by the standards of the literature. I find that such rapid transfers are necessary in order to prevent entrepreneurs from rapidly outgrowing financing constraints after a
Before writing down the entrepreneur’s optimization problem, it is convenient to simplify the bank’s break even constraint (23). Using (22) to substitute for $i_{t+1}$ in (23) and then using (27), (28) and (29) to substitute for the integral and for $b_t$, we obtain:

$$[1 - \mu F_t(\xi_{t+1})]R_{0,t+1}^K + [\Gamma_t(\xi_{t+1}) - \mu G_t(\xi_{t+1})]R_{1,t+1}^K = \frac{(L_t - 1)(1 + r_t)}{L_t}$$

(30)

### 4.10 Entrepreneur’s Optimization Problem

The entrepreneur wishes to maximize the present value of her household’s consumption. This amounts in her case to maximizing the expected present value of the wealth that flows from her to the workers. The entrepreneur’s expected net worth after production at time $t + 1$ is given by (26), of which fraction $\chi$ flows to the household.

Let $V(x_t, s_t)$ denote the continuation value of an entrepreneur at time $t$, after production and default have taken place, who has net worth $x_t$. $s_t$ denotes the aggregate state. At this time, the entrepreneur must choose her purchases of structures, intangibles and workers, these choices imply choices of leverage $L_t$, which, in combination with the bank’s break-even condition imply choices of $\xi_{t+1}$ for each state $s_{t+1}$. Then, the Bellman equation for the entrepreneur is:

$$V(x_t, s_t) = \max_{k_{S,t}, k_{I,t}, n_{t+1}, L_t, \xi_{t+1}(s_{t+1})} E_t[\chi m_{t+1}E[x_{t+1}] + (1 - \chi)V(x_{t+1}, s_{t+1})]$$

(31)

where the maximization is subject to the conditions that $E[x_{t+1}]$ evolves in accordance with (26), $L_t$ satisfies the definition (27), $R_{0,t}^K$ and $R_{1,t}^K$ satisfy their definitions (20) and (21), and, for each aggregate state $s_{t+1}$, $\xi_{t+1}(s_{t+1})$ satisfies the bank’s break even condition (30). Here $m_{t+1}$ denotes the household’s stochastic discount factor, given by:

$$m_{t+1} = \beta \frac{C_t}{C_{t+1}}$$

(32)

It is easy to verify that the linearity of the entrepreneur’s problem ensures that entrepreneurs will choose $k_{S,t}, k_{I,t}$ and $n_{t+1}$ proportional to $x_t$, and therefore that $L_t$ and $\xi_{t+1}$ will be invariant across entrepreneurs.\footnote{To see this, note that it must be the case that $V(x, s_t) \geq xV(1, s_t)$ because an entrepreneur with net worth $x$ can obtain $xV(1, s_t)$ simply by choosing $x$ times as much capital and workers each period as an entrepreneur who has net worth 1 at time $t$. By an analogous argument, we have that $V(1, s_t) \geq \frac{1}{x}V(x, s_t)$. Then, it must be that an entrepreneur with net worth $x$ can do no better, and no worse, than choosing $x$ times as much capital and workers as an entrepreneur with net worth 1. The strict concavity of the entrepreneur’s problem then implies that this must be the unique optimal decision.}

negative financial shock, due to the collapse in interest rates induced by the shock. A more elaborate model would provide developed microfoundations for why firms do not outgrow financing constraints so rapidly in practice; this may be related to the fact that the risk-free rate is much less volatile than implied by the model in this paper.

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4.11 Implications for Aggregates

In a slight abuse of notation, let \( \tilde{k}_{S,t}(x) \) denote the total stock of structures chosen by an entrepreneur with net worth \( x \) at the end of period \( t \). Define \( \tilde{k}_{I,t}(x) \) analogously. Since all entrepreneurs’ demands for capital and labor are linear in their individual net worth, the aggregate demands for capital and labor among entrepreneurs are given by:

\[
K_{S,t+1} = X_t \tilde{k}_{S,t}(1) \equiv \tilde{k}_{S,t}(X_t)
\]

(33)

\[
K_{I,t+1} = X_t \tilde{k}_{I,t}(1) \equiv \tilde{k}_{I,t}(X_t)
\]

(34)

\[
N_{t+1} = X_t n_{t+1}(1) \equiv n_{t+1}(X_t)
\]

(35)

Here \( X_t \) denotes the aggregate net worth of entrepreneurs. In deriving these expressions, I made use of the fact that the mean of \( \xi \) is 1, and that a law of large numbers holds so that the aggregate stock of capital evolves as if all entrepreneurs draw the shock \( \xi = 1 \).

Leverage of all entrepreneurs can therefore be obtained by aggregating equation (27):

\[
L_t = \frac{q_{S,t}K_{S,t+1} + q_{I,t}K_{I,t+1} + h_tN_{t+1}}{X_t}
\]

(36)

The aggregate quantity of the intermediate good \( Y_t \) sold by the entrepreneurs to the final goods producers is given by their total output, net of monitoring and hiring costs:

\[
Y_t = Z_t K_{S,t}^{\alpha_S} K_{I,t}^{\alpha_I} N_t^{1-\alpha_I-\alpha_S} - \frac{h_t}{P_{Y,t}} (N_{t+1} - (1 - \delta_S)N_t) - \frac{\mu}{P_{Y,t}} (q_{S,t-1}K_{S,t} + q_{I,t-1}K_{I,t} + h_{t-1}N_t)[F_{t-1}(\tilde{\xi}_t)R^K_{0,t} + G_{t-1}(\tilde{\xi}_t)R^K_{1,t}]
\]

(37)

Aggregate net worth at time \( t+1 \) will equal fraction \( 1 - \chi \) of the wealth earned by entrepreneurs, plus fraction \( 1 - \chi \) of the savings of workers. Aggregating equation (26), we have that

\[
X_{t+1} = (1 - \chi)X_t L_t R^K_{1,t+1}(1 - \Gamma_t(\tilde{\xi}_{t+1})) + (1 - \chi)(1 + r_t)(L_t - 1)X_t
\]

(38)

where the second right hand side term reflects that the savings of workers are equal to bank lending to entrepreneurs, given by \((L_t - 1)X_t\), and worker savings earn interest rate \( r_t \). Using (20), (21), (36) and the bank break-even condition (30) to substitute for \( r_t \), \( L_t \), \( R^K_{0,t} \) and \( R^K_{1,t} \), this can be rewritten as:

\[
X_{t+1} = (1 - \chi)\{P_{Y,t}Y_{t+1} - w_{t+1}N_{t+1} + q_{S,t+1}K_{S,t+1}(1 - \delta_S) + q_{I,t+1}(1 - \delta_I)K_{I,t+1} + h_{t+1}N_{t+2}\}
\]

That is, entrepreneurial net worth is always equal to fraction \( 1 - \chi \) of profits gross of hiring
costs, plus the total value of the stock of capital net of depreciation and next period labor. Here, in an abuse of notation, I use $w_t$ to refer to $E_\xi_0 w_0(\xi)$.

Since the total leverage $L_t$ and default cutoff $\bar{\xi}_{t+1}$ are the same for all entrepreneurs, it follows from (22) and (27) that all entrepreneurs pay the same interest rate on their debt, given by:

\[(R^K_t + R^K_{t+1} \xi_{t+1}) = \frac{L_t - 1}{L_t} (1 + i_{t+1})\]  

(39)

The gap between the expected borrowing interest rate $E_t[i_{t+1}]$ and the rate $r_t$ that workers save at is of particular significance. I shall refer to it as the credit spread in the model. That is the spread is defined as:

\[1 + \text{Spread}_t = \frac{E_t[1 + i_{t+1}]}{1 + r_t}\]

4.12 Worker Optimization and Value Functions

Workers can save in risk-free deposits in banks, earning interest rate $r_t$. The household may use its workers’ savings to consume. Therefore, household consumption will satisfy the following Euler equation:

\[u'(C_t) = \beta(1 + r_t)u'(C_{t+1})\]  

(40)

where $u'(C_t) = \frac{1}{C_t}$, due to the assumption of log utility in (1).

The wage earned by a privately employed worker each period will depend on the productivity shock $\xi$ of the entrepreneur he is matched with. Let $\mathcal{W}_t(\xi_t)$ represent the continuation value of a privately employed worker at the beginning of period $t$, conditional on being matched with an entrepreneur who has drawn shock $\xi_t$.

Then $\mathcal{W}_t(\xi_t, s_t)$ satisfies the following Bellman equation:

\[\mathcal{W}_t(\xi_t) = w_t(\xi_t) + (1 - \delta_N)E_t[m_{t+1}\mathcal{W}_{t+1}(\xi_{t+1})] + \delta_N E_t[m_{t+1}\mathcal{U}_{t+1}]\]  

(41)

That is, the worker’s continuation value depends on his wage and the probability of him remaining employed in the next period in the usual way. $\mathcal{U}_t$ is the continuation value of an unemployed worker at the start of $t$.

The continuation value of a worker employed in the public sector is similarly given by:

\[\mathcal{W}_{G,t} = w_{G,t} + (1 - \delta_N)E_t[m_{t+1}\mathcal{W}_{G,t+1}] + \delta_N E_t[m_{t+1}\mathcal{U}_{t+1}]\]  

(42)

Since all entrepreneurs are identical and make identical decisions in proportion to their net worth, wages across entrepreneurs will only vary due to variation in $\xi$. 

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where $w_{G,t}$ is the public sector wage at time $t$.

The value of an unemployed worker $\zeta_t$ satisfies:

$$\zeta_t = \nu C_t + f_t E_t[m_{t+1}\zeta_{t+1}(\xi_{t+1})] + (1 - f_t - f_{G,t})E_t[m_{t+1}\zeta_{t+1}] + f_{G,t}E_t[m_{t+1}w_{G,t+1}]$$  \hspace{1cm} (43)

The flow value of being unemployed $\nu C_t$ arises because the disutility of working is given by $\nu$ in equation (1). Since the marginal utility of consumption is $1/C_t$, it follows that the disutility of working, measured in units of consumption is $\nu C_t$. Note that there are no unemployment benefits.\footnote{The absence of unemployment benefits is consistent with the findings of Chodorow-Reich and Karabarbounis (2013) that effective benefits represent a very small fraction of the wage, once eligibility and cost of takeup are considered.}

4.13 Wage Bargaining

Wages are set immediately before production takes place, at the beginning of the period, but after aggregate and idiosyncratic shocks have been revealed. I consider two separate bargaining protocols for wages. As a baseline, I assume that wages are set by Nash bargaining. The surplus of a worker who is matched with an entrepreneur with idiosyncratic shock $\xi$ is $W_t(\xi) - \zeta_t$. The marginal value of the worker to the entrepreneur is equal to the worker’s marginal revenue product net of wages, plus the hiring cost, that is $(1 - \alpha_I - \alpha_S)\frac{Py_t(\xi)}{n_t} - w_t(\xi) + (1 - \delta_N)h_t$. The reason for this is that, if the worker and entrepreneur were suddenly to become unmatched, the entrepreneur would lose the worker’s contribution to production and would lose the ability to ‘sell’ the worker at price $h_t$, with probability $1 - \delta_N$, after production, but would not need to pay wages for the period.\footnote{In a number of recent contributions to the literature, such as Monacelli, Quadrini and Trigari (2011), the possibility that an entrepreneur may default affects wage bargaining. This mechanism is ruled out here, because whether an entrepreneur defaults is perfectly predictable from the shock $\xi$, which is known before wage bargaining takes place. Moreover, since the number of workers per entrepreneur is large, each worker acts in the knowledge that his own bargaining behavior will matter too little to affect whether or not the entrepreneur defaults.}

Letting $\vartheta$ denote the bargaining weight of the worker, the Nash-bargained wage therefore satisfies:

$$(1 - \vartheta)[W_t(\xi) - \zeta_t] = \vartheta[(1 - \alpha_I - \alpha_S)\frac{Py_t(\xi)}{n_t} - w_t(\xi) + (1 - \delta_N)h_t]$$  \hspace{1cm} (44)

Using (41) and (17) to substitute for $W_t(\xi)$ and $y_t$, we may verify that the wage matches the conjecture made in equation (19). Indeed,

$$w_t(\xi)n_t = w_{0,t}n_t + \vartheta(1 - \alpha_I - \alpha_S)y_t(\xi)$$  \hspace{1cm} (45)

$$= w_{0,t}n_t + \vartheta(1 - \alpha_I - \alpha_S)\tilde{y}_t$$  \hspace{1cm} (46)
where \( w_{0,t} \) solves:

\[
(1 - \vartheta)[W_t(0) - \mathcal{Z}_t] = \vartheta[-w_{0,t} + (1 - \delta_N)h_t]
\]

As an alternative to Nash bargaining, I also consider a protocol in which entrepreneur and worker may alternate in making wage offers which may delay the start of production, as in Hall and Milgrom (2008) and Eichenbaum, Eichenbaum and Trabandt (2013). The details of the bargaining procedure and the derivation of the bargaining solution are slightly complicated and so are left to the appendix. Here, I describe them only briefly. In the alternating offer procedure I consider, bargaining occurs in continuous time within the period – a period is divided into measure 1 of subperiods over which production and bargaining may take place.\(^{32}\)

The entrepreneur gets the opportunity to make an offer immediately at the beginning of the period. If the worker rejects this offer then agents must spend time developing new wage offers to propose. Opportunities for the worker to propose a wage offer to the entrepreneur arrive at a Poisson rate proportional to \( \vartheta \) throughout the period and opportunities for the entrepreneur to propose a wage offer to the worker arrive at a Poisson rate proportional to \( 1 - \vartheta \). Bargaining ends and production begins as soon as an offer is accepted. While agents bargain, the bargaining process may completely break down, in which case agents separate. This event arrives at Poisson rate \( \rho \). Therefore, if no offers are accepted for the entire period, then the probability that bargaining will have broken down by the end of the period is given by \( e^{-\rho} \). While bargaining, entrepreneurs must pay a flow cost \( \gamma \) to keep producing new wage offers. Workers do not face the disutility of working \( \nu \) while bargaining. In equilibrium, the entrepreneur’s initial wage offer at the beginning of the period is immediately accepted and production is not delayed.

This bargaining procedure leads to an equilibrium wage that satisfies:

\[
(1 - e^{-\rho}) \left[ \frac{h_t + \pi_t(\xi) - w_t(\xi)}{1 - \vartheta} - \frac{W_t(\xi) - \mathcal{Z}_t}{\vartheta} \right] + e^{-\rho} \left[ \frac{\gamma + \pi_t(\xi) - w_t(\xi)}{1 - \vartheta} - \frac{w_t(\xi) - \nu c_t}{\vartheta} \right] = 0
\]

(47)

where \( \pi_t(\xi) = \frac{p_{\xi,t}(1-\alpha_{y}-\alpha_{z})y_t(\xi)}{n_t} \) is the marginal revenue product of the worker.

In the limit as \( \rho \to \infty \) the second square bracketed term approaches zero, and the solution of the alternating offer procedure becomes identical to the Nash bargain. On the other hand, as

---

\(^{32}\)The main contrast between the alternating offer bargaining procedure assumed here and the approach of Christiano, Eichenbaum and Trabandt (2013) is that Christiano et. al. (and Hall and Milgrom (2008)) assume that the worker and employer take turns to make an offer – first the employer makes an offer, then the worker, then the employer and so on. I instead assume that the entrepreneur and worker make offers at a Poisson rate. The advantage of this is that it allows the entrepreneur and worker to have different bargaining strengths, depending on the speed at which they can make wage offers. This yields a solution that nests the generalized Nash bargaining solution (44) as a special case.
\( \rho \to \infty \) the alternating-offer wage simplifies to:

\[
w_t(\xi) = (1 - \vartheta) \nu C_t + \vartheta (\gamma + \pi_t(\xi))
\]

In that case, the bargained wage does not depend on the continuation value of the unemployed worker, and so does not depend on the job finding rate \( f_t \). This leads to a wage that is much more insensitive to cyclical fluctuations – that is, wages are endogenously more rigid. It may be verified that in the alternating offer bargaining procedure, the wage also satisfies (45) for any value of \( \rho \), provided \( w_{0,t} \) is defined accordingly.

### 4.14 Definition of Equilibrium

At any time \( t \), the aggregate state of the economy can be described by the variables \( K_{S,t}, K_{I,t}, N_t, N_{G,t}, Z_t, \sigma_t \). An equilibrium of the model consists of: a law of motion of these aggregate variables, depending on the realization of the shocks \( \epsilon_{\sigma}, \epsilon_z, \epsilon_G \); a decision rule for the households determining consumption \( C_t \) as a function of the aggregate state; decision rules for the entrepreneur determining capital and labor demand \( \tilde{k}_{S,t+1}, \tilde{k}_{I,t+1}, n_{t+1} \), default cutoff \( \bar{\omega}_{t+1} \) and leverage \( L_t \) as functions of the aggregate state and the state of the entrepreneur \((\xi, x)\); a function describing wages \( w_t \) paid by an entrepreneur as functions of the aggregate state and the state of the entrepreneur; value functions \( V(\cdot) \) and \( W(\cdot) \) describing the entrepreneur and privately employed workers’ continuation values as functions of the aggregate state and state of the entrepreneur; value functions \( W_G(\cdot) \) and \( U(\cdot) \) describing the continuation values of the publicly employed worker and unemployed worker as functions of the aggregate state, and functions of the aggregate state determining the variables \( m_t, I_{S,t}, I_{I,t}, Y_t, w_{G,t}, r_t, X_t, f_t, f_{G,t}, v_t, v_{G,t}, Y_{G,t}, \tau_t, R^K_{0,t}, R^K_{1,t}, q_{S,t}, q_{I,t}, p_{Y,t}, h_t \).

The functions describing these variables satisfy the following conditions:

- The stochastic shocks evolve according to equations (2)-(4).
- Capital and labor demands and total sales of intermediate goods are the aggregate of individual entrepreneur decisions, according to (33)-(35) and (37).
- Consumption and investment of each type are consistent with the final goods production function (5) and the laws of motion of capital (8)-(9).
- The entrepreneur’s value function and decisions jointly solve the entrepreneur’s optimization problem (31).
- Wages solve either the Nash bargaining solution (44) or the alternating offers solution (47).
- The worker’s value functions \( W(\cdot), W_G(\cdot) \) and \( U(\cdot) \) solve the Bellman equations (41)-(43).
• $Y_{G,t}$ satisfies the public goods production function (10), public sector wages satisfy $w_{G,t} = w_t$, and $\tau_t$ satisfies the government’s budget balance (11).

• Consumption satisfies the Euler equation (40) and the stochastic discount factor $m_t$ is given by (32).

• Entrepreneurial net worth is given by (38).

• Job finding rates and vacancies satisfy the accounting identities (12), (13), (15) and (16).

• For each entrepreneur the returns $R_{K_0,t}$ and $R_{K_1,t}$ are given by (20) and (21).

• The prices $q_{S,t}$, $q_{I,t}$ and $p_{Y,t}$ maximize profits for the final goods producer and hiring costs $h_t$ are given by (14).

5 Model Simulations

5.1 Calibration of the Nash Bargaining Model

First, I present the details of the calibration strategy and the results for the model with Nash bargaining over wages. As a baseline, I assume for now that the government sector employs no workers and only consider financial shocks. Other shocks are considered in subsequent sections.

I set the time period of the model to be one quarter. Table 1 shows the moments used and the parameter values chosen for the model with Nash bargaining. Most of the model parameters are calibrated so that the steady state of the model matches various data moments.

Particularly important are the factor shares and depreciation rates for intangibles and structures, $\alpha_I$, $\alpha_S$, $\delta_I$, $\delta_S$. I calibrate these so that the steady state of the model produces empirically reasonable values for the stock of and investment in structures, equipment and intangibles, and for the labor share. I take the average investment and stock of business structures and equipment from the NIPA and fixed asset tables, averaged over the period 1995-2007. I infer the level of investment in intangible capital and its stock using Corrado and Hulten’s (2010) estimate that, over the 1995-2007 period, intangible investment accounted for 55% of total business investment and intangible capital accounted for 34% of total business capital. I assume that

33 This can be calculated from Table 1 in Corrado and Hulten (2010). Corrado and Hulten calculate the level of investment in intangibles by summing over measurements of investment in a wide variety of assets including software, research and development, marketing activities and training. They infer the stock of intangibles based on estimates of the depreciation rates of these various assets. It is worth noting that the level of intangible capital calculated by Corrado and Hulten (2010) is actually smaller than many others in the literature. For instance, Eisfeldt and Papanikolaou (2014) estimate that the stock of ‘organization capital’ alone is greater than the stock of property plant and equipment. Their approach implies that more than 70% of business capital and an even larger fraction of business investment is intangible. See also Hulten and Hao (2008) and McGrattan and Prescott (2014) for methods that imply a somewhat larger fraction of the capital stock that is intangible than
Table 1: Calibration of Model with Nash Bargaining

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value Used</th>
<th>Target Moment</th>
<th>Moment Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Private Capital/Annual Output Ratio</td>
<td>1.89</td>
<td></td>
</tr>
<tr>
<td>$\delta_I$</td>
<td>0.049</td>
<td>Private Equipment &amp; Intangible Investment/Output</td>
<td>0.21</td>
<td>NIPA and Corrado and Hulten (2010)</td>
</tr>
<tr>
<td>$\delta_S$</td>
<td>0.011</td>
<td>Private Structural Investment/Output</td>
<td>0.036</td>
<td></td>
</tr>
<tr>
<td>$\alpha_I$</td>
<td>0.37</td>
<td>Private Labor’s Share of Business Output</td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td>$\alpha_S$</td>
<td>0.15</td>
<td>Ratio of Structural Capital to Total Capital</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>$\delta_N$</td>
<td>0.075</td>
<td>Inflow Rate into Unemployment</td>
<td>0.075</td>
<td>Davis et. al. (2010)/ CPS</td>
</tr>
<tr>
<td>$h_0$</td>
<td>0.31</td>
<td>Training Costs/Quarterly Wage</td>
<td>0.04</td>
<td>Silva and Toledo (2009)</td>
</tr>
<tr>
<td>$h_1$</td>
<td>0.019</td>
<td>Hiring Costs/Quarterly Wage</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.5</td>
<td>Hiring Cost Elasticity</td>
<td>0.5</td>
<td>Shimer (2005)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.31</td>
<td>Disutility of Working</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>0.285</td>
<td>Unemployment Rate</td>
<td>0.06</td>
<td>Bernanke, Gertler, Gilchrist (1999)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.37</td>
<td>Business Failure Rate</td>
<td>0.03</td>
<td>Gilchrist and Zakrajsek (2012)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.51</td>
<td>Annual Average Credit Spread</td>
<td>0.017</td>
<td>Masulis (1983)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.59</td>
<td>Equity-to-Debt Ratio</td>
<td>1.65</td>
<td>Modest compared to Christiano, Motto and Rostagno (2014)</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>5</td>
<td>Investment Adjustment Costs</td>
<td>5</td>
<td>Gilchrist and Zakrajsek (2012)</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>0.3</td>
<td>Capital adjustment costs</td>
<td>0.3</td>
<td>Hall (2001)</td>
</tr>
<tr>
<td>$\sigma_\sigma$</td>
<td>0.28</td>
<td>St. Dev. of Spread</td>
<td>0.0076</td>
<td>Gilchrist and Zakrajsek (2012)</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>0.84</td>
<td>Autocorrelation of Spread</td>
<td>0.84</td>
<td>Gilchrist and Zakrajsek (2012)</td>
</tr>
</tbody>
</table>
private output is equal to the level of private consumption plus business investment in structures, equipment and intangibles and calculate the implied ratios of investment/private output ratio and capital/private output ratio for equipment, structures and intangibles. Finally, using the NIPA measure of total worker compensation averaged over the 1995-2007 period, I also calculate private labor compensation as a share of private output.

I calibrate the values of $\beta$, $\alpha_I$, $\alpha_S$, $\delta_I$ and $\delta_S$ so that the steady state of the model matches the following five moments from these calculations: private total capital/private output; private equipment plus intangible investment/private output; private structures investment/private output, and private labor compensation/private output. The values of the moments used and the corresponding parameter values are shown in Table 1. The implied value of $\alpha_I$ is rather high: 0.37. This arises for two reasons. Firstly, the inclusion of intangible investment increases the value of output above the standard measure. This significantly depresses the fraction of output accruing to labor to 0.47, implying a higher $\alpha_S$ and $\alpha_I$. Furthermore, wage bargaining implies that some of the returns to an entrepreneur’s investment will transfer to her workers, who will be able to bargain for a higher wage if their employer invests more. This tends to distort the level of profits below the social return to capital, implying that a higher level of $\alpha_I$ and $\alpha_S$ is required to match the labor share in the data.

I calibrate the values of hiring and training costs $h_0$ and $h_1$ respectively to match estimates by Silva and Toledo (2009) for the fraction of the average wage spent by firms on hiring and training costs in the first quarter of employment. I set the separation rate $\delta_N$ to roughly match the inflow rate into unemployment, which Davis et. al. (2010) find to fluctuate between 2% and 2.5% monthly in the CPS over the 1995-2005 period. The labor bargaining weight $\vartheta$ is set to match a steady state unemployment rate of 6%. I set $\nu$ so that the flow value of unemployment evaluated in consumption units $\nu C_t$ is equal to 40% of the wage in the steady state, as in Shimer (2005).

In order to calibrate the financial parameters $\sigma$, $\mu$ and $\chi$, I follow a strategy similar to Bernanke, Gertler and Gilchrist (1999). I match a business failure rate of 3% annually, a credit spread of 1.7% annually and an entrepreneurial net-worth to debt ratio of 1.65. I set the parameters of the adjustment cost functions $s_1$ and $s_2$ to levels that are relatively modest by the approach used here. Calibrating to larger stocks and investment rates in intangibles would naturally imply a larger response of output to financial shocks in this paper.

34 See Acemoglu and Shimer (1999) for a discussion of this issue.
35 Calibrating $\nu$ to a significantly higher level than this implies that $\vartheta < 0.2$ is needed for the model to match the measured labor share in the data. Such a low bargaining weight for labor may be viewed as implausible. Hall (2014) argues in favor of a low disutility of working, based on the low Frisch elasticity of labor supply estimated in many empirical studies.
36 This is the average value of the credit spread measured by Gilchrist and Zakrajšek (2012) over the period 1973-2012.
37 Masulis (1983) finds that the equity value/debt varies between 1.3 and 2 for US corporations over the period 1937-1984.
standards of the DSGE literature. \( \kappa_1 = 5 \) implies convex investment adjustment costs of around one half of the level found by Christiano, Motto and Rostagno (2014). \( \kappa_2 = 0.3 \) implies capital adjustment costs at the lower end of the range suggested as plausible by Hall (2001).

Last, but not least, I calibrate the variance and autocorrelation of the shock \( \epsilon_\sigma \) in order to match the variance and autocorrelation of the credit spread in US data for the 1970-2012 period, where I use the measure of the credit spread constructed by Gilchrist and Zakrajšek (2012), which measures the average excess return on corporate bonds relative to treasury bonds of similar maturity.

I interpret the entrepreneurial risk shock as representing a financial shock and a severe increase in entrepreneurial risk as representing a financial crisis. This is for several reasons. First, the risk shock only affects the aggregate level of capital, output and employment insofar as it affects the costs of borrowing for entrepreneurs. That is, if we were to set monitoring costs equal to zero, thereby turning off financial frictions, the risk shock would have no effect on any aggregate quantities of interest. Second, the risk shock has an extremely strong effect on the credit spread as well as entrepreneurial leverage. Therefore, the risk shock induces strong comovements between credit spreads and financial conditions. Krishnamurthy and Muir (2015) have shown that movements in credit spreads are powerful predictors of the severity of financial crises. Third, Christiano, Motto and Rostagno (2014) have found in a estimated DSGE model featuring a similar entrepreneurial risk shock, that the risk shock accounts for 95% of the volatility in the credit spread at business cycle frequencies in the US and moreover that their measured risk shock rises and falls over time simultaneously with the standard deviation of the cross-section of stock returns of non-financial firms, suggesting it is indeed related to firm-level risk.

5.2 Results of the Nash Bargaining Model

Figure 5 shows the impulse responses of model variables to a one standard deviation innovation in the shock. All impulse responses are shown as percentage deviation from steady state values. The credit spread itself does not respond particularly persistently to the shock. This is unsurprising, since the shock process was calibrated to match the empirical behavior of spreads, which display little persistence. The bottom right panel shows the deviation of \( \sigma \) from its steady state value; this closely matches the behavior of the spread, indicating that the shock itself is no more persistent than the spread.

By contrast, the model generates a highly persistent response of output; output continues to decrease for the first four years after the shock hits and only recovers half way to trend ten years after the shock hits. Moreover, the decrease in output is quantitatively non-trivial: 0.23%}

\[38\text{In the model simulations, I define 'Output' to be the sum of the output of entrepreneurs plus government spending.}\]
Figure 5: Impulse Response to a Financial Shock: Nash Bargaining

![Impulse Response to a Financial Shock: Nash Bargaining](image1)

Figure 6: Impulse Response to a Financial Shock: Model with One Type of Capital

![Impulse Response to a Financial Shock: Model with One Type of Capital](image2)
for a one standard deviation innovation in the shock. Therefore, the model appears to provide a mechanism that could account for some of the large and persistent output decreases following financial crises. Note that the shape of the response of output in the model closely matches the shape of the response of intangibles. This is because the decrease in output is primarily caused by the decrease in intangible capital. While investment in structures actually drops much more than investment in intangibles after the financial shock, the stock of intangibles drops further and faster, due to the higher depreciation rate of intangibles. Since \( \alpha_I \) is relatively large, 0.37, this induces most of the drop in output.

In order to provide a sense of the quantitative importance of the high depreciation rate of intangible capital in the model, Figure 6 shows what happens if the model is re-solved and recalibrated to have only one kind of capital, with a depreciation rate of 6.7% and a steady-state capital-output ratio of 2.3, which Khan and Thomas (2011) find to be the averages in the US fixed asset tables over 1954-2002, after adjusting for growth. As is evident from figure, it is still the case in this setting that a financial shock decreases investment, leading to a deaccumulation of capital and a persistent decrease in output. However, the decrease in output is only around two-thirds as large as it is in the baseline model, in spite of a larger decrease in investment than in the baseline model. The reason is that the lower depreciation of capital in this setting means that the capital stock is more stable than the stock of intangibles is in the baseline model. Consequently, it decreases only around 3/4 as much as the stock of intangibles decreases in the baseline model, leading to a correspondingly smaller decrease in output.

However the financial shock in the model with Nash bargaining fails to produce a recession that looks at all like the data on two dimensions. First of all, in the model there is a consumption boom immediately after the shock impact, although consumption ultimately falls as output drops. This is due to a persistent decrease in interest rates after the shock hits. Such a consumption boom is a relatively common occurrence in real business cycle models with financial shocks. The second counterfactual prediction of the model is that the shock is followed by an increase in employment in the period after the shock, and only small decrease subsequently. Therefore, the fall in output caused by the shock is primarily a fall in productivity. However, in the data, decreases in employment are more important in recessions than productivity decreases.39

Fortunately, both these counterfactual predictions of the Nash bargaining model will be resolved in the model with alternating offer bargaining, discussed in the next section. With alternating offer bargaining, consumption rises for only one period after the shock hits, before falling, and employment falls immediately and substantially.

39The small rise in employment after the shock hits in this model is also caused by a decrease in interest rates, which serves to stimulate hiring.
5.3 Calibration of the Alternating Offers Bargaining Model

I calibrate the alternating offer bargaining model to match the same calibration targets as the model with Nash bargaining, leading to very similar values for the parameters. However, the alternating offer bargaining model has two more parameters than the Nash bargaining model: the cost to the entrepreneur of proposing wage offers \( \gamma \) and the rate at which wage bargaining breaks down \( \rho \). Since these parameters govern the rigidity of wages in response to aggregate shocks, I estimate them using a Bayesian approach based on the aggregate dynamics of wages in US data.

The alternating offer bargaining solution (47) implies that wages should depend upon current and expected future values of consumption, employment and productivity. Indeed, if the aggregate dynamic process governing these variables is known, values of the wage \( w_t \), hiring costs \( h_t \), vacancies \( v_t \) and the worker’s continuation values \( W_t \) and \( U_t \) can be calculated using only the model equations (41), (43), (14), (15), and (47), given the values of the parameters. Therefore, rather than conducting a Bayesian estimation of the whole model, I assume that productivity, consumption and employment follow a reduced-form VAR process with two lags, which I estimate against the data, and use only the VAR and the the five model equations just mentioned for the purposes of Bayesian estimation of the parameters \( \gamma \) and \( \rho \). The advantage of this approach is that my estimates of these two parameters will not be contaminated if other parts of the model, such as the equations governing financial markets, are mis-specified.

I assume that wages are measured with iid error and jointly estimate the parameters of the VAR, the error variance of wages and the parameters \( \rho \) and \( \gamma \). I use US data from 1948-2014, taking aggregate consumption, output and labor compensation in the NIPA and employment from the BLS. The measure of productivity I use is output per worker, and the measure of the wage is compensation per worker. In order to remove trends in the variables, I use the bandpass filter of Christiano and Fitzgerald (2003) to isolate variation in the 2-120 quarter frequency band.

Estimates of \( \rho \) and \( \gamma \) rely on a few of the other parameters of the model, notably the parameters governing hiring costs, \( \alpha_I \), \( \alpha_S \), the discount factor \( \beta \) and the parameters \( \nu \) and \( \vartheta \). For the purposes of the estimation, I fix the parameters \( \beta = 0.99 \), \( \alpha_I = 0.37 \), \( \alpha_S = 0.15 \), as calibrated for the Nash bargaining model in Table 1. For the remaining parameters, I fix them in the estimation to match the same target moments as in Table [1]

For the coefficients of the VAR, I use a normal prior with a standard deviation of 2 and a mean of 0, except for the coefficient of each variable on its own first lag, for which I use a

---

40120 quarters is of course much longer than what is usually considered the length of US business cycles. However, since the focus on the model is on persistent movements in aggregate variables, it is of importance that the response of wages to other variables is consistent with the data at frequencies below the business cycle as well as at high frequencies. This argues in favor of using data across a wide frequency band, such as 2-120 quarters.

41Estimates of the wage bargaining parameters are not sensitive to small changes in these choices.
normal prior with a mean of 0.9 and a standard deviation of 1. For the VAR shocks and the error term of wages, I use an inverse gamma-2 prior, with a mean of 0.01. For the parameter $\rho$, I use a lognormal prior. For the parameter $\gamma$ I assume that $\gamma = \gamma_0 T$, that is, it is equal to the proportion $\gamma_0$ of steady state productivity. For $\gamma_0$, I use a beta prior with mean of 0.6, a standard deviation of 0.12 a minimum of 0 and a maximum of 1.2 , which is close to a uniform prior on the range $[0, 1.2]$. Priors and estimates of these parameters are shown in Table 2.

Table 2: Estimates of Wage Bargaining Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Type</th>
<th>Prior Mean</th>
<th>Prior Standard Deviation</th>
<th>Posterior Mode</th>
<th>Posterior Stand. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log(\rho)$</td>
<td>Normal</td>
<td>$\log(0.5)$</td>
<td>3</td>
<td>$\log(0.0215)$</td>
<td>0.37</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>Beta</td>
<td>0.6</td>
<td>0.3</td>
<td>0.51</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Table 3: Calibration of Shocks for Alternating-Offer Bargaining Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value Used</th>
<th>Target Moment</th>
<th>Moment Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\sigma$</td>
<td>0.28</td>
<td>St. Dev. of Spread</td>
<td>0.0076</td>
<td>Gilchrist and Zakrajsek (2012)</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>0.84</td>
<td>Autocorrelation of Spread</td>
<td>0.84</td>
<td>Gilchrist and Zakrajsek (2012)</td>
</tr>
<tr>
<td>$\sigma_G$</td>
<td>0.023</td>
<td>St. Dev. of Gov. Employment</td>
<td>0.023</td>
<td>Quadrini and Trigari (2007)</td>
</tr>
<tr>
<td>$\rho_G$</td>
<td>0.94</td>
<td>Autocorrelation of Gov. Spending</td>
<td>0.94</td>
<td>Christiano, Motto and Rostagno (2014)</td>
</tr>
<tr>
<td>$\sigma_Z$</td>
<td>0.072</td>
<td>St. Dev. of TFP Innovations</td>
<td>0.072</td>
<td>King and Rebelo (1999)</td>
</tr>
<tr>
<td>$\rho_Z$</td>
<td>0.90</td>
<td>Autocorrelation</td>
<td>0.90</td>
<td>Michaillat (2014)</td>
</tr>
</tbody>
</table>

Although the priors are relatively loose, the posteriors are noticeably tighter around the modes $\log(0.021)$ and 0.55. Thus, I use $\rho = 0.0215$ and $\gamma = 0.51$ in the model simulations. $\rho = 0.0215$ is very far from the Nash bargaining solution (which is the special case $\rho = \infty$) and implies that wages are not very sensitive at all to unemployment. Values of $\rho$ above 0.05 can be rejected at the 5% level.

In order to compare the effects of the financial shock with those of other shocks, I also include the TFP and government employment shocks in the model with alternating-offer bargaining. I set the share of government employment to be 16.7% of total employment in the steady state, as found by Michaillat (2014) to be true in CES data. I set the variance and autocorrelation of the public employment shock $e_G$ so that the standard deviation of the autonomous component of public employment matches the value used by Quadrini and Trigari (2007), and the autocorrelation of government spending matches the estimate of Christiano, Motto and Rostagno (2014).
Finally, I set the variance of innovations in the TFP shock to match the value recommended by King and Rebelo (1999) and set the autocorrelation to match the autocorrelation of the cyclical component of TFP estimated by Michaillat (2014).

5.4 Results of the Model with Alternating-Offer Bargaining

Figure 7 shows the impulse responses to the financial shock in the alternating offer bargaining model. For ease of comparison, the impulse responses in the Nash bargaining model are also shown. The most dramatic difference between the two models concerns the response of employment. While employment rises slightly after the shock in the model with Nash bargaining, it falls even more than output in the model with alternating offer bargaining – productivity actually rises immediately after the shock in this model. The reason is that wages do not fall nearly as much in response to the falling capital stock in the alternating-offer model and so labor demand declines much more. Labor demand declines not only due to the lower level of capital, but also because entrepreneurs use external finance in order to pay hiring costs, and the shock makes this more expensive.

The decrease in employment in the alternating offer model is highly persistent, because labor demand remains relatively low as long as the stock of intangibles remains low. Moreover the drop in output is much larger than in the model with Nash bargaining. The reason is that the large drop in employment in the alternating offers model noticeably strengthens the propagation mechanism: since employment drops more, output drops more, so investment drops more, so the stock of intangibles drops even more, which further decreases labor demand. The net effect is a much larger decrease in output than the model with Nash bargaining and a highly persistent decrease in employment.

In this model, the level of consumption rises for one period after the shock and then falls. The initial rise in consumption after the shock is primarily an artifact of the assumption that time is discrete, with capital and labor chosen one period in advance. Therefore, output is for the most part set one period in advance. When the shock hits, investment decreases immediately, which forces consumption to increase for one period, since output is largely predetermined. However, after the first period, output falls and so consumption falls along with investment.

5.5 Effects of Other Shocks

Figure 8 shows the response of output to contractionary public employment and TFP shocks in the model with alternating offer bargaining. These shocks have much weaker effects on aggregate variables than the financial shock. Furthermore, they appear to have somewhat less persistent

42 For the most part, because government wage payments and monitoring costs are not set one period in advance.
effects than the financial shock – particularly on output. Neither the TFP nor the public employment shock generates much of a hump-shaped response of output, unlike the financial shock. Moreover, output and employment have largely returned to steady state levels 40 quarters after these shocks hit, in contrast to after a financial shock. Notice that this is true in spite of the fact that the shock process for both of these shocks was itself calibrated to be more persistent than the financial shock, as shown in Table 3. The reason for the lower persistence is that these shocks affect investment much less than the financial shock, and so the stock of intangibles decreases much less. Consequently, the mechanism leading to a persistent decrease in output following a financial shock operates much less strongly for the other shocks. That financial shocks in the model generate more persistent responses than non-financial shocks may account for why financial recessions appear to have more persistent output drops than other recessions.
From the perspective of most real business cycle models, it is surprising that government employment has such a large effect on aggregate employment and output in this setting. Notice that the increase in government employment has virtually no effect on private consumption or investment: the government spending multiplier is approximately equal to 1 in each period. The reason is that increased government employment increases labor market tightness, but this increase in labor market tightness has a very small effect on wages because of alternating offer bargaining. Consequently, job creation by the private sector is hardly hit.\footnote{The mechanism for the public employment multiplier here is therefore similar to Michaillat (2014).}
5.6 Comparison to the Great Recession

In order to evaluate the empirical relevance of the model, I compare the model predictions to the movements of aggregate variables in the US over the period during and after the Great Recession, that is over 2007-2014. I use the alternating offers model, because it is evident that the model with Nash bargaining will fail to match the collapse in employment that has been so characteristic of the Great Recession. In order to assess how far the poor performance of the US economy in this period can be attributed to the 2008 financial crisis, I feed into the model the sequence of financial shocks that would lead the model to exactly match the behavior of the credit spread in the US over this period. I assume that the economy was in the steady state in the first quarter of 2007. Given this sequence of shocks, I compare the implied behavior of other variables in the model to those in the data over this period. The results are shown in Figure 9. Data variables used in the figure are shown as percentage changes from their 2007 levels after eliminating a linear trend calculated over the period 1995-2006. Capital stocks at quarterly frequency were inferred using the perpetual inventory method, based on NIPA data on investment.

The model can replicate around one third of the collapse in employment and output over this period. It is suprisingly capable of replicating the failure of these variables to return to trend by 2015. The main failing of the model in this regard is that it predicts a very sharp decrease in output and employment starting in 2007, whereas the decrease in the data is more gradual. This suggests that there are other important forces causing economic sluggishness that are absent from the model. The model also accounts for around one half of the decrease in the stocks of structures and equipment and intangibles that is observed in the data. Here again, however, it predicts a more rapid decrease than occurs in the data. That said, the NIPA measures used in constructing the figure may significantly understate the size and rapidity of the decrease in the stock of intangible capital in recent years. The measurement of equipment and intangibles I used for Figure 9 comes from the NIPA, due to limited data availability for recent years. This excludes marketing expenditures, which appear to be one of the most cyclical types of intangible investment. The limited available data on advertising expenditures in recent years appear to suggest that these expenditures fell strongly and rapidly in the Great Recession also, suggesting that the decrease in equipment and intangibles was larger and faster than shown in Figure 9.

\[44\] In ongoing work, I construct an approximate quarterly series of the stock of intangible capital in the US in the post-war period based on data on advertising expenditures as well as the NIPA. I find that the stock of intangible capital is quite volatile over time and shows substantial decreases after spikes in the credit spread.
6 Conclusion

Movements in the stock of equipment and intangible capital may be a powerful source for propagation of shocks in the economy and may have a major role to play in the persistent output drops following financial crises. Based on a panel of OECD and emerging economies, I discuss several patterns which suggest that drops in investment during financial crises might be related to the persistent output drops following these crises. I argue that financial recessions appear to have more long-lasting effects than non-financial recessions, consistent with a large empirical literature. Furthermore, I argue that financial recessions seem to exhibit larger decreases in the investment-output ratio than non-financial recessions, even in the year of and immediately after the recession, and that the crises which did see more persistent output drops also saw larger drops in the investment-output ratio.
I construct a model in which a financial shock decreases investment, particularly hitting the stock of equipment and intangibles, due to their especially high depreciation rates. I show that, under Nash bargaining, this model is able to produce a persistent decrease in output around 50% larger than if capital is modeled in a traditional way. However, the model with Nash bargaining implies counterfactual predictions for employment. This suggests that deaccumulation of equipment and intangible capital likely cannot be a cause of large persistent drops in employment if wages are as elastic to the level of unemployment as they are with Nash bargaining.

However, if wages are less elastic to the level of unemployment than under Nash bargaining, movements in equipment and intangible capital could be a major cause of movements in employment. Using a Bayesian approach to estimate parameters of the wage bargaining process, I show that aggregate earnings data are more consistent with an alternating offer bargaining model in which wages are much less sensitive to the level of unemployment than occurs with Nash bargaining. This alternative offer bargaining model, in turn, implies much larger drops in output after a negative financial shock than occur with Nash bargaining. Moreover, in the alternating offer model, the decrease in equipment and intangible capital can lead to a highly persistent decrease in employment after a negative financial shock. This model also implies a government employment multiplier close to 1, which is more consistent with most empirical estimates than the near-zero government spending multiplier of standard RBC models.

Finally, I compare the behavior of aggregate variables in the US Great Recession to what would have occurred in the model with alternating offer bargaining, if movements in the credit spread in these years were caused by a sequence of financial shocks. I find that the alternating offer bargaining model is capable of replicating around one third of the large and persistent decline in output and employment that occurred in the Great Recession, although the model predicts a more rapid decrease in output, employment and capital than in the data. In sum, the model developed in this paper suggests that movements in equipment and intangible capital may be an important contributory force to fluctuations in output and employment and may play a key role in the propagation of some economic shocks. Furthermore, the model simulations suggest that the decrease in investment in equipment and intangible capital following financial crises may play an important role in leading these crises to have such persistent effects on output and employment.
Appendix: Alternating-Offer Bargaining

Bargaining Protocol

This section provides a precise microfoundation for the wage bargaining equation in \[ (47) \]. Bargaining occurs over time, simultaneously with production. Assume that a period \( t \) can itself be divided into a continuum of measure 1 subperiods. Therefore, within a period, time is continuous. I index time within the period by \( j \in [0, 1] \) to distinguish from \( t \), which is discrete. All shocks are revealed before \( j = 0 \). Production can occur over \( j \in [0, 1] \). Immediately after \( j = 1 \), investment, hiring, default and consumption all take place. The entrepreneur makes an initial wage offer to the worker at the instant \( j = 0 \). The offer will be a wage for the whole period. The worker can either accept or reject. If the worker accepts the offer, then production begins immediately and the offered wage is paid immediately at \( j = 1 \). If the worker rejects the offer, then the entrepreneur and worker start preparing new offers. This takes time. It is assumed that the entrepreneur becomes ready to make a new offer to the worker at Poisson rate \( \varsigma(1 - \vartheta) \). Similarly, the worker becomes ready to make an offer to the entrepreneur at Poisson rate \( \varsigma \vartheta \). Therefore, it is stochastic as to who will have a chance to make the next wage offer, the worker or the entrepreneur. The parameter \( \varsigma \) governs the speed at which any agent can make an offer, and the parameter \( \vartheta \) governs the relative speed at which workers and entrepreneurs can make offers.

Whenever the entrepreneur has the chance to make a new offer to the worker, then once again the worker can accept or reject. If the worker accepts then the worker commences work and production takes place for whatever is left of the period; total output produced will be lower than if an agreement had been reached at the beginning of the period because some time has passed. The agreed wage is paid at the end of the period. If the worker rejects again, then both entrepreneur and worker go back to preparing new offers and become ready to make a new offer at, respectively Poisson rates \( \varsigma(1 - \vartheta) \) and \( \varsigma \vartheta \).

Whenever the worker has a chance to make an offer, things proceed similarly. The entrepreneur can accept or reject. If she accepts then the worker starts work, output is produced in whatever is left of the period and the worker is paid at the end of the period. If the entrepreneur rejects, then both entrepreneur and worker go back to preparing new offers and can make another offer at Poisson rates \( \varsigma(1 - \vartheta) \) and \( \varsigma \vartheta \).

The entrepreneur and worker continue to make new offers in this way until an offer is accepted. However, the entrepreneur and worker would both prefer, all else equal, to reach an agreement as soon as possible because bargaining is costly in two ways. First, the longer time is spent bargaining the less time there is to produce output, and the less output is produced. The worker’s full marginal product of labor \( \pi_t(\xi_t) = (1 - \alpha_I - \alpha_S) \frac{y_t(\xi_t)}{n_t} \) is produced if a deal is made at the beginning of the period but output is only \( (1 - j)\pi_t \) if a deal is instead reached at point...
Second, while the entrepreneur and worker are continuing to bargain – until they reach an agreement – there is a flow probability $\rho$ that bargaining breaks down and the match separates. On the other hand, the worker benefits a little from bargaining in that he does not have to work while he is bargaining, and so does not face the disutility of working $\nu$.

If the end of the period is reached, i.e. $j = 1$, and no offer has yet been accepted but at the same time bargaining has not broken down, then no production takes place this period and the worker is not paid. At this point, the worker and entrepreneur separate with probability $\delta_N$, just as they would if they had reached agreement.

**Continuation Values**

Let $\hat{W}_t(\xi, j)$ denote the continuation value of an employed worker in sub-period $j$ who is matched with an entrepreneur of productivity $\xi$ and who has not yet reached agreement with his employer over wages this period. Similarly, let $V_{N,t}(\xi, j)$ denote the marginal value of the worker to the entrepreneur in sub-period $j$ if the entrepreneur is still matched with the worker but has not yet reached agreement with the worker over wages this period.

Recall that at the beginning of the period the entrepreneur makes an offer to the worker. As bargaining is totally wasteful, the entrepreneur maximizes her profits by offering the worker the wage $w_t(\xi)$ that makes the worker indifferent between accepting the offer and rejecting, and the worker will accept this offer. The value of an employed worker $W_t(\xi)$ at the beginning of the period is therefore the worker’s value of accepting this initial offer.

Worker indifference between accepting and rejecting the offer implies that:

$$W_t(\xi) = \hat{W}_t(\xi, 0) \quad (48)$$

Here, the left hand side denotes the value of accepting the offer and the right hand side denotes the value of rejecting.

Now suppose, hypothetically, that in some sub-period $j \in (0, 1)$ is reached and the worker and entrepreneur are still matched but no agreement has yet been reached. Suppose that the entrepreneur gets an opportunity to make an offer to the worker at $j$. Then the entrepreneur will make an offer that makes the worker indifferent between accepting and rejecting and the worker will accept. Let $\hat{w}_t(\xi, j)$ be the offer the entrepreneur would make in sub-period $j$. Then $\hat{w}_t(\xi, j)$ satisfies:

$$\hat{w}_t(\xi, j) + \hat{W}_t(\xi, 1) = W_t(\xi, j) \quad (49)$$

As is standard in the contract literature, it is assumed that the worker breaks indifference in favor of accepting the offer, since the resultant equilibrium can always be approached by a wage offer which the worker is strictly better off accepting.
where the left hand side denotes the worker’s value if he accepts, and the right hand side denotes his value if he rejects. The value to the worker of acceptance here is the value of the wage paid for the remainder of the period, plus the continuation value that the worker would have at the end of the period even if no agreement had been reached, i.e. $\hat{W}_t(\xi,1)$.

Since the worker will accept, then the entrepreneur correspondingly receives a value of:

$$(1 - j)\pi_t - \hat{w}_t(\xi,j) + \hat{V}_{N,t}(\xi,1)$$

That is, the entrepreneur gains a continuation value equal the value of the worker’s remaining marginal product that will be produced in the period, i.e. $(1 - j)\pi_t$ (because the worker will immediately start work at $j$), minus the wage, plus the marginal value the entrepreneur would have at the end of the period from the worker even if no agreement had been reached.

Suppose that some $j \in (0,1)$ is reached, the pair are still bargaining, and the worker gets an opportunity to make an offer to the entrepreneur. Then the worker will likewise make an offer that makes the entrepreneur indifferent between accepting and rejecting and the entrepreneur will accept. Let $w'_t(\xi,j)$ denote the offer the worker makes at $x$. Then $w'_t(\xi,j)$ satisfies:

$$(1 - j)\pi_t - w'_t(\xi,j) + \hat{V}_{N,t}(\xi,1) = \hat{V}_{N,t}(\xi,j)$$

where the left hand side is the entrepreneur’s continuation value if she accepts the offer and the right hand side is her value if she rejects. Since the entrepreneur will accept the offer, the worker will then get the value:

$$w'_t(\xi,j) + \hat{W}_t(\xi,1)$$

### 6.1 HJB Equations

The bargaining protocol therefore implies that $\hat{V}_{N,t}(\xi,j)$ and $\hat{W}_t(\xi,j)$ satisfy the following HJB equations:

$$0 = -\gamma - \rho \hat{V}_{N,t}(\xi,j) + \varsigma(1 - \vartheta)[(1 - j)\pi_t - \hat{w}_t(\xi,j) + \hat{V}_{N,t}(\xi,1) - \hat{V}_{N,t}(\xi,j)] + \varsigma \vartheta [(1 - j)\pi_t - w'_t(\xi,j) + \hat{V}_t(\xi,1) - V_t(\xi,j)] + \frac{\partial \hat{V}_{N,t}(\xi,j)}{\partial j}$$  \hspace{1cm} (51)

$$0 = \nu C_t + \rho [\hat{W}_t - \nu j C_t - \hat{W}_t(\xi,j)] + \varsigma(1 - \vartheta)[\hat{w}_t(\xi,j) + \hat{W}_t(\xi,1) - \hat{W}_t(\xi,j)] + \varsigma \vartheta [w'_t(\xi,j) + \hat{W}_t(\xi,1) - \hat{W}_t(\xi,j)] + \frac{\partial \hat{W}_t(\xi,j)}{\partial j}$$  \hspace{1cm} (52)
Equation (51) is the HJB equation of the entrepreneur while bargaining. During bargaining, the entrepreneur receives a flow value of $-\gamma$, that is, the cost of continuing to produce wage offers. With probability $\rho$ the match terminates and the entrepreneur loses the marginal value of the match, $\hat{V}_{N,t}(\xi,j)$. At rate $\varsigma(1-\vartheta)$ the entrepreneur gets to make an offer to the worker and the worker accepts, in which case the wage will be $\hat{w}_t(\xi,j)$ and the entrepreneur will get value $[(1-j)\pi_t - \hat{w}_t(\xi,j) + \hat{V}_{N,t}(\xi,1) - \hat{V}_{N,t}(\xi,j)]$, that is, the value of the deal minus the value of continuing to bargain. At rate $\varsigma\vartheta$ the worker makes an offer, the wage is $w'_t(\xi,j)$ and the entrepreneur gets $[(1-j)\pi_t - w'_t(\xi,j) + \hat{V}_t(\xi,1) - V_t(\xi,j)]$. Finally, even if bargaining continues without resolution, the entrepreneur’s value changes at rate $\frac{\partial V_{N,t}(\xi,j)}{\partial j}$.

The worker’s HJB equation (52) is very similar. The flow value of bargaining is given by $\nu C_t$, that is, the marginal value to the household of not working, evaluated in consumption units. If the match terminates — which it does at rate $\rho$ — the worker gets to enjoy being unemployed for the remainder of the period – i.e. gets $\nu C_t(1-x)$ from this and gets the value of being unemployed at the start of the recruitment phase $U_t - \nu j C_t$ but loses the value of being employed $W_t(\xi,j)$. If the entrepreneur gets to make an offer, the worker gets $[\hat{w}_t(\xi,j) + \hat{W}_t(\xi,1) - W_t(\xi,j)]$, and if the worker gets to make an offer, he gets $[w'_t(\xi,j) + \hat{W}_t(\xi,1) - W_t(\xi,j)]$. This gives rise to equation (52).

At the end of the period, if a deal has still not been reached, the worker and entrepreneur separate remain matched with probability $(1-\delta_N)$. In that case, the entrepreneur will be able to ‘sell’ the worker at price $h_t$. Therefore

$$\hat{V}_{N,t}(\xi,1) = (1-\delta_N)h_t$$

Substituting (49) and (50) into equations (51) and (52) gives rise to two linear differential equations that can be solved for $\hat{V}_{N,t}(\xi,j)$ and $W_t(\xi,j)$ given boundary conditions (53) and (48).

Using $w_t(\xi) = \hat{w}_t(\xi,0)$, and taking the limit as $\varsigma \to \infty$, the solution gives

$$(1-e^{-\rho}) \left[ \frac{h_t + \pi_t(\xi) - w_t(\xi)}{1-\vartheta} - \frac{W_t(\xi) - W_t}{\vartheta} \right] + e^{-\rho} \left[ \frac{\gamma + \pi_t(\xi) - w_t(\xi)}{1-\vartheta} - \frac{w_t(\xi) - \nu C_t}{\vartheta} \right] = 0$$

which was the wage bargaining solution considered in (47).

Taking the limit $\varsigma \to \infty$ implies that agents can make wage offers infinitely quickly, which is obviously implausible. However, if agents can each make offers on average at least once per week, and there are 13 weeks in a quarter, this implies that $\varsigma > 26$. At levels of $\varsigma$ this high

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46Macroeconomists are usually accustomed to seeing the discount rate multiplied by the continuation value on the left hand side of HJB equations. Here, this does not appear because I assume no discounting within the period — discounting occurs between periods not within them. So the left hand side of each HJB equation is just zero. The discount rate is so low at the quarterly level that the quantitative impact of this assumption is small.

47Details of the solution are available upon request.
or higher, the approximation error in assuming $\varsigma \to \infty$ is incredibly small, provided $\rho < 4$ and $\vartheta \in (0.05, 0.95)$. Thus I let $\varsigma \to \infty$ in order to produce a bargaining solution that is easily interpretable.

**References**


Hall, Robert E. 2014. “High Discounts and High Unemployment.”


