Quantifying the Macroeconomic Impact of Credit Expansions*

Corina Boar[†]
NYU and NBER

Matthew Knowles[§]
City, University of London

Kjetil Storesletten[‡]
University of Minnesota

Yicheng Wang[¶]
Peking University

June 17, 2025

Abstract

This paper studies how credit expansions affect economic activity and quantifies the relative role of household and firm-side channels in accounting for the aggregate effects. Using data from the quasi-natural experiment of bank deregulation across U.S. states in the 1980s, we document that deregulation led to an expansion of credit and economic activity. We use this causal evidence to estimate a small open-economy heterogeneous-agent New Keynesian model that replicates the effects of this shock. In the long run, reduced borrowing costs for firms account for the lion's share of the rise in output and employment. In the short run, the firm-side and household-side channels are equally important in driving the expansion of output and employment. Household demand is mostly associated with consumption whereas the firm side is mostly associated with investment. The finding that employment and production—including tradable output—experiences sustained growth after a credit expansion is consistent with the role we attribute to the firm-side channel.

*Yicheng Wang acknowledges financial support from the National Natural Science Foundation of China for grants 72473166, 72150003, 72373004. Storesletten acknowledges financial support from the Norwegian Research Council for grant 316301. We are grateful to Kaiji Chen, Russell Cooper, Wei Cui, Florian Ederer, Charles Engel, Hanming Fang, Dirk Krueger, Enrique Mendoza, Eric Swanson, Vincenzo Quadrini, Zhiwei Xu, Xiaodong Zhu, and participants at AEA 2021, Cologne, SED 2022, Econometric Society China 2022, 2024 PHBS-IER Conference, Shandong University, Jinan University, City, University of London, and Peking Univerity for helpful comments.

[†]Corina Boar: corina.boar@nyu.edu.

[‡]Kjetil Storesletten: kstoresl@umn.edu.

[§]Matthew Knowles: matthew.knowles@city.ac.uk.

[¶]Yicheng Wang (Corresponding author): wangyc@phbs.pku.edu.cn. Peking University HSBC Business School.

Keywords: credit market, New Keynesian, household heterogeneity, firm heterogeneity
JEL classifications: E3, E5, G2

1 Introduction

Changes in credit conditions affect the real economy through household-side and firm-side channels. Lower borrowing costs for firms increase investment whereas lower borrowing costs for households stimulate consumption. Both channels impact output and employment. The effects are shaped by assumptions about the economic environment, including financial frictions, tradability of goods, and price rigidities. Which of these channels is more important for the aggregate effects of looser credit conditions? This question is central to understanding the effects of credit expansions and guiding governments' and central banks' responses to changing credit conditions. For instance, if most of the macroeconomic effects of financial shocks are due to changes in household consumption, then supporting households through e.g. cash transfers might be a desirable response to contractionary financial shocks. On the other hand, such a policy makes less sense if the main effects of financial shocks are due to tighter financial constraints on firms.

This paper quantifies how loosening credit conditions affects the aggregate economy through each of these channels. To this end, we combine a Heterogeneous Agent New Keynesian (HANK) model with empirical evidence from the quasi-natural experiment of U.S. banking sector deregulation, which a large literature has argued induced a credit expansion in US states in the 1980s.[1] We assume a small open economy with heterogeneous households and firms and two sectors producing tradable and non-tradable goods, respectively. This economy, which is assumed to represent a U.S. state, is hit by an expansionary credit shock, modeled as a shock to external borrowing costs. The model incorporates many forces through which a credit shock may impact the economy, including effects on consumer demand, labor supply, investment, and firm entry and exit. The magnitudes of these various channels depend, unsurprisingly, on the parametrization of the model.

We identify the key model parameters using data on the credit expansion associated with the

[1]This literature begins with the seminal work of Jayaratne and Strahan (1996). For more recent references, see Rice and Strahan (2010), Kroszner and Strahan (2014), and citations therein.

US bank deregulation in the 1980s. This deregulation happened in different states at different times, in a way that the literature has argued is quasi-random conditional on controls. Using a local projection approach we estimate the effects of this deregulation on state-level financial and macroeconomic variables. We find that the deregulation caused a decrease in borrowing costs for households and firms, followed by an increase in aggregate output and employment that persisted for more than 10 years. We set key parameters of our HANK model to match the estimated impulse responses of financial and macroeconomic variables to this empirical credit shock.

We use the parameterized model to decompose the effect of the expansionary credit shock into two parts reflecting household-side versus firm-side channels: the effect of lower borrowing costs for households (and therefore on consumer demand), and the effect of lower borrowing costs for firms (i.e. effects through investment and firm entry). Our main finding is that the firm-side channel dominates: for parameter values that best fit our estimated impulse responses, the firm-side channel accounts for around 71% of the cumulative effect of the credit expansion on most variables during the first ten years after deregulation, while the household-side channel accounts for around 27%. The firm-side channel accounts for more than half of the short-run dynamics and grows in importance over time. This channel explains the cumulative rise of investment (accounting for 92%) and in the dynamics of firm entry and exit. The household-side channel nevertheless plays the dominant role in the rise of consumption (accounting for around 94%) and also plays an important role in the rise of output in the first five years after deregulation (accounting for 41%) .

The reason why the data points to the firm-side channel as the main driver of the aggregate effect of credit expansions in the medium term is that output and employment grow gradually after deregulation. This is consistent with the way firms respond to lower borrowing costs because sustained investments induce a gradual increase in firms' capital stock. The household channel, by contrast, induces a rapid rise in output and is therefore more important for the short-run dynamics. This narrative is consistent with the empirical dynamics of tradable output following a credit expansion at the state level. Using data on state-level data on tradable output from Mian and Sufi (2014) we find, using local projections, positive point estimates for tradable output growth after the deregulation. This is consistent with the importance of the firm-side channel because a lower borrowing cost for firms is the only mechanism that can stimulate tradable output: lower borrowing costs for firms boost investments and, hence, production and exports of tradable goods, whereas lower borrowing costs for households would *decrease* tradable output because the economy would satisfy this demand by producing more non-tradables and importing tradables.

To the best of our knowledge, our paper is the first to integrate a quantitative, heterogenous firm New-Keynesian model with causal empirical evidence from a natural experiment for credit market deregulation. Overall, our results point out the importance of both decreased borrowing costs for firms and households in accounting for the short-run effects, while in the longer run, reduced borrowing costs for firms account for the lion's share of the rise in output and employment.

Related Literature. Our paper builds on and contributes to several strands of literature. Closest to our paper is other work that seeks to evaluate the relative importance of different channels through which financial market changes impact real economic activity. Mian, Sufi, and Verner (2020) leverage the same natural experiment in bank deregulation. Using a qualitative model they argue that a number of empirical facts are consistent with deregulation mainly impacting the economy via demand-side rather than supply-side channels. We estimate dynamic impulse responses from the data and use our dynamic structural model—estimated to match these impulse responses—to *quantify* the role played by household-side versus firm-side channels.

Similar to us, Kehoe et al. (2020) study the relative importance of household-side and firm-side channels for the transmission of a financial shock. Unlike us, they study the tightening of financial markets, focusing on the US Great Recession. They use a different theoretical framework, based on search and matching models with human capital accumulation and no nominal rigidities. We instead study this question within a small open economy HANK model with endogenous firm creation, closer to the standard workhorse model in the business-cycle literature. We find a larger role for the firm channel than they do. Our results complement theirs and suggest that the importance of the household channel of credit shocks in the short run is a robust prediction of a variety of models; however, our results suggest that the effect of changes in firm borrowing conditions may still ultimately be more important, especially in the longer run.

More broadly, our paper relates to the large literature on the effects of financial shocks. Our theoretical approach is closest to models in the New Keynesian open economy literature that formalize credit shocks as a rise in the cost of external borrowing, such as Gertler, Gilchrist, and Natalucci (2007) and Cugat (2022).[2]

Our modeling framework builds upon the literature on open-economy heterogeneous agent New Keynesian (HANK) models. Our model features heterogeneity on both the household and firm side. On the household side, our model uses the TANK framework, which distinguishes between liquidity constrained and unconstrained households in a tractable way (Bilbiie, 2008, Boerma, 2014, Buffie and Zanna, 2018, Galí, López-Salido, and Vallés, 2007 in open-economy contexts). Rich firm-side heterogeneity of the kind we consider has only recently been introduced into New Keynesian DSGE models (Koby and Wolf, 2020, Ottonello and Winberry, 2020, Reiter, Sveen, and Weinke, 2013), with an almost exclusive focus on closed economies. An exception is the recent work of Caldara et al. (2020), who incorporate firm-side heterogeneity in an open economy New Keynesian model. Our modeling of firm-side heterogeneity follows Ottonello and Winberry (2020), although differently from their paper, we endogenize firm entry and exit. Our modeling of the open economy features follows Gali and Monacelli (2005) and Nakamura and Steinsson (2014).

In terms of methodology, our paper is in line with the recent emerging literature that uses

[2] Our approach and focus is different from other papers that study financial shocks, such as Gertler, Gilchrist, and Natalucci (2007), Bernanke, Gertler, and Gilchrist (1999) and Ottonello and Winberry (2020), which incorporate the role of asset prices in financial accelerator, and Jermann and Quadrini (2012) and Melcangi (2024), which highlight the role of working capital in the transmission of credit shocks.

identified estimates of causal effects to discipline structural macroeconomic models (Beraja, Hurst, and Ospina, 2019, Nakamura and Steinsson, 2014, Ottonello and Winberry, 2020).

2 Model

There are four types of agents in the domestic economy: households, intermediate goods firms, final goods firms, and capital-producing firms. Each intermediate goods firm operates in one of two sectors: a tradable goods sector, indexed by H, or a non-tradable goods sector, indexed by N. The final goods firms combine goods from the firms in the tradable and non-tradable sectors with foreign goods, indexed by F, to produce a non-differentiated final good. Households and capital-producing firms use final goods for consumption and investment. In mapping the model to the data, we interpret the domestic economy as a US state, and thus model it as a small open economy, and the foreign economy as the rest of the world.

Households. There are two types of households: a measure χ of "spenders", indexed by L, and a measure $1-\chi$ of "savers", indexed by S. Savers and spenders are identical within their group, so there is a representative household of each.[3] In each period t, households choose final goods for consumption. Savers also choose their next-period financial assets. Each household type is made up of a continuum of workers who each produce a differentiated labor supply indexed by $j \in [0,1]$.

Households seek to maximize

$$E_0 \sum_{t\geq 0} \beta^t U(C_t, \{N_{j,t}\}),$$

[3] We adopt a two-agent framework rather than a fully heterogeneous-agent representation of the household sector primarily for computational tractability. Our spender-saver dichotomy is a stylized representation of the hand-to-mouth behavior modeled in Kaplan and Violante (2014) and Kaplan, Moll, and Violante (2018). As Boar, Gorea, and Midrigan (2021) argue, this behavior effectively captures key predictions of richer housing models that explicitly incorporate institutional details of the mortgage market.

subject to their budget constraints. The budget constraint of a spender is

$$P_t C_t^L = \int w_{j,t} N_{j,t} \mathrm{d}j,$$

and the budget constraint of a saver is

$$\frac{B_t^H}{1-\chi} + P_t C_t^S = \frac{\mathrm{Div}_t}{1-\chi} + \int w_{j,t} N_{j,t} \mathrm{d}j + \frac{B_{t-1}^H}{1-\chi}\left(1 + r_{t-1}^H\right), \tag{1}$$

where P_t is the price of the final good, w_j is the nominal wage at which worker j supplies labor, Div_t is the total net dividend paid by all firms in the domestic economy, and $\frac{B_t^H}{1-\chi}$ is the saver's net financial assets at the start of period t. We assume that period utility takes the GHH form:

$$U(C_t, N_t) = \Gamma \frac{\left[C_t - \phi \int_{j=0}^{1} \frac{N_{j,t}^{1+\upsilon}}{1+\upsilon} \mathrm{d}j\right]^{1-\sigma}}{1-\sigma}.$$

We also assume labor is immobile across states and that wages feature nominal rigidity as in Galí (2015, Chapter 6) (see Appendix A.1 for details).

In a small open economy, there is a market for international borrowing and lending with an exogenous risk-free interest rate r. However, the domestic economy faces an exogenous wedge ψ_t^H which determines the spread between the domestic and foreign interest rates. Thus, the interest rate faced by savers is:

$$r_t^H = r + \psi_t^H. \tag{2}$$

When studying credit market liberalization we consider exogenous changes to ψ_t^H.

Capital Goods Producer. Capital in the local economy is produced by perfectly competitive capital goods producers who own the capital stock and lease it to intermediate goods producers.

The representative capital goods producer has the following production function:

$$K_{t+1} = (1-\delta_K)K_t + I_t - \kappa \left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t. \tag{3}$$

Capital is leased at the endogenous nominal rate $P_t(r_t^K + \delta_K)$, so nominal flow profit is:

$$\Pi_t^K = P_t(r_t^K + \delta_K)K_t - P_t I_t. \tag{4}$$

All firm owners maximize the present value of real profits, evaluated in units of the final good. The firms' stochastic discount factor M_t is consistent with the real interest rate faced by firms:

$$M_t = \frac{P_{t+1}}{P_t(1+r_t^F)}. \tag{5}$$

Analogously with the household interest rate r_t^H, we assume that $r_t^F = r + \psi_t^F$. In our baseline case, we assume that $\psi_t^F = \psi_t^H$ so that $r_t^F = r_t^H$.[4] In our counterfactuals, we will deviate from this.

Since the capital good's producer has constant returns to scale, its value at the start of the period is proportional to its capital stock. Denote the representative capital producer's value by $\mathcal{Q}_t K_t$ in units of the final good for which $\mathcal{Q}_t \equiv \mathcal{Q}(\mathbf{X}_t)$. As a result, the Bellman equation for the representative capital goods producer is:

$$\mathcal{Q}_t K_t = \max_{I_t}(r_t^K + \delta_K)(K_t) - I_t + M_t E_t\left[\mathcal{Q}_{t+1}\right] K_{t+1}. \tag{6}$$

Final Goods Firms. There is a representative, perfectly competitive, final goods firm with two constant returns to scale technologies. The first technology is that the final goods firm aggregates

[4] Given the household's Euler equation, this assumption is equivalent to setting $\beta\frac{u'(C_{t+1})}{u'(C_t)}$ as the firm's discount factor for real units in the next period.

output purchased from retailers in each of the two domestic sectors N and H into a composite good in that sector and then integrates the output of retailers in the foreign economy into a composite imported good. Let I_S denote the set of retailers in sector $S \in \{N; H; F\}$, where F denotes the foreign economy. Each retailer $i \in I_S$ produces a separate variety i, sold at price p_i.

Specifically, the final goods firm uses the first technology to produce output Y_S^d of the composite good of sector $S \in \{N; H; F\}$. This technology uses the following production function:[5]

$$Y_S^d = \left[\int_{i \in I_S} \left(y_i^d \right)^{\frac{\eta - 1}{\eta}} di \right]^{\frac{\eta}{\eta - 1}}, \tag{7}$$

where y_i^d is the output of the variety produced by retailer $i \in I_S$ and $\eta > 1$ is the elasticity of substitution across varieties.[6]

The second technology is that the final goods firm aggregates some amount of domestically produced tradable goods Y_H^d with imported tradable goods Y_F^d and non-tradable goods Y_N^d to produce a final good, based on the following production function:

$$Y_t^d = \left(\gamma^\gamma (1-\gamma)^{1-\gamma} \right)^{-1} \left[\left(\omega_H^{\frac{1}{\zeta}} \left(Y_{H,t}^d \right)^{\frac{\zeta - 1}{\zeta}} + (1 - \omega_H)^{\frac{1}{\zeta}} \left(Y_{F,t}^d \right)^{\frac{\zeta - 1}{\zeta}} \right)^{\frac{\zeta}{\zeta - 1}} \right]^\gamma \left(Y_{N,t}^d \right)^{1-\gamma}.$$

This production function entails that tradable goods from the home and foreign economy can be substituted for one another with a constant elasticity of substitution ζ. Tradable and non-tradable goods can be substituted for one another with a constant elasticity of substitution of 1. Here, $\omega_H \in (0, 1)$ measures the degree of home bias in demand, and $\left(\gamma^\gamma (1-\gamma)^{1-\gamma} \right)^{-1}$ is a normalization factor that ensures the price of final goods satisfies $P = 1$ in the steady state.

Final goods firms purchase composite goods at price P_S, for $S \in \{N; H; F\}$ and sell final goods at price P. We adopt the normalization that the price of imported composite goods (which

[5]For the moment, we omit the time index.
[6]We typically use the superscript d to denote demand.

depends on conditions in the foreign economy) satisfies $P_{F,t} = 1$ for all t.

Final goods firms choose the input of each variety i and the output of each composite good, to maximize profits from the first technology, $P_S Y_S^d - \int_{i \in I_S} p_i y_i^d di$, for each $S \in \{N; H; F\}$. Their choice of y_i^d satisfies $P_S \left(Y_S^d\right)^{\frac{1}{\eta}} \left(y_i^d\right)^{\frac{-1}{\eta}} = p_i$. Furthermore, final goods firms choose the input Y_S^d of each composite good S and output of final goods to maximize the profit from the second technology, $PY^d - \sum_S P_S Y_S^d$. These choices satisfy: $P_N Y_N^d = (1-\gamma) PY^d$, $P_H Y_H^d = \omega_H \gamma Y^d P \left(P_H^\gamma P_N^{1-\gamma} P^{-1}\right)^{\frac{1-\zeta}{\gamma}}$, and $Y_F^d = (1-\omega_H)\gamma Y^d P \left(P_N^{1-\gamma} P^{-1}\right)^{\frac{1-\zeta}{\gamma}}$. The last equation uses our normalization of the price for the foreign goods to one and where the final goods price satisfies:

$$P = P_N^{1-\gamma} \left(\left[\omega_H P_H^{1-\zeta} + (1-\omega_H) \right]^{\frac{1}{1-\zeta}} \right)^{\gamma}.$$

We let $Y_{S,t}$ denote the total real output of retailers in sector $S \in \{N, H\}$. In the non-tradable sector, market clearing requires that this equals the real input of the final goods firm, so $Y_{N,t}^d = Y_{N,t}$.

Exports. Final goods firms in the foreign economy behave similarly to those in the domestic economy. Thus, the foreign demand y_i^* for each tradable domestic variety i satisfies $P_H \left(Y_H^*\right)^{\frac{1}{\eta}} \left(y_i^*\right)^{\frac{-1}{\eta}} = p_i$, where the aggregate foreign demand for domestic tradable goods Y_H^* satisfies $P_H Y_H^* = (1-\omega_F)\gamma Y^* P^* \left(P_H^\gamma \left(P_N^*\right)^{1-\gamma} \left(P^*\right)^{-1}\right)^{\frac{1-\zeta}{\gamma}}$. In a small-open economy foreign expenditure Y^*, foreign price levels P^*, and P_N^* are all exogenous and constant, so we can write $Y_H^* = \overline{X} P_H^{-\zeta}$, where $\overline{X}$ is a constant. We will calibrate the model so that, in the steady state, $P_H = 1$ and $\overline{X}$ is the steady state level of exports. The home economy's level of (nominal) net exports is:

$$NX_t = P_H Y_H^* - Y_{F,t}^d = \overline{X} P_H^{1-\zeta} - (1-\omega_H)\gamma Y^d P \left(P_N^{1-\gamma} P^{-1}\right)^{\frac{1-\zeta}{\gamma}}. \tag{8}$$

Market clearing in the home tradable sector implies that the output of retailers in this sector is equal to the demand of the final goods firms at home and abroad, so that $Y_{H,t}^d + Y_{H,t}^* = Y_{H,t}$.

Retailers. There is a continuum of measure one of retailers in each sector S. Retailers do not enter or exit. Each retailer uses one unit of the intermediate good in its sector to produce one unit of its variety $i \in [0, 1]$. The price of the intermediate good in sector S is $P_{W,S,t}$. Retailers set their price p_i to maximize present discounted profits, given domestic and foreign demand function for their variety. However, they face Calvo pricing frictions: only a fraction $1 - \vartheta_P$ of retailers can change their price each period, as in Galí (2015, Chapter 3). See Appendix A.1 for details.

Intermediate Goods Producers. Intermediate goods producers have idiosyncratic productivity and produce homogeneous products for each sector $S \in \{N, H\}$. For these firms, both input and output markets are perfectly competitive. We explicitly model endogenous firm entry, as in Hopenhayn (1992). Potential new firms pay a fixed entry cost at the start of the period and then draw productivity z from a distribution with density $g(z)$. Each period a continuing firm keeps its productivity z with probability ρ_z; and with probability $1 - \rho_z$, it draws a new z from the distribution g. For tractability, we make the following distributional assumption.

Assumption 1. *The distribution g is Pareto, so that $g(z) = \xi z^{-\xi-1}$ for $z \geq 1$ and zero otherwise.*

A firm i produces with $y_i = z_i^{1-\theta}(k_i^{1-\alpha} n_i^{\alpha})^{\theta}$, where n_i is a composite of the quantities of labor of each worker j that is used by the firm, and it satisfies $n_i^{\frac{\epsilon_W - 1}{\epsilon_W}} = \int_0^1 n_{i,j}^{\frac{\epsilon_W - 1}{\epsilon_W}} \mathrm{d}j$, and $n_{i,j}$ denotes firm i's use of labor j. Each period, if the firm chooses to stay in business, it must pay c^F units of final goods as fixed cost; or the firm must exit.

The optimization problem for the firm at time t is described by the following Bellman equation:

$$\begin{aligned} W(z; \mathbf{X}_t, S) \;&=\; \pi^W(z; \mathbf{X}_t, S) + M_t \rho_z \max\{W(z; \mathbf{X}_{t+1}, S); 0\} \\ &\quad + M_t(1 - \rho_z) E[\max\{W(z'; \mathbf{X}_{t+1}, S); 0\}] \end{aligned} \tag{9}$$

where $\mathbf{X}_t$ is the aggregate state, $\pi^W(z; \mathbf{X}_t, S)$ is the optimal flow profit, and M_t is the discount factor, the same as for the capital goods producing firms. The static profit $\pi^W(z; \mathbf{X}_t, S)$ is:

$$P_t \pi^W(z; \mathbf{X}_t, S) \equiv \max_{k, \{n_{i,j}\}} \left\{ P_{W,S,t} y_i - (r_t^K + \delta_K) P_t k_i - c^F P_t - \int_0^1 w_{t,j} n_{i,j} \mathrm{d}j \right\}. \tag{10}$$

In equilibrium, the value of an intermediate goods firm $W(z; \mathbf{X}_t, S)$ is increasing in z. Therefore, there is a unique cutoff $z^\star_{S,t}$ such that the firm will exit iff its productivity falls below $z^\star_{S,t}$.

We assume that the entry cost is paid in units of final goods, but that the cost of entry is an increasing convex function of the number of entrants. (see Bergin, Feng, and Lin (2018) and references therein). In particular, we assume that the entry cost in a sector is equal to $\nu P_t \left(\frac{M^e_{S,t}}{M^W_{S,t-1}} \right)^\Theta$, where $\Theta \geq 0$ is the elasticity of entry costs with respect to the number of entrants. In equilibrium, the following conditions for free entry and exit must be satisfied:

$$\nu \left(\frac{M^e_{S,t}}{M^W_{S,t-1}} \right)^\Theta = E[\max\{W(z_t; \mathbf{X}_t, S); 0\}] \tag{11}$$

$$W(z^\star_{S,t}; \mathbf{X}_t, S) = 0, \text{ or } W(1; \mathbf{X}_t, S) > 0 \text{ and } z^\star_{S,t} = 1. \tag{12}$$

The latter case here corresponds to the case in which no firms exit.

We let $M^W_{S,t}$ denote the mass of active intermediate goods firms in period t in sector S, after both entry and exit decisions have been made, $M^e_{S,t}$ denote the mass of firms that enter, even though some of these firms will exit immediately, and $\mu_{S,t}(z)$ denote the density function for active firms' productivity in period t in sector S. We characterize in Appendix A the law of motion for the measure of intermediate goods produces with any productivity $\hat{z}$.

Aggregate Profits. Total nominal profits (net of entry costs) are given by:

$$\Pi_t = \int_{i\in I_H \cup I_N} P_t \pi^W_{i,t} di + \Pi^K_t + \Pi^R_t - \nu P_t \left(\frac{M^e_{H,t}}{M^W_{H,t-1}}\right)^\Theta M^e_{H,t} - \nu P_t \left(\frac{M^e_{N,t}}{M^W_{N,t-1}}\right)^\Theta M^e_{N,t}. \tag{13}$$

where $\pi^W_{i,t}$ is the period profit of the intermediate goods producer i in units of the final good, according to the function in equation (10), and Π^K_t and Π^R_t are the total profits of capital goods firms and retailers given in equation (4) and equation (A.3). $\Pi_t = \Pi^R_t$ at the steady state since all firms apart from retailers are either competitive or face free entry. Since final goods firms make zero profits every period, we do not need to consider their profits here.

Exogenous Dividend Rules and Firm Borrowing. As discussed above, firms in the economy are owned by saver households, to whom they pay dividends equal to Div_t in the aggregate. The aggregate assets of firms, B^F_t, evolve according to:

$$B^F_t = \Pi_t - \mathrm{Div}_t + B^F_{t-1}(1 + r^F_{t-1}), \tag{14}$$

where Π_t is the total net profits as defined in equation (13).

In our benchmark case, where $r^F_t = r^H_t$, the level of Div_t is neutral for the aggregate allocation, in line with the Modigliani-Miller theorem: if firms borrow more at time t and issue higher dividends this has no effect on savers. In our counterfactuals, when firms and consumers face different interest rates, the level of dividends matters for the allocation. For instance, if $r^F_t < r^H_t$ then households in the domestic economy are better off if firms borrow as much as possible and issue as high as possible dividends to households. Conversely, if $r^H_t < r^F_t$, households in the domestic economy are better off if dividends are as low as possible. An implication of this is that if firms optimally chose their dividends to maximize the present discounted utility of their owners, then there would be a corner solution or no finite solution for optimal dividends when $r^H_t \neq r^F_t$.

To avoid such corner solutions, we follow Chen, Karabarbounis, and Neiman (2017) and assume that firms smooth dividends over time, setting them according to an exogenous distribution rule. That firms smooth dividends has been demonstrated by a large empirical literature, and may reflect asymmetric information between managers and investors, agency problems, or external financing constraints, among other factors (Leary and Michaely, 2011). In particular, we assume that firms set dividends equal to a constant fraction Ξ of the output of final goods: $\text{Div}_t = \Xi Y_t^d$.

Equilibrium. In Appendix A we define the equilibrium of the economy and characterize the decision rules of each agent. In line with the small-open-economy assumption, the interest rates $\{r_t^H, r_t^F\}$ and the price of foreign goods P_F are treated as exogenous. The domestic prices $\{w_t, r_t^K, \mathcal{Q}_t, P_{H,t}, P_{N,t}, P_t\}$ ensure market clearing in the markets for labor, capital, and the domestic tradable, nontradable, and final goods.[7]

3 Parameterization

In this section, we discuss how we parameterize the model. We set a number of parameters exogenously to conventional values. We estimate the rest to match features of the data, notably causal estimates of credit expansion on macroeconomic aggregates obtained from a quasi-natural experiment, namely the deregulation of the banking sector in the United States in the 1980s.

3.1 Parameters Set Exogenously

The parameters we set exogenously are listed in Table 1. A model period is one year. On the household side of the economy, we set the discount factor β to 0.96, the risk aversion parameter σ to 2, and the parameter υ to 0.25; these values are typically used in the literature (e.g., Arellano,

[7]Our small open economy has the foreign tradeable goods as the numéraire. Moreover, there is no local government (hence, no link between fiscal policy and the interest rate) and bonds are denominated in foreign goods with an exogenous interest rate. The equilibrium is therefore determinate (Hagedorn, 2023).

Bai, and Kehoe, 2019). We normalize the disutility of labor ϕ to 1. The fraction of spender households, χ, is set to be 0.25 (Aguiar, Bils, and Boar, 2024). The interest rate r is equal to $1/\beta - 1$, consistent with the existence of a steady state level of consumption when $\psi_t^H = 0$.

On the production side of the economy, we set the depreciation for physical capital δ_K to 0.1 and $\alpha = 0.64$ so that the share of labor income relative to capital income is consistent with standard values. We set the returns to scale parameter for intermediate goods firms at $\theta = 0.85$, so that the economic profit share absent fixed costs (which is $1 - \theta$ in the model) corresponds to the share of rents going to entrepreneurs in the data (Cagetti, De Nardi et al., 2006, Quadrini, 2000). We calibrate γ so that the share of tradable goods is about 0.7 in the steady state (Nakamura and Steinsson, 2014). Similarly, we set the degree of home bias in the demand of tradable goods, ω_H, to 0.17. In the Pareto distribution $g(z) = z_m^{\xi} \frac{\xi}{z^{\xi+1}}$ we normalize z_m to 1 and we use data on the US distribution of firm size to discipline our choice of ξ.[8] Figure 9 in the Appendix shows that our model matches the firm size distribution in the data reasonably well. To calibrate parameters related to sticky prices and wages, we follow the literature (Galí, 2015, Chapter 3, Nakamura and Steinsson, 2013, Basu and House, 2016) and set the elasticity for type-specific labor variety ϵ_w to 20, θ_w to 0.3895, which is consistent with a probability of nominal wage changes per quarter of 21%, and also set ϑ_P to 0.2470, which is consistent with a probability of nominal price changes per month of 11%. Lastly, we calibrate Ξ so that the initial firm-side debt is roughly about 62% of total debt, consistent of the data in 1980. The implied value of Ξ is 0.245.

3.2 Estimated Parameters

We estimate the remaining parameters to match several features of the data. These parameters are: κ, the capital adjustment cost parameter for capital goods producers; c^F, the flow operating cost for intermediate goods producers; ρ_z, the persistence for firm-level productivity among intermediate

[8]We used the data from Restuccia and Rogerson (2008) and Rossi-Hansberg and Wright (2007).

Table 1: Parameters Set Exogenously

Parameter	Description	Value
β	Discount factor	0.96
σ	Risk aversion	2.0
υ_L	Labor supply elasticity parameter	0.25
γ	Preference over Tradable goods	0.7
χ	Share of HHs hand-to-mouth	0.25
δ_K	Capital depreciation	0.1
α	Labor share for Intermediate Goods Firms	0.64
ξ	Scale for Productivity Pareto Dist.	1.3
θ	Return to scale for Intermediate Goods Firms	0.85
ω_H	Home bias within tradable goods demand	0.17
ϵ_w	Elasticity for type-specific labor variety	20.0
θ_w	Prob. for wages not changing	0.3895
ϑ_P	Prob. for prices not changing	0.2470
Ξ	Dividend distribution ratio	0.245

goods producers; ν, the level coefficient for new firms' entry costs; Θ, the elasticity of entry costs with respect to aggregate entry rates; η, the demand elasticity for intermediate goods; ζ, the constant elasticity of substitution between home tradable goods and foreign tradable goods; and the average level of steady state debt, $\bar{B}$.

To estimate these parameters, we choose the values that minimize the distance between a number of moments in the model and in the data. As we will argue below, the dynamic effect of credit market conditions on a number of economic variables in the model is informative for the values of these parameters. To estimate such dynamic effects in the data, we leverage the credit expansion induced by the graduate deregulation of the banking sector across US states the 1980s. We focus specifically on the deregulation of intrastate bank branching, so as to minimize concerns regarding spillover effects across state borders. This event has been studied extensively in the literature (see Rice and Strahan, 2010 and Kroszner and Strahan, 2014 and the references therein) and the established consensus is that it can be plausibly considered to be exogenous (conditional on a few controls, which we also incorporate), allowing quasi-experimental identification of the effect of the deregulation at the state level.[9] In Appendix B, we review the institutional details of this reform and describe all our data sources.

We estimate the impulse responses at the state level after bank branching deregulation for lending rates, household debt, output, employment, wages, and firm entry and exit rates. Our data on lending rates and household debt are from the Call Report data of institutions regulated by the Federal Deposit Insurance Corporation (FDIC). Data on output, employment, and wages are from Bureau of Economic Analysis (BEA), and data on firm (establishment) entry and exit rates are from the Business Dynamics Statistics database.

Our estimation of impulse responses follows a local projection approach, which does not require us to first specify an underlying dynamic model, and is known to be relatively robust to mis-specification of the data-generating process (Jorda, 2005). We estimate specifications of the form:

$$g_{i,t+k} = \alpha_{i,k} + \bar{\alpha}_{t,k} + \sum_{j=1}^{J} \beta_{j,k} g_{i,t-j} + \sum_{j=0}^{J} \gamma_{j,k} D^{intra}_{i,t-j} + \sum_{j=0}^{k-1} \delta_{j,k} D^{intra}_{i,t+k-j} + \Gamma \mathbf{X}_{it} + \epsilon_{i,t+k} \quad (15)$$

where $g_{i,t+k}$ is the growth rate of the outcome variable that we consider in state i between $t+k-1$ and $t+k$; $D^{intra}_{i,t}$ is a dummy variable equal to 1 if state i lifted its intrastate branching restrictions in year t and zero otherwise; $\mathbf{X}_{it}$ is a vector of controls for output and employment growth, employment share of major sectors; $\alpha_{i,k}$ and $\overline{\alpha}_{t,k}$ are state and year fixed effects, respectively. The specification also includes lags of the dependent variable to control for potential pre-trends, with the coefficients $\beta_{j,k}$ capturing the autoregressive structure of the outcome variables' growth. We set the number of lags J to 4 and estimate equation (15) in growth rates to account for non-stationarity

[9] The easing of intra-state branching restrictions occurred gradually between 1970 and 1994, with states dominated by well-capitalized large banks lifting restrictions sooner (Rice and Strahan, 2010). To mitigate concerns that such states might be different in terms of economic outcomes, we follow the literature and include in our specification lags of our outcome variables of interest, as well as controls for indicators of economic activity.

in some of our outcome variables.[10] Although controlling for lags of the dependent variable and state fixed effects may, in principle, bias the estimation in small samples, Nickell (1981) shows that the order of bias is $1/T$, which is small for our dataset (a panel of 51 states observed over 57 years). We cluster standard errors at the state level to account for serial correlations within state and across time.

Given this specification, the estimated impulse response function for the level of an outcome variable is $IRF\left(T\right) = \sum_{k=0}^{T} \gamma_{0,k}$. Using the estimated impulse response function for the lending rate and equation (2), we back out the sequence of underlying shocks ψ_t^H, which we then feed into the model. Since our empirical analysis only focuses on the first 10 years after the deregulation, we also focus on the first 10 years' responses in our model, and to capture the persistence of shocks we assume that the underlying shock of ψ_t^H gradually returns to its initial level with a fixed persistence parameter of 0.99.[11] We jointly calibrate all the parameters using the simulated method of moments (SMM) to match the impulse responses to bank deregulation of real GDP, employment, (real) wages, firm entry rates and exit rates, and household debt, as well as steady state moments (the average level of debt to output across states, about 27%; the entry and exit rates in the steady state).[12] We use the inverse of the empirically estimated variances (for each variable and at each horizon of the impulse response function) as weights for the distance between model and data moments.[13]

Table 2 lists the estimated parameters. We briefly discuss the intuition for how they are pinned

[10]We show in Appendix B that our results are robust to estimating (15) in levels rather than growth rates and to not including lags of the deregulation variable in the specification.

[11]Numerically, we also experimented with many other settings, such as persistence parameter of 0.95, or letting the shocks of ψ_t^H remain at their lowest point for an additional 10 years or 50 years and then gradually reverting to the initial level of 0. Since our empirical and quantitative analysis focuses on the first 10 years, we find that our results are essentially not affected when we change our assumptions regarding what happens in the very long-run future in the model.

[12]We assume that aggregate labor in the model, N_t, corresponds to aggregate employment in the data.

[13]Nominal GDP in our model economy is $P_t Y_t^d + NX_t$ and Real GDP is nominal GDP deflated by P_t. Note that GDP components include total consumption, total investment (including physical capital investments, the entry costs for all new firms, and the total operating costs for active firms), and net exports.

down. Given other parameters, $\bar{B}$ is mostly relevant for the steady state level of debt to output (of course, $\bar{B}$ also matters for how households respond to interest rates shocks). We use 1.2% as the target for the debt service to output ratio in the steady state (the debt to output ratio is about 27% as in the data from around the 1980s). κ affects the speed of capital adjustment and hence output and employment dynamics. The parameters c^F, ρ_z, ν, and Θ affect firms' productivity, profits, and therefore firms' decisions to enter and exit. Thus, entry rates and exit rates in the steady state and their dynamic responses can provide important information about these parameters. Lee and Mukoyama (2015) report that in the Census of Manufacturers, 1972-1997, the average entry and exit rates are approximately 6.2% and 5.5%, respectively. The demand elasticity for intermediate goods η is mostly relevant for firms' demand changes, and thus output dynamics will provide important information to discipline these. The elasticity of substitution between home tradable goods and foreign tradable goods, ζ, matters for foreign demand in responses to domestic price changes.

Comfortingly, a number of the parameters are estimated to take values close to what is typically used in the literature. For example, with our estimated capital adjustment cost, the implied elasticity of the capital price with respect to the investment capital ratio is fairly close to that defended by Bernanke, Gertler, and Gilchrist (1999). Additionally, the constant elasticity of substitution between home tradable goods and foreign tradable goods, ζ, is estimated to be about 2.15, which is fairly close to the value of 2 used by Gali and Monacelli (2005) and Nakamura and Steinsson (2014). Furthermore, our value for η is about 5.58, so the implied markup is 21%, largely consistent with values typically used in the literature (Basu and Fernald, 1997, Burstein and Hellwig, 2008 and Atkeson and Kehoe, 2005).

Figure 1 shows our estimated impulse responses in the model and in the data. Overall, our model can match these responses fairly well, despite the small number of parameters we use to target these statistics. In Appendix C we provide further intuition for identification by examining

Table 2: Parameters Calibrated Internally

Parameter	Description	Value	Relevant Moments
κ	capital adj. costs	0.4868	Employment and GDP dynamics
c^F	flow operating costs	8.3664	Entry and exit rates
ρ_z	persistence for firm-level productivity	0.7210	Entry and exit rates
ν	entry cost, linear coefficient	46.0895	Entry and exit rates
Θ	elasticity of entry costs w.r.t. to Agg. entry rate	0.4488	Entry and exit rates
η	demand elas. for intermediate goods	5.5836	Employment and GDP dynamics
ζ	elasticity between Home and Foreign tradables	2.1567	Employment and GDP dynamics
$\bar{B}$	HH steady state debt level	-0.8723	Debt service to output ratio; Debt dynamics

how the impulse responses vary with the internally calibrated parameters. Specifically, in Figures 10-17 we compare our benchmark impulse responses with those obtained with higher and lower values of the parameters $\eta, \upsilon_L, \zeta, \kappa, c^F, \rho_z, \nu$ and Θ, and in Figure 18 we show that the moments implied by the model are monotonic in parameter values.

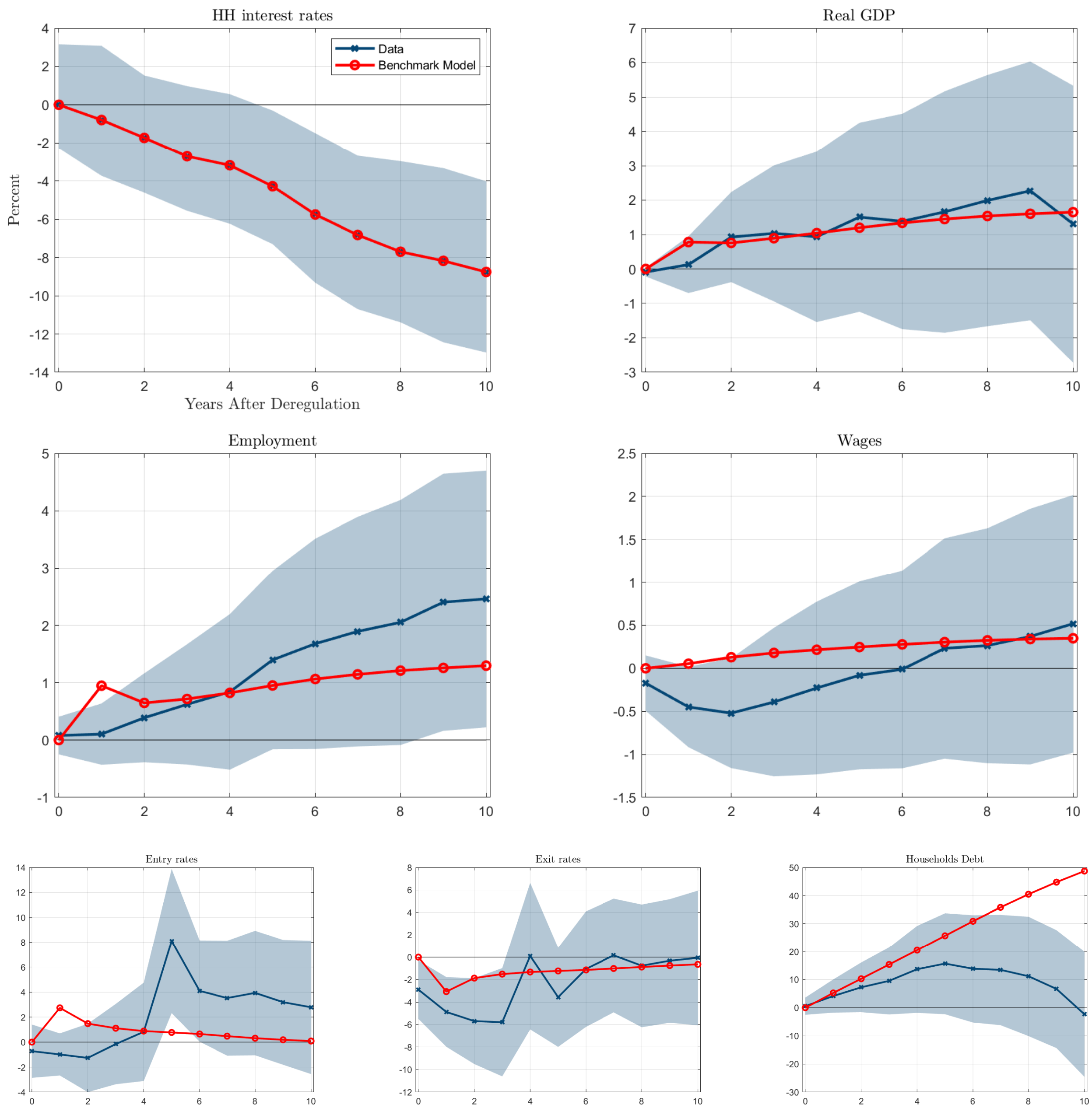
4 Effects of Bank Deregulation

Having estimated the model, in this section we perform our main decomposition exercise aimed at quantifying the relative importance of household and firm-side channels in explaining the aggregate effect of the credit expansion. We also discuss the role of different model ingredients and discuss alternative transmission mechanisms.

4.1 Quantifying the Relative Importance of Different Channels

To quantify the relative importance of household and firm-side channels in explaining the aggregate effect of the credit expansion, we consider the case in which there are interest rate shocks ψ_t that apply only to households or only to firms in the economy. This allows us to decompose the relative importance of household and firm-side channels and investigate the extent through which deregulation acts through the production and the household side of the economy. In the benchmark model, we assumed that ψ_t was the same for households and firms, implying that all

Figure 1: Impulse Responses from Bank Deregulation, Model vs. Data



Notes: The shaded areas are 90% confidence intervals.

agents face the same interest rate ($r_{F,t} = r_{H,t}$). By contrast, in the decomposition analysis, $r_{F,t}$ will no longer equal $r_{H,t}$.

Figure 2: Shocks to Firms vs. Households Only: Impulse Responses

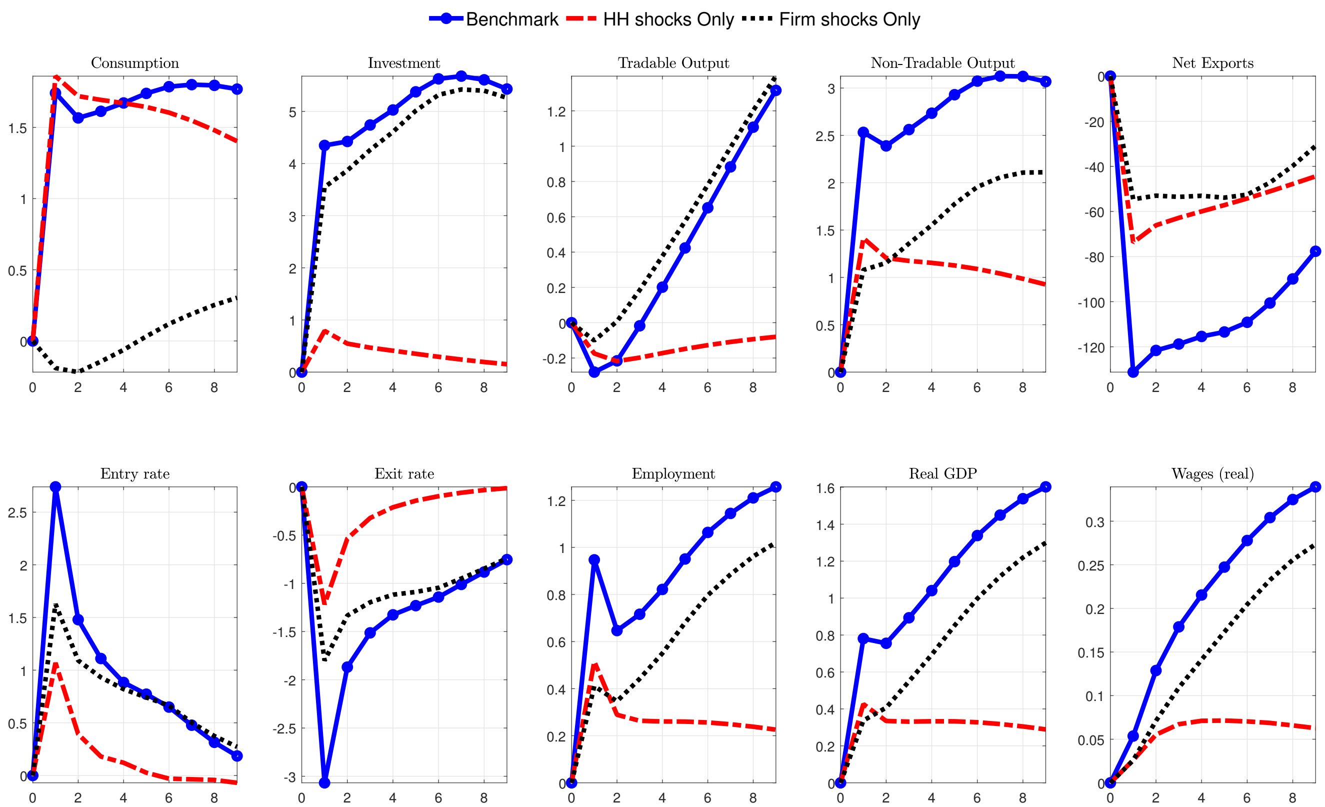


Figure 2 plots the impulse responses for three different situations: (1) the benchmark model which we label "Benchmark"; (2) shocks only to interest rate that households face, $r_{H,t}$, while keeping $r_{F,t}$ is fixed at its initial level, which we label "HH shocks Only"; and (3) shocks only to the interest rate firms face, $r_{F,t}$, while keeping $r_{H,t}$ fixed its initial level, which we label "Firm shocks Only." All variables are expressed in percent deviations from their steady state values.

By comparing the three different impulse responses and inspecting the dynamics, we find that the increases in GDP, employment and consumption are due to a combination of household and firm-side channels. In the short run, firm-side and household-side channels are roughly equally important. In the long run, the firm-side channel dominates. We now discuss the precise mechanisms operating in the two channels.

The Firm Channel. The mechanism of how an interest rate shock propagates through the firm-side channel is that, as capital goods producers face lower interest rates and thus lower costs for investment, they will supply more capital goods to the rental market, reducing the rental rate of capital. However, the response is dampened by the capital goods adjustment costs and by the fact that some intermediate inputs must be produced at home and the boom causes the prices for the home goods to increase. The lower interest rate and capital rental rate will also raise the profitability and present value of a firm, which in turn implies an increase in firm entry and a decrease in firm exit.

The Household Channel. An interest rate shock operates as follows through the household channel: when interest rates are exogenously reduced, the financially unconstrained are on the Euler Equation and will borrow more from abroad, front-loading their consumption from the future to the present. The hand-to-mouth agents will also increase their consumption because current wages go up. Thus, the nominal demand for goods in both sectors increases in the aftermath of the shock. This raises prices in both sectors.

In the non-tradable sector, the increase in prices makes it profitable for firms to increase production by hiring more labor and renting more capital. This in turn will drive up factor prices (real wages and capital goods renting prices r_K). Rising real wages in turn raise labor supply, aggregate employment and output. In addition to their effect on production, rising prices in the non-tradable sector tend to cause a rise in firm profits; in turn, fewer incumbent firms exit and more new firms enter.

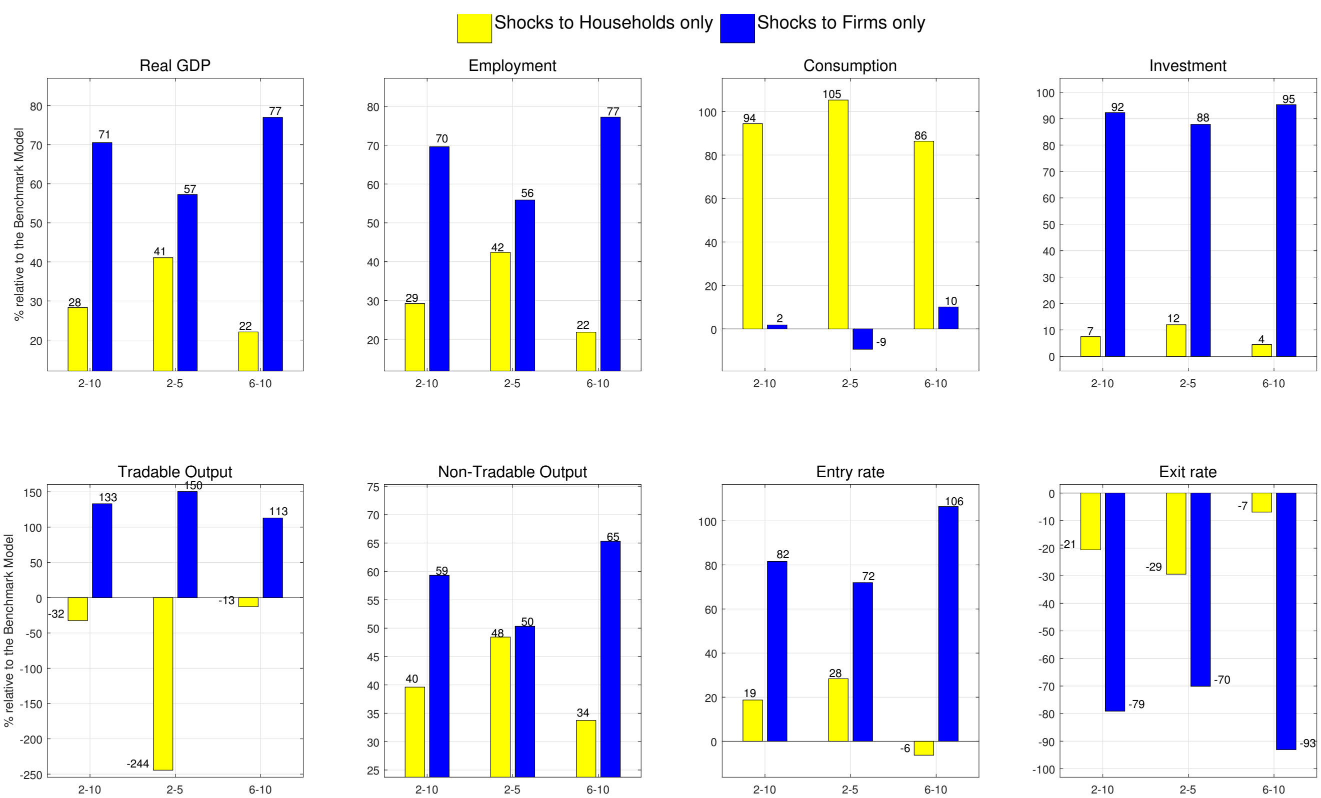
The growth in output in the non-tradable sector is exacerbated in the first few periods by nominal rigidities in goods and labor markets, which cause rising demand to lead to higher production rather than price increases. At the same time, as we discuss in Section 4.2 below, the core mechanisms of the household channel are not radically changed when we reduce nominal rigidities, in contrast to what one might expect from closed economy New Keynesian models. As such, the core household demand channel that drives the rise in output and employment is not the usual New Keynesian channel that operates via nominal rigidities. The reason that output and employment can rise even in the absence of nominal rigidities is that real wages can rise due to real exchange rate appreciation (i.e. a higher relative price of non-tradables compared to imported tradables). Since the price of imported tradables is determined by conditions outside the domestic economy, this price remains fixed (normalized at the value 1) after the interest-rate shock. Thus, final goods prices are also less responsive to the shock, since these partly depend on the price of imports. As such, rising nominal wages after the shock translate into rising real wages, and so labor supply, employment and, therefore, output increase, even in the absence of nominal rigidities.

For the tradable goods sector, however, rising domestic household demand propagates differently. Increased domestic demand for tradable goods is met by importing more foreign tradable goods. Demand for foreign tradables rises while demand for domestically produced tradables falls because higher domestic demand pushes up domestic relative to foreign prices. The shift towards importing tradables frees up labour to move to the the non-tradable sector to meet higher non-tradables demand, which in turn lowers domestic production of tradable goods—a mechanism reminiscent of the J-curve effect in the international business cycle literature (e.g., see Kehoe, Kydland, and Backus (1992)). As a result, a shock that lowers the interest rate of households only leads tradable good production to fall and remain below steady state for more than 10 years, while non-tradables production rises (see Figure 2).

Comparing the Overall Importance of the Two Channels. Figure 2 quantifies the importance of the two channels for various aggregate variables. The figure shows that firm shocks are more important for GDP, investment, and employment, whereas household shocks account for most of the dynamics of consumption.

Figure 3 computes averages for the percentage deviation from steady state for each variable across different horizons (years 2 to 10, years 2 to 5, and years 6 to 10). In each panel the sum of the two bars is approximately 100%. We interpret these bars as representing the fraction of the overall response of the variable that is due to each of the two channels. Figure 4 provides more information for other economic variables' decomposition over years 2 to 10, such as the price for capital goods, prices for different sectors' output goods, real wages, employment and output in different sectors.

Figure 3: Shocks to Firms vs. Households Only: Accounting



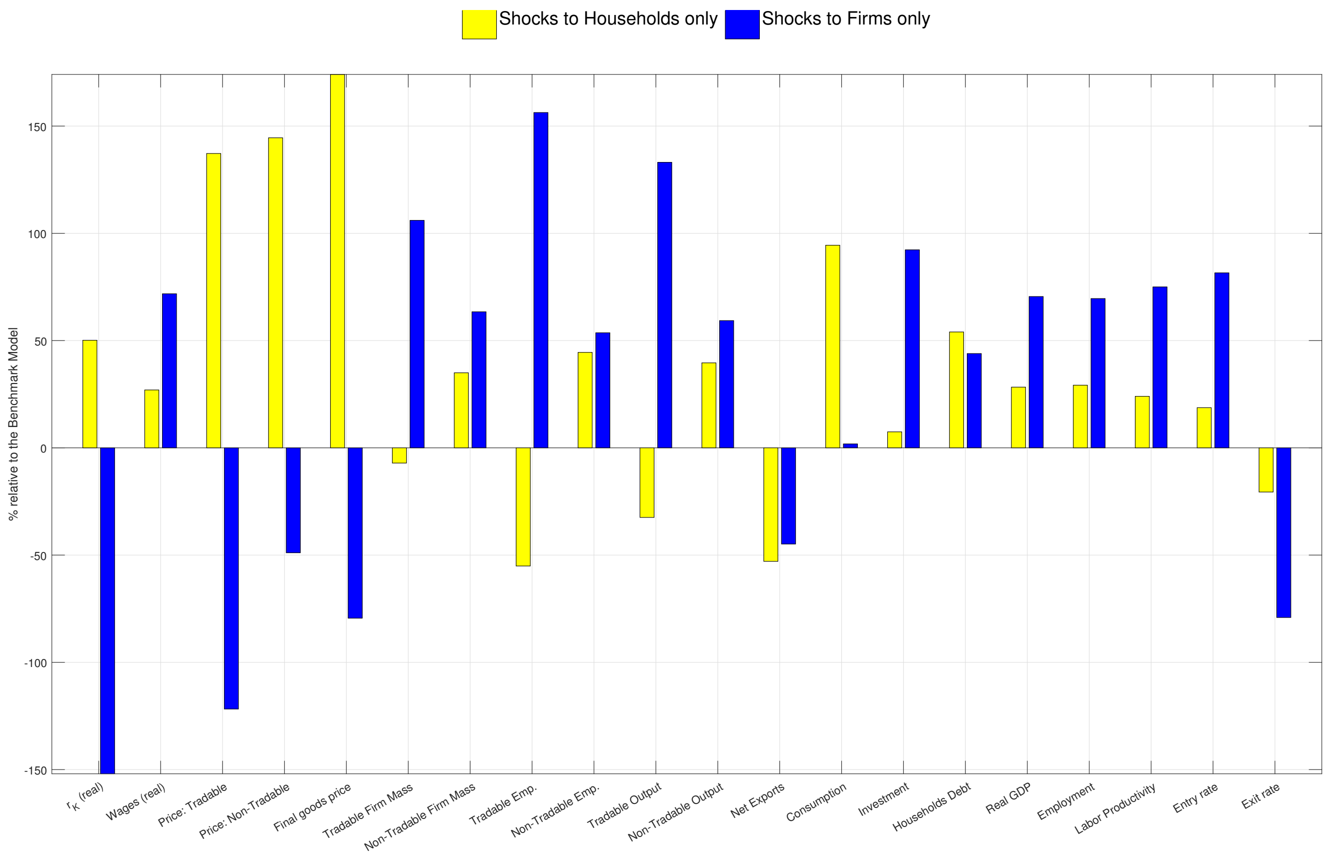
This accounting exercise suggests that over 10 years, shocks to the firms account for the lion's share of movements in output and employment following a credit deregulation: The firm channel accounts for 71% of the GDP increase, 70% for total employment, and 92% for investment, whereas shocks to the households can account for just 28% of the GDP increase over 10 years. The patterns for non-tradable output, total employment, and wages are similar to that of GDP. For aggregate consumption, however, household-side shocks are dominant, accounting for 94% overall, and even larger in the short run. Closely related, the household channel is important for price dynamics: this channel is dominant for the price dynamics of non-tradable goods, domestic tradable goods, and final goods (see Figure 4). For investment, entry and exit, firm-side shocks have shares larger than that for GDP, and firm-side shocks are more important at longer horizons (years 6-10).

The reason why the data identifies the firm-side channel as the main driver of the aggregate effect of credit expansions in the medium term is the fact that output and employment grow gradually after deregulation. This is consistent with the way firms respond to lower borrowing costs because sustained investments induce a gradual increase in firms' capital stock. The household channel, by contrast, induces a rapid rise in output and is therefore more important for the short-run dynamics.

Identification Through Dynamics of Tradable Output. Note that the household channel and the firm channel exhibit opposite responses for tradable goods output: tradable output falls in response to household-only shocks and increases in response to firm-only shocks (as the rental rate of capital falls, firms in both sectors increase capital, employment, and production). The dynamics for net exports follows the pattern of tradable output and drops more in the case of household-only shocks.

This suggests that the dynamics of tradable goods after credit deregulation could provide identification for the household channel versus the firm channel (see Mian and Sufi (2014) and Mian, Sufi, and Verner (2020) for a similar argument). Unfortunately, the BEA, our data source for state-level time series for macro aggregates, only publishes industry-level GDP by state going back to 1997, long after the branch deregulation completed. We thus rely on the state-level tradable and non-tradable output data provided by Mian and Sufi (2014), which goes back to 1963.

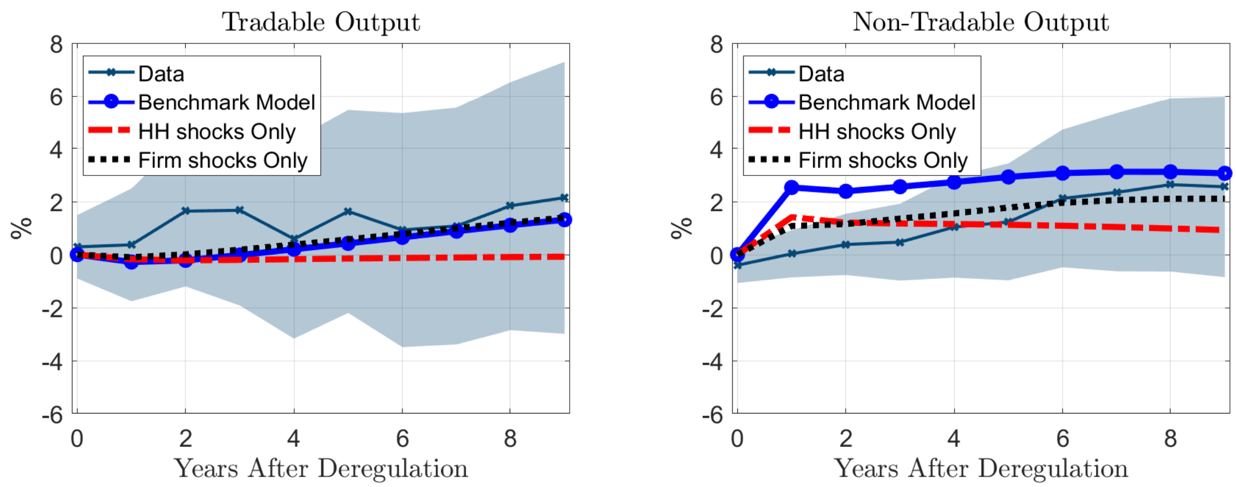
Figure 4: Shocks to Firms vs. Households Only: Other Variables



In these data, tradable output is produced by industries with gross trade higher than $10,000 per worker, or higher than $500M in absolute value, whereas non-tradable output is produced only by retail and restaurants. As a consequence, the tradable sector accounts for 11% of total employment and the non-tradable sector for 20%, with the majority of employment unaccounted for in the output measures.

With this caveat in mind, we estimate impulse responses of the Mian and Sufi (2014) tradable and non-tradable output data to the intrastate branching deregulation following the same strategy as in Section 3. Figure 5 plots the empirical impulse responses against the model counterparts from Figure 2. We find positive point estimates for the responses of both tradable and non-tradable output to the lifting of the intrastate branching restrictions, although the 90% confidence interval includes zero. This is consistent with our finding that firm-side shocks is the main channel for the macroeconomic responses, at least in the medium term. Note also that our model matches the point estimates for the impulse responses quantitatively, even though these impulse response functions were not used in the estimation of the model. We interpret this as auxiliary evidence in support of our model.

Figure 5: Tradable and Non-Tradable Outputs in the Model and Data



Simple AR(1) Shock to the Interest rate. Our benchmark experiment features a highly persistent shock to the interest rate. To understand the role of persistence of interest rate shock, we now vary the persistence by considering the effects of interest rate shocks that follow AR(1) processes. The economy starts from the initial steady state. In period 1 there is an unanticipated shock to the interest rate that decays with persistence ι, so that $\psi_t^H = \iota^{t-1}\psi_1^H$. We consider two cases – one with high persistence ($\iota = 0.95$) and one transitory case ($\iota = 0$). The persistent shock ψ_1^H is 0.35 percentage points on impact—the same as in our estimated ψ^H at year 10. The transitory is three times larger on impact in order to ease the comparison. The impulse responses are compared in Figure 6. All variables are denoted in percent deviations from their corresponding initial steady state values, and we only report the first 20 periods.

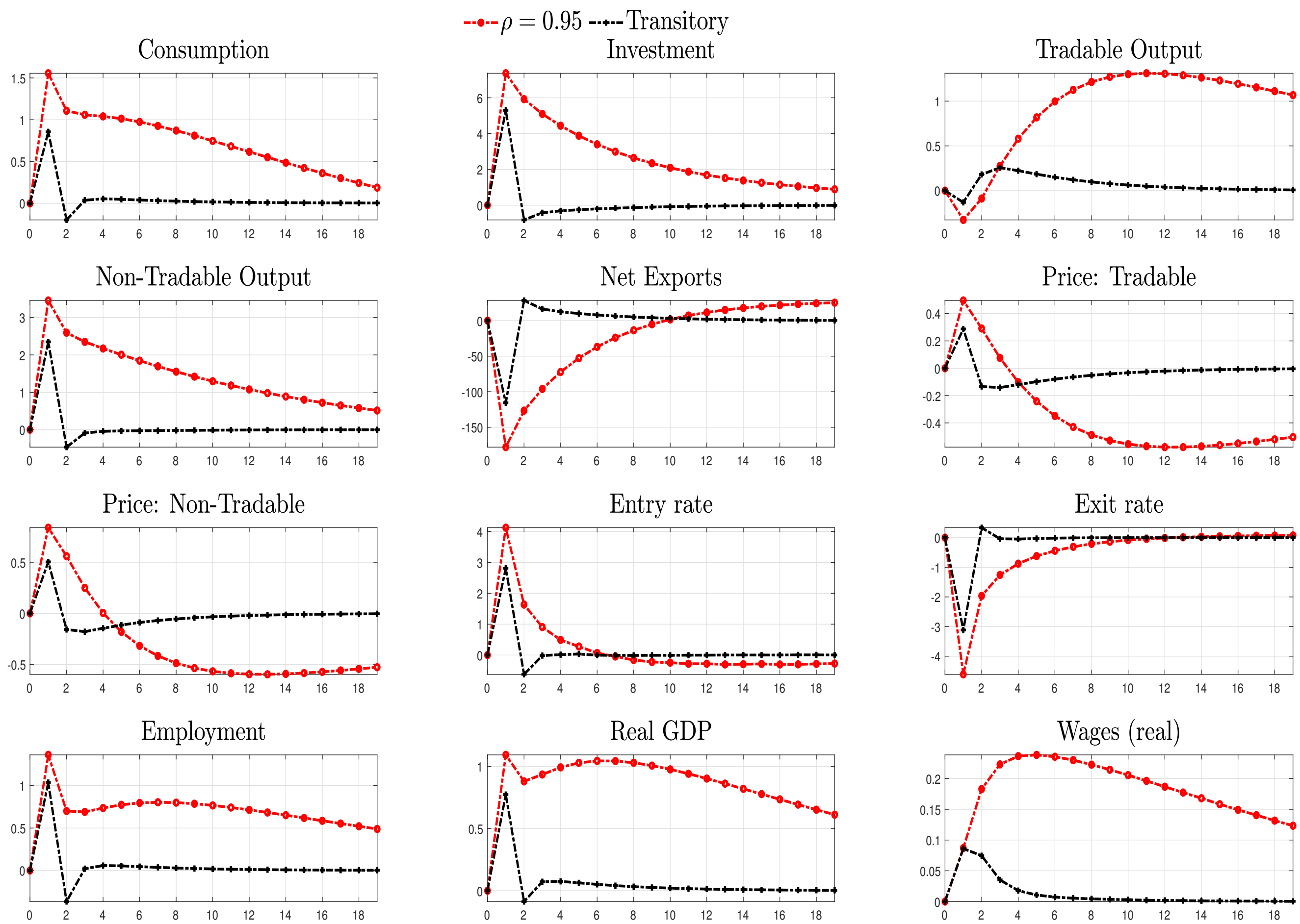
The effects of the persistent AR(1) interest rate shock are qualitatively similar to those of the calibrated interest rate shock shown in Figure 2. Consumption largely rises via the household demand channel, and investment by the firm-side channel. Since interest rate shocks are mean-reverting by design, there are no permanent changes in interest rates, rental rates for capital, firm technology, or any other economic fundamentals in the long run. Consequently, after an initial burst in household debt, households gradually repay the debt burden and reduce their consumer demand. Since firms expect that their profits only increase in the first few periods following shocks, the incentives for long-term investment and for new firms' entry are also relatively transient.[14]

For a transitory interest-rate shock the economic effects are, as one can expect, short lived and propagate through the increases in household and firm debt, as well as through price and wage stickiness. For investment, the initial responses have similar patterns with a slightly smaller magnitude; however, the main difference is that investment quickly reverts in the case of transitory shocks, as the capital stock in the first period builds up and firms have to lower investments when interest rates are back to normal levels.

For the transitory case, a notable feature is that in the short run the initial boom caused by the lower interest rate is immediately followed by a recession. The reason is that prices and wages are stuck on too high levels when the stimulus is over. During the period with low interest rates some firms get to increase wages and domestic prices. However, the period after, when the stimulus is over, prices are too high relative to the price of the tradeable foreign good, whose price is constant. This may cause a short-lived recession despite the fact that capital remains above its steady state level.

[14]Numerically, after about 50 years, the economy gradually converges back to the initial steady state. Along the transition the economy features a slightly lower consumption and non-tradable output/employment, higher tradable goods output/employment, and a slightly higher net export. We solve for the model with a very large number of time periods (800) periods; our results for the first 10 or 20 periods (our main focus) essentially remain the same when we further increase the number of periods.

Figure 6: Model Responses to AR(1) shocks in interest rates: persistent vs. transitory



4.2 Robustness to Parameter Changes

In Appendix C.4, we inspect how changes to our model parameters affect ourmodel impulse responses and decomposition results. We explore, in turn, the effects of varying the Calvo parameters governing wage and price rigidity, of varying the size of capital adjustment costs, of varying the firm entry cost parameters ν and Θ, of varying the elasticity of intertemporal substitution and Frisch elastictity of labor supply, and of varying the intial level of firm debt. As shown in Appendix C.4, our main conclusions are very robust to moderate changes in the values of almost all these parameters. Impulse responses for aggregate variables change a little as parameters vary, but are ultimately very similar across the different parametrizations. Our main decomposition results likewise hold up: firm-side shocks account for about 70% of the responses in GDP, employment, and non-tradable outputs over 10 years; shocks to households are more important for consumption dynamics, and they are more important in the short run; shocks to firms are important for investment and even more important in the longer run. Tradable and non-tradable output have opposite responses, and firm dynamics are affected by both shocks but relatively more by firm-side shocks. The only parameter we consider where results turn out to be more sensitive is the elasticity of intertemporal substitution: when households have lower inter-temporal substitution incentives (higher value of σ), the household-side effects are greatly reduced, and shocks to firms largely account for responses in GDP, employment, although consumption is still mainly accounted by the household side. Intuitively, for interest rate shocks on households to work, it is important that households are able to adjust their consumption path over time.

The limited effect of nominal rigidity on impulse responses reflects the fact that the expansionary credit shock is close to permanent. Consequently, the response of real output due to nominal rigidities is small compared to the response of inflation, because the New Keynesian Phillips curve is relatively steep in response to permanent shocks. In Appendix C.4, we show that higher levels of price rigidity do induce a larger rise of output and employment in the short run after the credit expansion, but that this effect dies out within about 2 years. The lack of importance of nominal wage and price rigidity for results also draws attention to the fact that the key household demand channel in our model is not the usual New Keynesian channel involving nominal rigidities, as discussed in Section 4.1 above. The limited effect of entry and adjustment costs on results likewise reflects the extreme persistence of the shock.

4.3 Other forms of financial or interest rate shocks

In this paper we studied the effects of an expansionary financial shock by modeling the deregulation of intrastate bank branching succinctly as an exogenous decrease in the interest rate faced by households and firms in the state. While this simplified representation of financial shocks follows a long tradition in the business-cycle literature (Neumeyer and Perri, 2005, Smets and Wouters, 2007), it begs some deeper questions about the effects of financial deregulation. First, is it possible to provide microfoundations for how intrastate bank branching deregulation reduced interest rates? Second, does the main effect work through a reduction in interest rates, which is our focus, or are there other effects? In this section, we suggest that the answer to each of these questions is 'yes'.

Microfoundations of interest rate decrease after deregulation. We document that the empirical effects of bank branching deregulation involved increasing debt and lower interest rates on this debt. A standard micro foundation for these effects is that the deregulation caused a rightward shift in the overall supply curve for credit. Intuitively, if each bank branch has an upward sloping supply curve for lending, then removing legal restrictions on new branches forming would shift the market supply curve of lending rightwards, reducing the interest rate on loans and increasing the quantity of debt. In Appendix A.5, we provide a simple formalization of this idea.

Different channels for economic effects of deregulation. A potential concern with our approach is that the macroeconomic effects of banking deregulation might have operated through channels entirely different from the interest rate channel we study. For instance, many models of financial frictions assume that agents face quantity rationing in the amount they can borrow (e.g., via a collateral constraint), and banking deregulation might have influenced the extent of such credit rationing. Alternatively, banking deregulation could have introduced more generous mortgage contracts, leading to higher house prices, which might, in turn, affect household demand through wealth effects or by relaxing collateral constraints.

While these other channels are undoubtedly worth exploring, we view the primacy of the interest rate channel as a useful working assumption for several reasons. First, our empirically estimated impulse responses in Figure 1 show that loan interest rates declined after deregulation, allowing us to quantify the size of the financial shock. In contrast, reductions in credit rationing are harder to observe and measure directly. Second, it is straightforward to imagine that branching deregulation, by encouraging the entry of new bank branches, increased loan supply and lowered interest rates. It is less obvious that it would have loosened collateral constraints. Third, the interest rate channel in our model predicts a slightly larger increase in household debt than what we estimate in the data, suggesting that the observed rise in debt can be explained without invoking changes in credit rationing. Fourth, many effects of reduced credit rationing resemble those of lower interest rates, as both operate as wedges in the first-order conditions for households and firms. Thus, relaxing credit rationing would likely amplify the effects of the interest rate channel rather than introduce qualitatively different dynamics. For these reasons, we focus on interest rates rather than modeling changes in credit rationing.

4.4 Additional Channels for Amplification

Our main finding—that most of the effects of credit deregulation worked through the firm channel—would be strengthened if we were to incorporate two standard extensions to our model: working capital constraints and endogenous technical change.

Working capital constraints on hiring and other inputs. Working-capital constraints would increase firms' reliance on external capital. A fall in firms' borrowing costs would then lead to a larger rise in output as it would be easier for firms to finance their working capital and hire more labor.[15] We conjecture that such extension would lead to a greater role for the firm-side channel relative to the household channel, particularly in the short run.

Long-run technological change. An alternative mechanism could be the effect of the credit expansion on long-run technological progress. Our model does not feature endogenous technological change, but could be extended along the lines of the endogenous growth literature so that the rate of technological progress depends on firm investments in R&D. Queralto (2020) develops a model where entrepreneurs finance R&D by borrowing from banks, and R&D leads to endogenous growth. An increase in the cost of borrowing reduces entrepreneurs' R&D expenditures, reducing the growth rate and leading to a permanent decrease in output. Our model could readily be extended to incorporate this channel. Doing so would presumably not overturn our conclusions that the firm-side channel accounts for roughly half of the output increase in the short term, but would strengthen the long-run impact of this channel.

This mechanism could be even more potent if investment in R&D is risky. For instance, Cui, Xie, and Zhang (2024) study how interest rates affect the risk taking behavior of firms. If a decrease in interest rates leads to a greater willingness by firms to take on risks, then this could

[15] If firms have variable capital utilisation, then in this case it is also possible that capital utilization could increase when the marginal product of capital increases.

increase their incentive to engage in risky R&D investment as opposed to safer investments in physical capital, potentially increasing long run growth.

5 Conclusion

This paper explores, both theoretically and empirically, the effects of changes in credit conditions on the real economy. We ask whether these effects work primarily through households or firms. To study this issue we focus on the bank deregulation in the U.S. in the 1980s and interpret deregulation as a fall in borrowing rates. To derive theoretical predictions we propose an open-economy New-Keynesian model with heterogeneous firms and households. The model incorporates tradable and non-tradable goods sectors, nominal frictions in both goods prices and in wages, and endogenous firm entry and exit. Following a large literature in finance, we view the state-level bank deregulation as a staggered natural experiment. We use the causal evidence from these credit market deregulation episodes to estimate the model. The model generates impulse responses for key economic variables consistent with causal empirical evidence.

Using our model as a laboratory, we quantify the relative importance of household-side and firm-side effects when there are changes in interest rates. The firm-side channel accounts for around 70% of the cumulative response of most variables in the first 10 years after the shock, while the household channel explains roughly the remaining 30%. The household channel is more relevant in the short run as it accounts for about half the short-run output movements and most of the consumption dynamics. The identification of the firm-side shocks as the main driver of medium-term dynamics stems from the fact that macroeconomic aggregates such as output and employment expanded gradually following the credit expansions—consistent with sustained investments and growing capital stocks associated with firm-side shocks. Lower borrowing costs for households induce instead an immediate increase in consumption but not a gradual increase.[16]

We have studied the effects of credit shocks at the state level and therefore abstract from potential spillover effects across state borders. The main mechanism works through firm investments and households' demand for tradable and nontradable goods and the quantitative effects could potentially be different for changes in credit conditions that apply to the U.S. as a whole. We leave it for future work to extend the analysis to shocks that affect the entire U.S.

Our decomposition into household and firm channels contrasts with some previous work including (Mian, Sufi, and Verner, 2020), who studied the relative importance of demand and supply channels for the transmission of credit shocks. The task of decomposing supply and demand effects in a large dynamic model such as ours is not straightforward, since, for instance, the effect of the shock on firms' investment can be viewed as affecting both the demand for capital goods and the subsequent supply of output through capital accumulation. We leave this task to future work. At the same time, our results suggest that any such decomposition is likely to find an important role for both demand and supply channels. This is because the effects that we ascribe to household shocks operate through consumer demand, and therefore presumably would mainly qualify as a demand channel, whereas the effects on firms are a mixture of demand-side effects (due to investment demand) and supply-side effects (due to capital accumulation). Since we have found that the household channel is quantitatively quite important in the short run, this is suggestive that demand-side effects are likely to be significant. On the other hand, in the long run, the effect of capital accumulation is important in our model, highlighting the importance of the supply side.

[16]Preferences incorporating habit formation might be an alternative mechanism that could induce more gradual consumption responses, thereby strengthening the role of the household channel even in the medium term.

References

Aguiar, Mark, Mark Bils, and Corina Boar. 2024. "Who Are the Hand-to-Mouth?" *The Review of Economic Studies* 92 (3):1293–1340.

Arellano, Cristina, Yan Bai, and Patrick J Kehoe. 2019. "Financial frictions and fluctuations in volatility." *Journal of Political Economy* 127 (5):2049–2103.

Atkeson, Andrew and Patrick J Kehoe. 2005. "Modeling and measuring organization capital." *Journal of Political Economy* 113 (5):1026–1053.

Basu, Susanto and John G Fernald. 1997. "Returns to scale in US production: Estimates and implications." *Journal of political economy* 105 (2):249–283.

Basu, Susanto and Christopher L House. 2016. "Allocative and remitted wages: New facts and challenges for keynesian models." In *Handbook of macroeconomics*, vol. 2. Elsevier, 297–354.

Beraja, Martin, Erik Hurst, and Juan Ospina. 2019. "The aggregate implications of regional business cycles." *Econometrica* 87 (6):1789–1833.

Bergin, Paul R, Ling Feng, and Ching-Yi Lin. 2018. "Firm entry and financial shocks." *The Economic Journal* 128 (609):510–540.

Bernanke, Ben S, Mark Gertler, and Simon Gilchrist. 1999. "The financial accelerator in a quantitative business cycle framework." *Handbook of macroeconomics* 1:1341–1393.

Bilbiie, Florin O. 2008. "Limited asset markets participation, monetary policy and (inverted) aggregate demand logic." *Journal of economic theory* 140 (1):162–196.

Boar, Corina, Denis Gorea, and Virgiliu Midrigan. 2021. "Liquidity Constraints in the U.S. Housing Market." *The Review of Economic Studies* 89 (3):1120–1154.

Boerma, Job. 2014. "Openness and the (inverted) aggregate demand logic." *De Nederlandsche Bank Working Paper* .

Buffie, Edward F and Luis-Felipe Zanna. 2018. "Limited asset market participation and determinacy in the open economy." *Macroeconomic Dynamics* 22 (8):1937–1977.

Burstein, Ariel and Christian Hellwig. 2008. "Welfare costs of inflation in a menu cost model." *The American Economic Review* 98 (2):438–443.

Cagetti, Marco, Mariacristina De Nardi et al. 2006. "Entrepreneurship, frictions, and wealth." *Journal of political Economy* 114 (5):835–870.

Caldara, Dario, Matteo Iacoviello, Patrick Molligo, Andrea Prestipino, and Andrea Raffo. 2020. "The economic effects of trade policy uncertainty." *Journal of Monetary Economics* 109:38–59.

Chen, Peter, Loukas Karabarbounis, and Brent Neiman. 2017. "The global rise of corporate saving." *Journal of monetary economics* 89:1–19.

Cugat, Gabriela. 2022. "Emerging markets, household heterogeneity, and exchange rate policy." *Northwestern University, working paper* .

Cui, Wei, Richard Cong Xie, and Renbin Zhang. 2024. "Risk-taking with Financing Constraints." Mimeo, UCL.

Galí, Jordi. 2015. *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications.* Princeton University Press.

Galí, Jordi, J David López-Salido, and Javier Vallés. 2007. "Understanding the effects of government spending on consumption." *Journal of the european economic association* 5 (1):227–270.

Gali, Jordi and Tommaso Monacelli. 2005. "Monetary policy and exchange rate volatility in a small open economy." *The Review of Economic Studies* 72 (3):707–734.

Gertler, Mark, Simon Gilchrist, and Fabio M Natalucci. 2007. "External constraints on monetary policy and the financial accelerator." *Journal of Money, credit and Banking* 39 (2-3):295–330.

Hagedorn, Marcus. 2023. "Local Determinacy in Incomplete-Markets Models." CEPR working paper DP18642.

Hopenhayn, Hugo A. 1992. "Entry, exit, and firm dynamics in long run equilibrium." *Econometrica: Journal of the Econometric Society* :1127–1150.

Jayaratne, Jith and Philip E Strahan. 1996. "The finance-growth nexus: Evidence from bank branch deregulation." *The Quarterly Journal of Economics* 111 (3):639–670.

Jermann, Urban and Vincenzo Quadrini. 2012. "Macroeconomic effects of financial shocks." *American Economic Review* 102 (1):238–271.

Jorda, Oscar. 2005. "Estimation and Inference of Impulse Responses by Local Projections." *American Economic Review* 95 (1):161–182.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante. 2018. "Monetary policy according to HANK." *American Economic Review* 108 (3):697–743.

Kaplan, Greg and Giovanni L. Violante. 2014. "A Model of the Consumption Response to Fiscal Stimulus Payments." *Econometrica* 82 (4):1199–1239.

Kehoe, Patrick J, Finn E Kydland, and David K Backus. 1992. "International Real Business Cycles." *Journal of Political Economy* .

Kehoe, Patrick J, Pierlauro Lopez, Virgiliu Midrigan, and Elena Pastorino. 2020. "On the importance of household versus firm credit frictions in the Great Recession." *Review of Economic Dynamics* 37:S34–S67.

Koby, Yann and Christian Wolf. 2020. "Aggregation in heterogeneous-firm models: Theory and measurement." *Princeton, Manuscript* .

Kroszner, Randall S. and Philip E. Strahan. 2014. *Regulation and Deregulation of the U.S. Banking Industry: Causes, Consequences, and Implications for the Future*. University of Chicago Press, 485–543.

Leary, Mark T and Roni Michaely. 2011. "Determinants of dividend smoothing: Empirical evidence." *The Review of Financial Studies* 24 (10):3197–3249.

Lee, Yoonsoo and Toshihiko Mukoyama. 2015. "Entry and exit of manufacturing plants over the business cycle." *European Economic Review* 77:20–27.

Melcangi, Davide. 2024. "Firms' precautionary savings and employment during a credit crisis." *American Economic Journal: Macroeconomics* 16 (1):356–386.

Mian, Atif and Amir Sufi. 2014. "What Explains the 2007–2009 Drop in Employment?" *Econometrica* 82 (6):2197–2223.

Mian, Atif, Amir Sufi, and Emil Verner. 2020. "How Does Credit Supply Expansion Affect the Real Economy? the productive capacity and household demand channels." *The Journal of Finance* 75 (2):949–994.

Morgan, Donald P., Bertrand Rime, and Philip E. Strahan. 2004. "Bank Integration and State Business Cycles." *The Quarterly Journal of Economics* 119 (4):1555–1584.

Nakamura, Emi and Jón Steinsson. 2013. "Price rigidity: Microeconomic evidence and macroeconomic implications." *Annu. Rev. Econ.* 5 (1):133–163.

Nakamura, Emi and Jon Steinsson. 2014. "Fiscal stimulus in a monetary union: Evidence from US regions." *American Economic Review* 104 (3):753–92.

Neumeyer, Pablo A and Fabrizio Perri. 2005. "Business cycles in emerging economies: the role of interest rates." *Journal of monetary Economics* 52 (2):345–380.

Nickell, Stephen. 1981. "Biases in Dynamic Models with Fixed Effects." *Econometrica* 49 (6):1417–26.

Ottonello, Pablo and Thomas Winberry. 2020. "Financial heterogeneity and the investment channel of monetary policy." *Econometrica* 88 (6):2473–2502.

Park, Ki Young. 2012. "Interstate Banking Deregulation and Bank Loan Commitments." *The B.E. Journal of Macroeconomics* 12 (2).

Quadrini, Vincenzo. 2000. "Entrepreneurship, saving, and social mobility." *Review of Economic Dynamics* 3 (1):1–40.

Queralto, Albert. 2020. "A model of slow recoveries from financial crises." *Journal of Monetary Economics* 114:1–25.

Reiter, Michael, Tommy Sveen, and Lutz Weinke. 2013. "Lumpy investment and the monetary transmission mechanism." *Journal of Monetary Economics* 60 (7):821–834.

Restuccia, Diego and Richard Rogerson. 2008. "Policy Distortions and Aggregate Productivity with Heterogeneous Plants." *Review of Economic Dynamics* 11 (4):707–720.

Rice, Tara and Philip E. Strahan. 2010. "Does Credit Competition Affect Small-Firm Finance?" *The Journal of Finance* 65 (3):861–889.

Rossi-Hansberg, Esteban and Mark LJ Wright. 2007. "Establishment size dynamics in the aggregate economy." *American Economic Review* 97 (5):1639–1666.

Smets, Frank and Rafael Wouters. 2007. "Shocks and frictions in US business cycles: A Bayesian DSGE approach." *American economic review* 97 (3):586–606.

Sylla, Richard, John B. Legler, and John J. Wallis. 1987. "Banks and State Public Finance in the New Republic: The United States, 1790-1860." *The Journal of Economic History* 47 (2):391–403.

Teulings, Coen N. and Nikolay Zubanov. 2014. "Is Economic Recovery A Myth? Robust Estimation of Impulse Responses." *Journal of Applied Econometrics* 29 (3):497–514.

Online Appendix: Not Intended for Publication

A Equilibrium and Numerical Solution

In this section we define the equilibrium of the economy and characterize the solution to each agent's problem.

A.1 Additional Model Elements

Nominal wage rigidity. In line with our small open-economy assumption, the labor market is closed in the sense that there is no labor migration across state borders. We assume that wages feature nominal rigidity along the lines of Galí (2015, Chapter 6). Each household is made up of a continuum of workers who each produce a differentiated labor supply indexed by $j \in [0, 1]$. Each worker j sets a nominal wage at which she will work, and firms choose how much of the worker's labor to demand at this wage. Worker nominal wage setting is subject to a Calvo-type rigidity; that is, at the start of period t, a worker can set a new nominal wage with probability $1 - \theta_w$, and with probability θ_w she must continue to work at the nominal wage from the previous period.

If a worker j is free to set a new wage in period t, then she is assumed to set this wage to maximize the present discounted value of the worker's contribution to the household's utility. Therefore, the new wage $w^{\star}_{j,t}$ solves:

$$\max \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{w^{\star}_{j,t} N_{j,t+k}}{P_{t+k}} - \phi \frac{N_{j,t+k}^{1+\upsilon}}{1+\upsilon} \right), \tag{A.1}$$

where $N_{j,t+k}$ is the demand for j's labor in period $t + k$, given the wage rate $w^{\star}_{j,t}$.[17]

Nominal price rigidity for retailer. Retailers set their price p_i to maximize present discounted profits, given domestic and foreign final goods firms' respective demand function for their variety. However, they face Calvo pricing frictions; only a fraction $1 - \vartheta_P$ of retailers can change their price each period, as in standard literature (such as Galí (2015, Chapter 3)).

As shown in Galí (2015, Chapter 3), these assumptions, combined with the first-order conditions of the final goods firm, imply that the price of each composite good Y_S, $S \in \{N, H\}$, satisfies:

$$\left(\frac{P_{S,t}}{P_{S,t-1}} \right)^{1-\eta} = \vartheta_P + (1 - \vartheta_P) \left(\frac{P^{\star}_{S,t}}{P_{S,t-1}} \right)^{1-\eta}, \tag{A.2}$$

[17] We will not introduce notation to specify if worker j is a member of a spender or saver household because, as we will show later, a worker j will choose the same $w_{j,t}$ and $N_{j,t}$ regardless of which of the two household types she belongs to.

where $P^{\star}_{S,t}$ is the price chosen by firms that reset prices in period t. This is given by the first-order condition:

$$\sum_{k=0} \vartheta_P^k \mathbb{E}_t \left[\prod_{j=0}^{k} M_{t+j} \left(\frac{P^{\star}_{S,t}}{P_{S,t+k}} \right)^{-\eta} Y_{S,t+k} \left(\frac{1}{P_{t+k}} \right) \left(P^{\star}_{S,t} - \frac{\eta}{\eta-1} P_{W,S,t+k} \right) \right] = 0.$$

Total nominal profits of the retailers Π_t^R are as follows:

$$\Pi_t^R = \left(\sum_{S \in \{N,H\}} P_{S,t} Y_{S,t} - P_{W,S,t} Y_{W,S,t} \right). \tag{A.3}$$

where $Y_{W,S,t} = \int_{I \in S} y_i di$ is the intermediate goods output of sector $S \in \{N; H\}$, which is used as an input for the retailers in that sector. By integrating the demand curves for retailers we find that, $Y_{W,S,t} = Y_{S,t} \left(\mathcal{D}_{S,t} \right)^{-\eta}$, where $\mathcal{D}_{S,t}$ is a measure of price dispersion, $\mathcal{D}_{S,t} = \left(\int p_i^{-\eta} di \right)^{\frac{-1}{\eta}} / P_{S,t}$. Then, analogously with $P_{S,t}$, $\mathcal{D}_{S,t}$ evolves according to the following expression:

$$\left(\frac{P_{S,t} \mathcal{D}_{S,t}}{P_{S,t-1} \mathcal{D}_{S,t-1}} \right)^{-\eta} = \vartheta_P + (1 - \vartheta_P) \left(\frac{P^{*}_{S,t}}{P_{S,t-1} \mathcal{D}_{S,t-1}} \right)^{-\eta}. \tag{A.4}$$

A.2 Equilibrium

We first define the equilibrium of the economy.

Market Clearing Conditions. The following equations ensure that all markets clear:

- The market for labor of each type j clears:

$$\sum_{S \in \{H,N\}} M^W_{S,t} \int I_{\{z \geq z^{\star}(S, \mathbf{X}_t)\}} n_j^{\star}(S,z) \mu_{S,t}(z) dz = N_{j,t} \tag{A.5}$$

- The market for capital goods clears:

$$\sum_{S \in \{H,N\}} M^W_{S,t} \int I_{\{z \geq z^{\star}(S, \mathbf{X}_t)\}} k^{\star}(S,z) \mu_t(S,z) dz = K_t. \tag{A.6}$$

- The market for domestically produced tradable goods clears (total supply $Y_{H,t}$ equals total demand $Y^d_{H,t} + Y^{*}_{H,t}$):

$$Y_{H,t} = Y^d_{H,t} + Y^{*}_{H,t} \tag{A.7}$$

- The market for domestically produced non-tradable goods clears ($Y^d_{N,t}$ is the total demand):

$$Y_{N,t} = Y^d_{N,t} = \frac{(1-\gamma)P_t Y^d_t}{P_{N,t}} \tag{A.8}$$

- The market for final goods clears (Y^d_t is the total supply):

$$Y^d_t = C_t + I_t + \sum_{S\in\{H,N\}} c^F M^W_{S,t} + \sum_{S\in\{H,N\}} \nu M^e_{S,t}\left(\frac{M^e_{S,t}}{M^W_{S,t-1}}\right)^{\Theta} \tag{A.9}$$

Equilibrium Definition. Denote the total mass of continuing firms in sector S as $M^W_{S,t}$, denote the pdf for continuing firms as $\mu_{S,t}(z)$, and denote the total mass of new entrants at the end of each period t as $M^e_{S,t}$. The competitive equilibrium consists of prices $\{P_{Nt}, P_t, w_t\}_{t=0}^{\infty}$ and value functions of $W(z;\mathbf{X}_t, S)$ such that,

- C^L_t, C^S_t, B^H_t and $N_{j,t}$ for each j solve the spender's and saver's problems.
- The interest rate faced by savers is given by (2).
- The discount factor of firms is given by (5), and the aggregate profit of firms is given by (13), and retailers' profits are given by (A.3).
- For Y^d_t and $Y^d_{S,t}$ of each $S \in \{N, H\}$, final goods firms maximize their period profits.
- $P_{N,t}$, $P_{H,t}$ and w_t evolve according to the Calvo price-setting and wage-setting conditions (A.2) and (A.1).
- Aggregate investment I_t solves the optimization problem of the capital goods firm (6). The total profits of capital goods firms are given by (4). Total capital stock evolves according to the law of motion in (3).
- Intermediate goods firms' value function $W(\cdot)$ evolves according to the Bellman equation (9). For each intermediate goods producer i, its profit $\pi^W_{i,t}$, output $y_{i,t}$ and input uses $k_{i,t}$, $n_{i,j,t}$ (for each j) are given by the solution of its within-period problem in (10). $z^\star_{S,t}$ and $M^E_{S,t}$ are consistent with the free entry and exit conditions (11) and (12).
- The distribution and total mass of intermediate goods firms evolves according to (A.10) and (A.11) as follows:

  The measure of intermediate goods producers with any productivity $\hat{z}$, $\hat{z} \geq z^\star(\mathbf{X}_{S,t+1})$, evolves according to:

$$
\begin{aligned}
& M^{W}_{S,t+1}\int I_{\{z'\geq z^{\star}(\mathbf{X}_{S,t+1})\}} I_{\{z'\leq \hat{z}\}}\mu_{S,t+1}(z')dz' \\
= \; & M^{W}_{S,t}\rho_z \int I_{\{z\geq z^{\star}(\mathbf{X}_{S,t})\}} I_{\{z\leq \hat{z}\}} I_{\{z\geq z^{\star}(\mathbf{X}_{S,t+1})\}}\mu_{S,t}(z)dz \\
& + \left[M^{W}_{S,t}(1-\rho_z)+M^{e}_{S,t+1}\right]\int I_{\{z'\leq \hat{z}\}} I_{\{z'\geq z^{\star}(\mathbf{X}_{S,t+1})\}} dG_z(z'). \qquad \text{(A.10)}
\end{aligned}
$$

and the measure of all active and exiting firms evolve according to:

$$
\begin{aligned}
M^{W}_{S,t+1} &= M^{W}_{S,t}\rho_z \int I_{\{z\geq z^{\star}(\mathbf{X}_{S,t})\}} I_{\{z\geq z^{\star}(\mathbf{X}_{S,t+1})\}}\mu_{S,t}(z)dz \\
&\quad + \left[M^{W}_{S,t}(1-\rho_z)+M^{e}_{S,t+1}\right]\times\int I_{\{z'\geq z^{\star}(\mathbf{X}_{S,t+1})\}} dG_z(z') \\
M^{Exit}_{S,t+1} &= M^{W}_{S,t}\rho_z \int I_{\{z\geq z^{\star}(\mathbf{X}_{S,t})\}} I_{\{z\leq z^{\star}(\mathbf{X}_{S,t+1})\}}\mu_{S,t}(z)dz \\
&\quad + \left[M^{W}_{S,t}(1-\rho_z)+M^{e}_{S,t+1}\right]\times\int I_{\{z'\leq z^{\star}(\mathbf{X}_{S,t+1})\}} dG_z(z') \qquad \text{(A.11)}
\end{aligned}
$$

- Total firm assets evolve according to (14), with dividend payments given by ΞY^d_t.
- Labor, capital, non-tradable goods markets, and final goods markets clear, according to Eqs. (A.5), (A.6),(A.9) and (A.8).

A.3 Characterization for the Economy

We next characterize the decision rules of the agents in the economy.

A.3.1 Dynamics of the Number of Intermediate Goods Firms, and Their Average Productivity

To better describe the dynamics of average productivity and the total measure of intermediate goods producers, we first define the average TFP of active intermediate goods producers as $\overline{z}_t = \int_{z>z^{\star}_t} z\mu_t(z)dz$. The exact dynamics of $\overline{z}_t$ and the total mass of intermediate goods producers M^W_t are described by the following proposition.[18]

Proposition 1. *Suppose that Assumption 1 holds. Let*

$$
j_t = \max\left[\left\{k\in\{0,1,...,t-1\}\middle| z^{\star}_k > z^{\star}_t \;\; or \;\; z^{\star}_k \geq z^{\star}_i,\ \forall i<k\right\}\cup\{0\}\right] \qquad \text{(A.12)}
$$

[18]For convenience, we omit the subscript S for sector in the statement of the proposition.

Then the values of $\overline{z}_t$ *and* M_t^W *evolve over time according to the following equations:*

$$\overline{z}_t = \frac{\xi z_t^\star}{\xi-1} + \frac{\rho_z^{t-j_t} M_{j_t}^W}{M_t^W}\left(\overline{z}_{j_t} - \frac{\xi \min\{z_t^\star; z_{j_t}^\star\}}{\xi-1}\right)$$

$$M_t^W = \rho_z^{t-j_t} M_{j_t}^W \min\left\{1; \left(\frac{z_t^\star}{z_{j_t}^\star}\right)^{-\xi}\right\} + (z_t^\star)^{-\xi}\sum_{i=1}^{t-j_t}\rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e)$$

Proof. See Appendix A.4.1. □

Intuitively, there are two facts are important for the analytical results. First, we assume that a firm's productivity does not change in the next period with probability ρ_z, and with probability $1-\rho_z$, the firm draws a new productivity from the same, exogenous distribution g). Second, intermediate good firms are perfectly competitive, so that the firm's optimal decision is relatively simple (maximizing static profits for a given z, and deciding on exits or not). As a result, the dynamics of optimal cutoffs and the average TFP of active firms are relatively easy and tractable.

A.3.2 Closed-Form Solution for Entry and Exit

In our model, heterogenous firms have endogenous entry and exit behavior. We find that convergence of the computation is vastly improved when we use the following closed-form solution for the free entry and free exit conditions. In particular, we can solve for the optimal cutoffs and the mass of entrants, $z_t^\star$ and M_t^e.

Proposition 2. $z_t^\star$ *and* M_t^e *satisfy:*

$$\nu\left(\frac{M_{S,t}^e}{M_{S,t-1}^W}\right)^\Theta = \sum_{j=0}^{\infty}\mathcal{A}_{t,j}\left(\frac{z_{t,j}^{\star-(\xi-1)}}{\xi-1}\right) \tag{A.13}$$

and

$$z_t^\star = \frac{c_F - M_t(1-\rho_z)\sum_{j=0}^{\infty}\mathcal{A}_{t+1,j}\left(\frac{z_{t+1,j}^{\star-(\xi-1)}}{\xi-1}\right)}{(1-\rho_Z M_t)A_t} - \left(\frac{1}{(1-\rho_z M_t)A_t}\right)\sum_{j=1}^{\infty}\mathcal{A}_{t,j}\left(z_{t,j}^\star - z_{t+1,j-1}^\star\right) \tag{A.14}$$

where

$$A_{S,t} = \frac{\overline{\pi}_{S,t}^W}{1-\rho_z M_t} \tag{A.15}$$

$$z_{S,t,j}^\star = \max\{z_{S,\tau}^\star\}_{\tau=t}^{\tau=t+j} \tag{A.16}$$

$$\mathcal{A}_{t,j} = (1-\rho_Z M_{t+j})A_{t+j}\frac{\Pi_{k=0}^{k=j}(M_{t+k}\rho_z)}{M_{t+j}\rho_z} \tag{A.17}$$

Proof. See Appendix A.4.2 for proofs and more details. □

A.3.3 Solving the Household's Problem

We then consider the optimality conditions of the households. Let $\Gamma\lambda_t^S P_t$ denote the marginal utility of consumption for savers at time t. Then, we have:

$$\lambda_t^S P_t = \left[C_t^S - \phi \int_{j=0}^{1} \frac{N_{j,t}^{1+\upsilon}}{1+\upsilon} \mathrm{d}j\right]^{-\sigma} \tag{A.18}$$

$$\lambda_t^S - \beta(1+r_t^H)E_t\lambda_{t+1}^S = 0. \tag{A.19}$$

A.3.4 Retail Price Setting

Recall that retail prices of each composite good evolve according to (A.2), which we repeat for convenience:

$$\left(\frac{P_{S,t}}{P_{S,t-1}}\right)^{1-\eta} = \vartheta_P + (1-\vartheta_P)\left(\frac{P_{S,t}^{\star}}{P_{S,t-1}}\right)^{1-\eta}$$

where $P_{S,t}^{\star}$ satisfies the first-order condition:

$$\sum_{k=0} \vartheta_P^k \mathbb{E}_t \left[\prod_{j=0}^{k} M_{t+j} \left(\frac{P_{S,t}^{\star}}{P_{S,t+k}}\right)^{-\eta} Y_{S,t+k} \left(\frac{1}{P_{t+k}}\right) \left(P_{S,t}^{\star} - \frac{\eta}{\eta-1} P_{W,S,t+k}\right)\right] = 0.$$

To simplify this, we let:

$$M_t\mathcal{A}_{P,S,t} = \frac{\eta}{\eta-1} \sum_{k=0} \vartheta_P^k \mathbb{E}_t \left[\prod_{j=0}^{k} M_{t+j} P_{S,t+k}^{\eta} \left(\frac{1}{P_{t+k}}\right) Y_{S,t+k} P_{W,S,t+k}\right]$$

$$M_t\mathcal{B}_{P,S,t} = \sum_{k=0} \vartheta_P^k \mathbb{E}_t \left[\prod_{j=0}^{k} M_{t+j} P_{S,t+k}^{\eta} \left(\frac{1}{P_{t+k}}\right) Y_{S,t+k}\right].$$

Then, we can write the first-order condition as:

$$\mathcal{B}_{P,S,t} P_{S,t}^{\star} - \mathcal{A}_{P,S,t} = 0$$

or

$$P_{S,t}^{\star} = \frac{\mathcal{A}_{P,S,t}}{\mathcal{B}_{P,S,t}}.$$

Substituting this into the equation for the evolution of aggregate composite good prices, we find that these prices must satisfy:

$$\left(\frac{P_{S,t}}{P_{S,t-1}}\right)^{1-\eta} = \vartheta_P + (1-\vartheta_P)\left(\frac{\mathcal{A}_{P,S,t}}{\mathcal{B}_{P,S,t} P_{S,t-1}}\right)^{1-\eta}. \tag{A.20}$$

That is, for any given t, when adjusting prices, all retailers in sector S choose the same price, $P^{\star}_{S,t} = \frac{\mathcal{A}_{P,S,t}}{\mathcal{B}_{P,S,t}}$.

The equations above also imply that $\mathcal{A}_{P,S,t}$ and $\mathcal{B}_{P,S,t}$ satisfy:

$$\mathcal{A}_{P,S,t} = \left(\frac{\eta}{\eta-1}\right) P^{\eta}_{S,t} P^{-1}_{t} Y_{S,t} P_{W,S,t} + \vartheta_P M_{t+1} \mathbb{E}_t \mathcal{A}_{P,,S,t+1}$$

$$\mathcal{B}_{P,S,t} = P^{\eta}_{S,t} P^{-1}_{t} Y_{S,t} + \vartheta_P M_{t+1} \mathbb{E}_t \mathcal{B}_{P,S,t+1}.$$

A.3.5 Wage Setting

Following the arguments of (Galí, 2015, Chapter 6), it is straightforward to show that firm i's demand for labor j will satisfy:

$$n_{i,j,t} = \left(\frac{w_{j,t}}{w_t}\right)^{-\epsilon_w} n_{i,t}$$

where w_t is an aggregate wage index:

$$w_t = \left(\int_0^1 w_{j,t}^{1-\epsilon_w} \mathrm{d}j\right)^{\frac{1}{1-\epsilon_w}}$$

Aggregating across intermediate goods firms, we therefore have:

$$N_{j,t} = \left(\frac{w_{j,t}}{w_t}\right)^{-\epsilon_w} N_t = w_{j,t}^{-\epsilon_w} \hat{w}_t$$

where $\hat{w}_t = w_t^{\epsilon_w} N_t$ is an aggregate variable, and

$$N_t = \int_{i \in I_N cup I_H} n_{i,t} di = \int_{i \in I_N \cup I_H} \left(\int_0^1 n_{i,j}^{\frac{\epsilon_w - 1}{\epsilon_w}} \mathrm{d}j\right)^{\frac{\epsilon_w}{\epsilon_w - 1}} di$$

Recall that a worker j who is free to choose their wage in period t sets $w_{j,t} \equiv w^{\star}_{j,t}$ to maximize (A.1):

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{w^{\star}_{j,t} N_{j,t+k}}{P_{t+k}} - \phi \frac{N_{j,t+k}^{1+\upsilon}}{1+\upsilon}\right),$$

Substituting in the labor demand function, this is:

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{w_{j,t}^{\star 1-\epsilon_w} \hat{w}_{t+k}}{P_{t+k}} - \phi \frac{\hat{w}_{t+k}^{1+\upsilon} w_{j,t}^{\star -\epsilon_w(1+\upsilon)}}{1+\upsilon}\right),$$

The first order condition for optimization is:

$$0 = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{(1-\epsilon_w) w_{j,t}^{\star -\epsilon_w} \hat{w}_{t+k}}{P_{t+k}} + \epsilon_w (1+\upsilon)\phi w_{j,t}^{\star -\epsilon_w(1+\upsilon)-1} \cdot \frac{\hat{w}_{t+k}^{1+\upsilon}}{1+\upsilon} \right)$$

which rearranges to:

$$0 = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{(1-\epsilon_w) w_{j,t}^{\star 1+\epsilon_w \upsilon} \hat{w}_{t+k}}{P_{t+k}} + \epsilon_w \phi \hat{w}_{t+k}^{1+\upsilon} \right)$$

which rearranges to:

$$w_{j,t}^{\star 1+\epsilon_w \upsilon} = \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\epsilon_w \phi \hat{w}_{t+k}^{1+\upsilon} \right)}{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left((\epsilon_w - 1) P_{t+k}^{-1} \hat{w}_{t+k} \right)}$$

Now, let

$$\mathcal{A}_{w,t} = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\epsilon_w \phi \hat{w}_{t+k}^{1+\upsilon} \right)$$

$$\mathcal{B}_{w,t} = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k (\epsilon_w - 1) P_{t+k}^{-1} \hat{w}_{t+k}$$

These satisfy the recursive equations:

$$\mathcal{A}_{w,t} = \epsilon_w \phi \hat{w}_t^{1+\upsilon} + \beta\theta_w \mathbb{E}_t \mathcal{A}_{w,t+1}$$

$$\mathcal{B}_{w,t} = (\epsilon_w - 1) P_t^{-1} \hat{w}_t + \beta\theta_w \mathbb{E}_t \mathcal{B}_{w,t+1}$$

So we use these recursions to solve for $\mathcal{A}_{w,t}$ and $\mathcal{B}_{w,t}$, and let:

$$w_{j,t}^{\star} = \left(\frac{\mathcal{A}_{w,t}}{\mathcal{B}_{w,t}} \right)^{\frac{1}{1+\epsilon_w \upsilon}}$$

Thus, we have that, for both savers and spenders:

$$\mathcal{A}_{w,t} = \epsilon_w \phi \left(w_t^{\epsilon_w} N_t \right)^{1+\upsilon} + \beta\theta_w \mathbb{E}_t \mathcal{A}_{w,t+1} \tag{A.21}$$

$$\mathcal{B}_{w,t} = (\epsilon_w - 1) P_t^{-1} w_t^{\epsilon_w} N_t + \beta\theta_w \mathbb{E}_t \mathcal{B}_{w,t+1} \tag{A.22}$$

$$w_{j,t}^{\star} = \left(\frac{\mathcal{A}_{w,t}}{\mathcal{B}_{w,t}} \right)^{\frac{1}{1+\epsilon_w \upsilon}}$$

and we see that $w_{j,t}$ and N_t are the same for each j and the same for savers and spenders.

Following the arguments of (Galí, 2015, Chapter 6), the evolution of the aggregate wage w_t is given by:

$$w_t = \left(\theta_w w_{t-1}^{1-\epsilon_w} + (1-\theta_w) w_t^{\star 1-\epsilon_w} \right)^{\frac{1}{1-\epsilon_w}}$$

Substituting in the equation for $w^{\star}_{j,t}$ above, we can write:

$$w_t = \left(\theta_w w_{t-1}^{1-\epsilon_w} + (1-\theta_w)\left(\frac{\mathcal{A}_{w,t}}{\mathcal{B}_{w,t}}\right)^{\frac{1-\epsilon_w}{1+\epsilon_w \upsilon}}\right)^{\frac{1}{1-\epsilon_w}} \tag{A.23}$$

We showed above that the saver's marginal utility of consumption at time t is:

$$\lambda_t^S P_t = \left[C_t^S - \phi \int_{j=0}^{1} \frac{N_{j,t}^{1+\upsilon}}{1+\upsilon} \mathrm{d}j\right]^{-\sigma}$$

Using the labor demand curve above, this is:

$$\lambda_t^S P_t = \left[C_t^S - \frac{\phi N_t^{1+\upsilon}}{1+\upsilon} \int_{j=0}^{1} \left(\left(\frac{w_{j,t}}{w_t}\right)^{-\epsilon_w}\right)^{1+\upsilon} \mathrm{d}j\right]^{-\sigma}$$

which is

$$\lambda_t^S P_t = \left[C_t^S - \frac{\phi N_t^{1+\upsilon} \mathcal{D}_{w,t}^{-\epsilon_w(1+\upsilon)}}{1+\upsilon}\right]^{-\sigma} \tag{A.24}$$

where the wage dispersion term $\mathcal{D}_w$ satisfies:

$$\mathcal{D}_{w,t} = \frac{\left[\int_0^1 w_{j,t}^{-\epsilon_w(1+\upsilon)} dj\right]^{\frac{-1}{\epsilon_w(1+\upsilon)}}}{w_t}$$

The dispersion term evolves according to:

$$\mathcal{D}_{w,t} w_t = \left(\theta_w \left(\mathcal{D}_{w,t-1} w_{t-1}\right)^{-\epsilon_w(1+\upsilon)} + (1-\theta_w)\left(\frac{\mathcal{A}_{w,t}}{\mathcal{B}_{w,t}}\right)^{\frac{-\epsilon_w(1+\upsilon)}{1+\epsilon_w \upsilon}}\right)^{\frac{-1}{\epsilon_w(1+\upsilon)}} \tag{A.25}$$

A.3.6 Solving the problem of the capital goods producer

Substituting the capital goods production function into the Bellman equation for the capital goods producer, we obtain:

$$\begin{aligned} \mathcal{Q}_t K_t = {} & (r_t^K + \delta_K)(K_t) + M_t E_t\left[\mathcal{Q}_{t+1}\right] K_{t+1} \\ & - \left(K_{t+1} - (1-\delta_K)K_t + \kappa\left(\frac{K_{t+1}}{K_t} - 1\right)^2 (K_t)\right) \end{aligned}$$

The first order condition for K_{t+1} is: (omitting the S subscript)

$$E_t\left[M_t \mathcal{Q}_{t+1}\right] - 1 - 2\kappa\left(\frac{K_{t+1}}{K_t} - 1\right) = 0$$

The envelope condition is:

$$\mathcal{Q}_t = \delta_K + r_t^K + \left((1-\delta_K) - \kappa\left(\frac{K_{t+1}}{K_t} - 1\right)^2 + 2\kappa\left(\frac{K_{t+1}}{K_t} - 1\right)\frac{K_{t+1}}{K_t}\right)$$

which simplifies to:

$$\mathcal{Q}_t = \delta_K + r_t^K + \left(1 - \delta_K + \kappa\left(\left(\frac{K_{t+1}}{K_t}\right)^2 - 1\right)\right)$$

Combining the first order condition and the envelope condition to eliminate $\mathcal{Q}$, we get:

$$\begin{aligned} E_t[M_t(\delta_K + r_{S,t+1}^K)] + E_t\left[M_t\left(1 - \delta_K + \kappa\left(\left(\frac{K_{S,t+2}}{K_{S,t+1}}\right)^2 - 1\right)\right)\right] \\ = \left(1 + 2\kappa\left(\frac{K_{S,t+1}}{K_{S,t}} - 1\right)\right) \end{aligned} \tag{A.26}$$

A.3.7 Solving the Intermediate Goods Producer's Problem

For the static profit of intermediate goods producers firms. $\pi^W(z;\mathbf{X}_t)$, we have

$$\begin{aligned} \pi^W(z;\mathbf{X}_t, S) &\equiv \max_{k,n}\left\{\left(\frac{P_{W,S,t}}{P_t}\right) y_i - (r_t^K + \delta_K)k - \frac{w_t n}{P_t} - c^F\right\} \\ &= \max_{k,n}\left\{\left(\frac{P_{W,S,t}}{P_t}\right) z_i^{1-\theta}(k_i^{1-\alpha} n_i^{\alpha})^{\theta} - (r_t^K + \delta_K)k - \frac{w_t n}{P} - c^F\right\} \end{aligned} \tag{A.27}$$

The first order conditions are:

$$(r_t^K + \delta_K)k = (1-\alpha)\theta\left(\frac{P_{W,S,t}}{P_t}\right) z_i^{1-\theta} \cdot (k_i^{1-\alpha} n_i^{\alpha})^{\theta}$$

$$\left(\frac{w_t n}{P_t}\right) = \alpha\theta\left(\frac{P_{W,S,t}}{P_t}\right) z_i^{1-\theta} \cdot (k_i^{1-\alpha} n_i^{\alpha})^{\theta}$$

Then

$$k^{1-\alpha} n^{\alpha} = \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r_t^K + \delta_K)^{1-\alpha}\left(\frac{w_t}{P_t}\right)^{\alpha}}\right)\theta\left(\frac{P_{W,S,t}}{P_t}\right) z_i^{1-\theta} \cdot (k_i^{1-\alpha} n_i^{\alpha})^{\theta}$$

and so

$$\left(k^{1-\alpha}n^{\alpha}\right)^{\theta} = \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r_t^K+\delta_K)^{1-\alpha}\left(\frac{w_t}{P_t}\right)^{\alpha}}\right)^{\frac{\theta}{1-\theta}} \left(\theta\left(\frac{P_{W,S,t}}{P_t}\right)\right)^{\frac{\theta}{1-\theta}} z_i^{\theta}$$

and so

$$\pi^W(z;\mathbf{X}_t,S) = (1-\theta)\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r_t^K+\delta_K)^{1-\alpha}\left(\frac{w_t}{P_t}\right)^{\alpha}}\right)^{\frac{\theta}{1-\theta}} \theta^{\frac{\theta}{1-\theta}}\left(\frac{P_{W,S,t}}{P_t}\right)^{\frac{1}{1-\theta}} z_i - c^F$$

Therefore, we may write:

$$\pi^W(z;\mathbf{X}_t,S) = \overline{\pi}^W_{S,t} z_i - c^F$$

where

$$\overline{\pi}^W_{S,t} = (1-\theta)\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{(r_t^K+\delta_K)^{1-\alpha}\left(\frac{w_t}{P_t}\right)^{\alpha}}\right)^{\frac{\theta}{1-\theta}} \theta^{\frac{\theta}{1-\theta}}\left(\frac{P_{W,S,t}}{P_t}\right)^{\frac{1}{1-\theta}} \tag{A.28}$$

The total revenue of intermediate goods producers i is:

$$P_{W,S,t}y_i = P_t\left(\frac{1}{1-\theta}\right)\left(\pi^W(z;\mathbf{X}_t,S)+c_F\right) = \frac{P_t\overline{\pi}^W_{S,t}z_i}{1-\theta}$$

A.4 Proofs of Propositions

A.4.1 Proof of Proposition 1

Recall that, for the dynamics of the measures of firms, we have:

$$\begin{aligned}
& \forall \hat{z} \geq z^{\star}(\mathbf{X}_{t+1}), M_{t+1}^F \int saI_{\{z' \geq z^{\star}(\mathbf{X}_{t+1})\}} I_{\{z' \leq \hat{z}\}} \mu_{t+1}(z')dz' \\
= \;& M_t^F \rho_z \int I_{\{z_t \geq z^{\star}(\mathbf{X}_t)\}} I_{\{z_t \leq \hat{z}\}} I_{\{z_t \geq z^{\star}(\mathbf{X}_{t+1})\}} \mu_t(z_t)dz_t \\
& + \left[M_t^F(1-\rho_z) + M_{t+1}^e\right] \int I_{\{z' \leq \hat{z}\}} I_{\{z' \geq z^{\star}(\mathbf{X}_{t+1})\}} dG_z(z')
\end{aligned} \tag{A.29}$$

$$M_{t+1}^F = M_t^F \rho_z \int I_{\{z_t \geq z^{\star}(\mathbf{X}_t)\}} I_{\{z_t \geq z^{\star}(\mathbf{X}_{t+1})\}} \mu_t(z_t)dz_t + \left[M_{t+1}^F(1-\rho_z) + M_{t+1}^e\right] \times \int I_{\{z' \geq z^{\star}(\mathbf{X}_{t+1})\}} dG_z(z') \tag{A.30}$$

$$M_{t+1}^{Exit} = M_t^F \rho_z \int I_{\{z_t \geq z^{\star}(\mathbf{X}_t)\}} I_{\{z_t \leq z^{\star}(\mathbf{X}_{t+1})\}} \mu_t(z_t)dz_t + \left[M_t^F(1-\rho_z) + M_{t+1}^e\right] \times \int I_{\{z' \leq z^{\star}(\mathbf{X}_{t+1})\}} dG_z(z') \tag{A.31}$$

To simplify these expressions, let $H_t(z)$ be the cdf of continuing firms associated with μ_t. That is,

$$H_t(y) \equiv \frac{\int_{z_t^\star}^{y} \mu_t(z)dz}{\int_{z_t^\star}^{\infty} \mu_t(z)dz}$$

Then, from the expressions above, we can say that H_t and M_t^W evolve according to:

$$M_{t+1}^W(1 - H_{t+1}(z)) = \begin{cases} \rho_z M_t^W(1 - H_t(z)) + ((1-\rho_z)M_t^W + M_{t+1}^e)(1 - G(z)) & \text{if } z \geq z_{t+1}^\star \\ M_{t+1}^W & \text{if } z \leq z_{t+1}^\star \end{cases} \tag{A.32}$$

Here, there is no $z \geq z_t^\star$ in these expressions because $H_t(z_t^\star) = 0$, so all the necessary information is already contained in H_t.

At $z_{t+1}^\star$, both cases must give the same term on the left hand side, and so:

$$M_{t+1}^W = \rho_z M_t^W(1 - H_t(z_{t+1}^\star)) + ((1-\rho_z)M_t^W + M_{t+1}^e))(1 - G(z_{t+1}^\star)) \tag{A.33}$$

Finally,

$$M_t^{Exit} = M_t^W \rho_z H_t(z_{t+1}^\star) + ((1-\rho_z)M_t^W + M_{t+1}^e)G(z_{t+1}^\star) \tag{A.34}$$

Differentiating equation (A.32) with respect to z, we have:

$$M_{t+1}^W \mu_{t+1}(z) = \begin{cases} \rho_z M_t^W \mu_t(z) + ((1-\rho_z)M_t^W + M_{t+1}^e)g(z) & \text{if } z > z_{t+1}^\star \\ 0 & \text{if } z \leq z_{t+1}^\star \end{cases} \tag{A.35}$$

These equations fully describe the evolution of the firm distribution given M_t^e and $z_t^\star$, for each t.

We may guess and verify that, for any z, the solution has the following form:

$$M_t^W \mu_t(z) = \begin{cases} g(z) \sum_{i=1}^{k+1} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e) & \text{if } z > z_t^\star \\ 0 & \text{if } z \leq z_t^\star \end{cases} \tag{A.36}$$

where k is the number of *consecutive* periods for which $z_t^\star < z$.

That is, $\max\{z_{t-i}^\star\}_{i=0}^{i=k} < z$, but $z_{t-k}^\star > z$. If $z_t^\star < z$ for all t, then $k = \infty$.

It can be proved by induction that this is the solution. Assume it holds for some t, then plug it into equation (A.35) and we see that it also holds for $t+1$.

Recall that, for the dynamics of the measures of firms, we have:

$$
\begin{aligned}
& \forall \hat{z} \geq z^{\star}(\mathbf{X}_{t+1}), M_{t+1}^F \int I_{\{z' \geq z^{\star}(\mathbf{X}_{t+1})\}} I_{\{z' \leq \hat{z}\}} \mu_{t+1}(z') dz' \\
= \; & M_t^F \rho_z \int I_{\{z_t \geq z^{\star}(\mathbf{X}_t)\}} I_{\{z_t \leq \hat{z}\}} I_{\{z_t \geq z^{\star}(\mathbf{X}_{t+1})\}} \mu_t(z_t) dz_t \\
& + \left[M_t^F (1-\rho_z) + M_{t+1}^e \right] \int I_{\{z' \leq \hat{z}\}} I_{\{z' \geq z^{\star}(\mathbf{X}_{t+1})\}} dG_z(z')
\end{aligned} \tag{A.37}
$$

$$
\begin{aligned}
M_{t+1}^F &= M_t^F \rho_z \int I_{\{z_t \geq z^{\star}(\mathbf{X}_t)\}} I_{\{z_t \geq z^{\star}(\mathbf{X}_{t+1})\}} \mu_t(z_t) dz_t \\
& \quad + \left[M_{t+1}^F (1-\rho_z) + M_{t+1}^e \right] \times \int I_{\{z' \geq z^{\star}(\mathbf{X}_{t+1})\}} dG_z(z') \qquad \text{(A.38)} \\
M_{t+1}^{Exit} &= M_t^F \rho_z \int I_{\{z_t \geq z^{\star}(\mathbf{X}_t)\}} I_{\{z_t \leq z^{\star}(\mathbf{X}_{t+1})\}} \mu_t(z_t) dz_t \\
& \quad + \left[M_t^F (1-\rho_z) + M_{t+1}^e \right] \times \int I_{\{z' \leq z^{\star}(\mathbf{X}_{t+1})\}} dG_z(z') \qquad \text{(A.39)}
\end{aligned}
$$

To simplify these expressions, let $H_t(z)$ be the cdf of continuing firms associated with μ_t. That is,

$$
H_t(y) \equiv \frac{\int_{z_t^\star}^{y} \mu_t(z) dz}{\int_{z_t^\star}^{\infty} \mu_t(z) dz}
$$

Then, from the expressions above, we can say that H_t and M_t^W evolve according to:

$$
\begin{aligned}
& M_{t+1}^W (1 - H_{t+1}(z)) \\
= & \begin{cases} \rho_z M_t^W (1 - H_t(z)) + ((1-\rho_z) M_t^W + M_{t+1}^e)(1 - G(z)) & \text{if } z \geq z_{t+1}^\star \\ M_{t+1}^W & \text{if } z \leq z_{t+1}^\star \end{cases}
\end{aligned} \tag{A.40}
$$

Here, there is no $z \geq z_t^\star$ in these expressions because $H_t(z_t^\star) = 0$, so all the necessary information is already contained in H_t.

At $z_{t+1}^\star$, both cases must give the same term on the left hand side, and so:

$$
M_{t+1}^W = \rho_z M_t^W (1 - H_t(z_{t+1}^\star)) + ((1-\rho_z) M_t^W + M_{t+1}^e))(1 - G(z_{t+1}^\star)) \tag{A.41}
$$

Finally,

$$
M_t^{Exit} = M_t^W \rho_z H_t(z_{t+1}^\star) + ((1-\rho_z) M_t^W + M_{t+1}^e) G(z_{t+1}^\star) \tag{A.42}
$$

Differentiating equation (A.40) with respect to z, we have:

$$M_{t+1}^{W}\mu_{t+1}(z) = \begin{cases} \rho_z M_t^W \mu_t(z) + ((1-\rho_z)M_t^W + M_{t+1}^e)g(z) & \text{if } z > z_{t+1}^\star \\ 0 & \text{if } z \leq z_{t+1}^\star \end{cases} \tag{A.43}$$

These equations fully describe the evolution of the firm distribution given M_t^e and $z_t^\star$, for each t.

Now suppose that for some $k \geq 0$, that $z_t^\star \geq \max\{z_{t-i}^\star\}_{i=0}^{k}$. Then, iterating equation (A.35) forward, we have that

$$M_t^W \mu_t(z)$$

$$= \begin{cases} \rho_z^{k+1} M_{t-k-1}^W \mu_{t-k-1}(z) + g(z)\sum_{i=1}^{k+1}\rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e) & \text{if } z > z_t^\star \\ 0 & \text{if } z \leq z_t^\star \end{cases} \tag{A.44}$$

More generally, we may guess and verify that, for any z, the solution has the following form:

$$M_t^W \mu_t(z) = \begin{cases} g(z)\sum_{i=1}^{k+1}\rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e) & \text{if } z > z_t^\star \\ 0 & \text{if } z \leq z_t^\star \end{cases} \tag{A.45}$$

where k is the number of *consecutive* periods for which $z_t^\star < z$.

That is, $\max\{z_{t-i}^\star\}_{i=0}^{i=k} < z$, but $z_{t-k-1}^\star > z$. If $z_t^\star < z$ for all t, then $k = \infty$.

It can be proved by induction that this is the solution. Assume it holds for some t, then plug it into equation (A.35) and we see that it also holds for $t+1$.

Using equations (A.44) and (A.36) we now describe the evolution of $\overline{z}$ and M_t^W. First note that under Assumption 1, it is easy to show that

$$\int_{z>z^\star} z g(z)dz = \left(\frac{\xi}{\xi-1}\right)(z^\star)^{1-\xi}$$

and that

$$\int_{z>z^\star} g(z)dz = (z^\star)^{-\xi}$$

We now prove the Proposition by considering two separate cases: first, the case in which $z_{j_t}^\star \leq z_t^\star$ and, second, the case in which $z_{j_t}^\star > z_t^\star$, with j_t defined in both cases according to equation (A.12).

Case 1: Suppose that $z_{j_t}^\star \leq z_t^\star$. Then, by the definition of j_t in equation (A.12), it must be that $z_t^\star \geq z_{j_t}^\star \geq z_i^\star$, for all $i < j_t$. Furthermore, it must be that there exists no $k > j_t$, with $k < t$ that satisfies $z_k^\star > z_t^\star$, since otherwise it would be the case that $j_t = k$. In combination, these inequalities imply that it must be the case that for all $i < t$, $z_t^\star \geq z_i^\star$. Then, integrating equation

(A.36) and setting $k = \infty$ we get:

$$M_t^W \int_{z>z^\star} z\mu(z)dz = \left(\sum_{i=1}^{\infty} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e) \right) \int_{z>z^\star} zg(z)dz \tag{A.46}$$

That is,

$$M_t^W \overline{z}_t = \left(\sum_{i=1}^{\infty} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e) \right) \left(\frac{\xi}{\xi-1} \right) (z_t^\star)^{1-\xi}. \tag{A.47}$$

This expression is problematic in that M_t^W is not defined for $t < 0$. However, we can make this expression meaningful by assuming that at $t = 0$ the economy is in a steady state, so that $M_t^W = M_0^W$, $M_t^e = M_0^e$ and $z_t^\star = z_0^\star$ for all $t < 0$.

Similarly, we have that with $z_t^\star \geq z_{t-i}^\star$ for all $i \geq 0$,

$$M_t^W \int_{z>z^\star} \mu(z)dz = \left(\sum_{i=1}^{\infty} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e) \right) \int_{z>z^\star} g(z)dz. \tag{A.48}$$

That is,

$$M_t^W = \left(\sum_{i=1}^{\infty} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e) \right) (z_t^\star)^{-\xi}. \tag{A.49}$$

Comparing equations (A.47) and (A.49), we see that, in this case,

$$\overline{z}_t = \left(\frac{\xi}{\xi-1} \right) z_t^\star. \tag{A.50}$$

Equally, since by assumption $z_{j_t}^\star \geq z_i^\star$, for all $i < j_t$, we can also use equation (A.36), setting $k = \infty$, to conclude that:

$$\overline{z}_{j_t} = \left(\frac{\xi}{\xi-1} \right) z_{j_t}^\star \tag{A.51}$$

$$M_{j_t}^W = \left(\sum_{i=1}^{\infty} \rho_z^{i-1}((1-\rho_z)M_{j_t-i}^W + M_{j_t-i+1}^e) \right) (z_{j_t}^\star)^{-\xi} \tag{A.52}$$

Now, combining equations (A.50) and (A.51), we see immediately that

$$\overline{z}_t = \frac{\xi z_t^\star}{\xi-1} = \frac{\xi z_t^\star}{\xi-1} + \frac{\rho_z^{t-j_t} M_{j_t}^W}{M_t^W} \left(\overline{z}_{j_t} - \frac{\xi z_{j_t}^\star}{\xi-1} \right) \tag{A.53}$$

Likewise, combining equations (A.49) and (A.52), we get:

$$M_t^W = \rho_z^{t-j_t} M_{j_t}^W \left(\frac{z_t^\star}{z_{j_t}^\star} \right)^{-\xi} + (z_t^\star)^{-\xi} \sum_{i=1}^{t-j_t} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e) \tag{A.54}$$

Since $z_t^\star \geq z_{j_t}^\star$, equations (A.53) and (A.54) correspond to the results of Proposition 1 in this case.

Case 2: Now consider instead the alternative case, where $z_{j_t}^\star > z_t^\star$. By the definition of j_t in equation (A.12), it must then be the case that $z_i^\star \leq z_t^\star$ for all i satisfying $j_t < i \leq t$. Then, setting $k = t - j_t - 1$ it follows that $z_t^\star \geq \max\{z_{t-i}^\star\}_{i=0}^k$. Then, integrating equation (A.44) with $k = t - j_t - 1$ we have that:

$$M_t^W \int_{z>z_t^\star} z\mu_t(z)dz = \rho_z^{t-j_t} M_{j_t}^W \int_{z>z_t^\star} z\mu_{j_t}(z)dz + \int_{z>z_t^\star} zg(z)dz \sum_{i=1}^{t-j_t} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e)$$

Since in this case it holds that $\int_{z>z_t^\star} z\mu_{j_t}(z)dz = \int_{z>z_{j_t}^\star} z\mu_{j_t}(z)dz = \overline{z}_{j_t}$, we may simplify this to:

$$M_t^W \overline{z}_t = \rho_z^{t-j_t} M_{j_t}^W \overline{z}_{j_t} + \left(\frac{\xi}{\xi-1}\right)(z_t^\star)^{1-\xi} \sum_{i=1}^{t-j_t} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e) \tag{A.55}$$

Similarly, in this case we may also integrate (A.44) with $k = t - j_t - 1$ to get that

$$M_t^W \int_{z>z_t^\star} \mu_t(z)dz = \rho_z^{t-j_t} M_{j_t}^W \int_{z>z_t^\star} \mu_{j_t}(z)dz + \int_{z>z_t^\star} g(z)dz \sum_{i=1}^{t-j_t} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e).$$

This simplifies to:

$$M_t^W = \rho_z^{t-j_t} M_{j_t}^W + (z_t^\star)^{-\xi} \sum_{i=1}^{t-j_t} \rho_z^{i-1}((1-\rho_z)M_{t-i}^W + M_{t-i+1}^e). \tag{A.56}$$

Dividing equation (A.55) by equation (A.56) implies that

$$\overline{z}_t = \frac{\xi z_t^\star}{\xi-1} + \frac{\rho_z^{t-j_t} M_{j_t}^W}{M_t^W}\left(\overline{z}_{j_t} - \frac{\xi z_t^\star}{\xi-1}\right) \tag{A.57}$$

Since, by assumption, $z_{j_t}^\star > z_t^\star$, equations (A.56) and (A.57) correspond to the results of Proposition 1 in this case.

The intuition for the proof is as follows. Let j_t denote the latest period $k < t$ such that $z_k^\star > z_t^\star$ (the definition of j_t has to be slightly more complicated than this in practice, to account for the cases where no such period k exists). The dynamics of intermediate goods firms productivity imply that, in order to know the distribution of intermediate goods firms productivities at time t, it suffices to know only the distribution at time j_t, along with $z_t^\star$ and the number of new entrants in periods $j_t + 1, ...t$. This is because each wholesaler either retains the same productivity z as it had in the previous period, or draws a new productivity according to the Pareto distribution g, with all new entrants drawing a productivity according to g. Then, since $z_{j_t}^\star > z_t^\star$, it follows that all intermediate goods producers who were active in period j_t who have not drawn a new productivity will still be active in period t. Now, the measure $\mu(z_t)$ of intermediate goods producers with a given z_t will be the sum of the measure of intermediate goods producers who have retained the same z since period j_t and the measure of intermediate goods producers who either entered or drew a new productivity between j_t and t. Intermediate goods producers who draw a new productivity or entered between these periods will be active in period t if and only if their current productivity exceeds $z_t^\star$ (since $z_k^\star \leq z_t^\star$ for all periods $j_t < k \leq t$). Therefore, the distribution of z_t across intermediate goods producers who drew a new z between j_t and t will be the Pareto distribution g truncated at $z_t^\star$. Combining this with the measure of intermediate goods producers who have not changed productivity since j_t gives the distribution of z_t across all intermediate goods producers. Integrating this to get $\overline{z}_t$ and M_t^F then gives the results of Proposition 1.

A.4.2 Proof of Proposition 2

To begin with, we substitute $\pi^W(z, \mathbf{X}_t) = z\overline{\pi}_t^W - c_F$ into the intermediate goods producer Bellman equation to get:

$$\begin{aligned} W(z; \mathbf{X}_t, S) &= z\overline{\pi}_{S,t}^W - c_F + M_t\rho_z \max\{W(z; \mathbf{X}_{t+1}, S); 0\} \\ &\quad + M_t(1-\rho_z)E_{G_z}[\max\{W(z'; \mathbf{X}_{t+1}, S); 0\}] \end{aligned} \tag{A.58}$$

It turns out to be much more convenient to define $\hat{W}_S(z, \mathbf{X}_t)$ as:

$$\hat{W}_S(z, \mathbf{X}_t) = \max\{W(z; \mathbf{X}_t, S); 0\}$$

Then, we can write the free entry and free exit conditions as:

$$\nu \left(\frac{M_{S,t}^e}{M_{S,t-1}^W} \right)^\Theta = E_{G_z}\hat{W}_S(z, \mathbf{X}_t) \tag{A.59}$$

$$z_{S,t}^\star = \inf\{z \geq 1 : \hat{W}_S(z, \mathbf{X}_t) > 0\} \tag{A.60}$$

To prove the Proposition, we first show the following Lemma giving a closed form solution for $\hat{W}$-

Lemma 1. *Suppose that $z_{S,t}^\star > 1$ always. The maximized firm's value function $\hat{W}_S(z, \mathbf{X}_t)$ satisfies:*

$$\hat{W}_S(z, \mathbf{X}_t) = \sum_{j=0}^{\infty}(1 - M_{t+j}\rho_z)A_{S,t+j} \max\left\{z - z_{S,t,j}^\star; 0\right\} \frac{\Pi_{k=0}^{k=j}(M_{t+k}\rho_z)}{M_{t+j}\rho_z} \tag{A.61}$$

where

$$A_{S,t} = \frac{\overline{\pi}^W_{S,t}}{1 - \rho_z M_t} \tag{A.62}$$

$$z^\star_{S,t,j} = \max\{z^\star_{S,\tau}\}_{\tau=t}^{\tau=t+j} \tag{A.63}$$

In the steady state, these equations become:

$$\hat{W}_S(z) \equiv A^* \max\{z - z^\star_S; 0\} \tag{A.64}$$

$$A^*_S = \frac{1}{1 - \rho_z M} \overline{\pi}^W_S z \tag{A.65}$$

Proof. The proof below frequently omits the S subscript, but the proof is unchanged if this is included.

Using the definition of $\hat{W}$, the intermediate goods firm Bellman equation implies that:

$$\hat{W}_S(z, \mathbf{X}_t) =$$
$$\max\left\{0; \overline{\pi}^W_{S,t} z - c_F + M_t[\rho_z \hat{W}_S(z; \mathbf{X}_{t+1}) + (1 - \rho_z) E_{G_z} \hat{W}_S(z'; \mathbf{X}_{t+1})]\right\}$$

We can use the free entry condition to simplify the recursive equation for $\hat{W}_S(z, \mathbf{X}_t)$ further:

$$\hat{W}_S(z, \mathbf{X}_t) =$$
$$\max\left\{0; \overline{\pi}^W_{S,t} z - c_F + M_t[\rho_z \hat{W}_S(z; \mathbf{X}_{t+1}) + (1 - \rho_z)\nu \left(\frac{M^e_{S,t+1}}{M^W_{S,t}}\right)^\Theta]\right\} \tag{A.66}$$

We first solve for the intermediate goods firm's maximized value function $\hat{W}_S(\mathbf{X}_t, z)$ in the steady state.

First note that free exit means that $\hat{W}_S(z^\star) = 0$ in the steady state ($z^\star > 1$ must hold in the steady state since otherwise no firms would exit). Since the continuing firm's profits $\pi^W_t(z)$ are linear in z, we may guess (and will verify) that for the continuing firm the maximized value function should also be linear in z in the steady state. Since $\hat{W}_S(z^\star) = 0$, this implies that:

$$\hat{W}_S(z) \equiv A^* \max\{z - z^\star; 0\},$$

for some constant A^*.

Substituting this into the Bellman equation and assuming a steady state, we get:

$$\begin{aligned}\hat{W}_S(z) &= z\overline{\pi}_S^W z - c_F + M\rho_z A^* \max\{z - z^\star; 0\} \\ &+ M(1-\rho_z)A^* \int\limits_{\{z' \geq z^\star(\mathbf{X}_{t+1}, S)\}} (z' - z^\star) dG,\end{aligned}$$

This is consistent with our guess that $\hat{W}_S(z) = A^* \max\{z - z^\star; 0\}$ provided that $\frac{\partial \hat{W}_S(z)}{\partial z} = A^*$ for $z > z^\star$, which holds as long as:

$$A^* = \overline{\pi}_S^W z + M\rho_z A^*$$

i.e.

$$A^* = \frac{1}{1-\rho_z M}\overline{\pi}_S^W z$$

Now we prove the dynamic solution for $\hat{W}_S(z, \mathbf{X}_t)$ by induction. The dynamic solution for $\hat{W}_S(z, \mathbf{X}_t)$ equivalent to the steady state solution just derived, in steady state, so the dynamic solution is definitely correct once we reach steady state. Let

$$S_t(z) = \overline{\pi}_{S,t}^W z - c_F + M_t(1-\rho_z)\nu \left(\frac{M_{S,t+1}^e}{M_{S,t}^W}\right)^\Theta$$

Now, suppose the dynamic solution holds for $t+1$, then the Bellman equation for W implies that:

$$\begin{aligned}&\hat{W}_S(z, \mathbf{X}_t) \\ &\quad = \max\Bigg\{0; S_t(z) + M_t\rho_z(1 - M_{t+1}\rho_z)A_{t+1}\max\{z - z_{t+1}^\star; 0\} \\ &\qquad + M_t\rho_z \sum_{j=1}^{\infty}(1 - M_{t+1+j}\rho_z)A_{t+1+j}\max\{z - z_{t+1,j}^\star; 0\}\Pi_{\tau=t+1}^{\tau=t+j}(M_\tau \rho_Z)\Bigg\}\end{aligned}$$

which is:

$$\hat{W}_S(z, \mathbf{X}_t) = 1(z > z_t^\star)\left(S_t(z) + \sum_{j=0}^{\infty}(1 - M_{t+1+j}\rho_z)A_{t+1+j}\max\{z - z_{t+1,j}^\star; 0\}\Pi_{\tau=t}^{\tau=t+j}(M_\tau\rho_Z)\right)$$

and, since $\hat{W}_S(z^\star, \mathbf{X}_t) = 0$, it must also be the case that:

$$0 = S_t(z_t^\star) + \sum_{j=0}^{\infty}(1 - M_{t+1+j}\rho_z)A_{t+1+j}\max\{z_t^\star - z_{t+1,j}^\star; 0\}\Pi_{\tau=t}^{\tau=t+j}(M_\tau\rho_Z)$$

Combining these, we have:

$$\hat{W}_S(z, \mathbf{X}_t) = \mathbb{1}(z > z_t^\star)\left(S_t(z) - S_t(z_t^\star)\right) + 1(z > z_t^\star) \times \left(\sum_{j=0}^{\infty}(1 - M_{t+1+j}\rho_z)A_{t+1+j}\left(\max\{z - z_{t+1,j}^\star; 0\} - \max\{z_t^\star - z_{t+1,j}^\star; 0\}\right)\Pi_{\tau=t}^{\tau=t+j}(M_\tau \rho_Z)\right),$$

which is

$$\hat{W}_S(z, \mathbf{X}_t) = A_t(1 - M_t\rho_z)\max\{z - z_t^\star; 0\} + \sum_{j=0}^{\infty} \left(A_{t+1+j}(1 - M_{t+1+j}\rho_z)\mathbb{1}(z > z_t^\star)\left(\max\{z - z_{t+1,j}^\star; 0\} - \max\{z_t^\star - z_{t+1,j}^\star; 0\}\right)\Pi_{\tau=t}^{\tau=t+j}(M_\tau \rho_Z)\right),$$

which is

$$\hat{W}_S(z, \mathbf{X}_t) = A_t(1 - M_t\rho_z)\max\{z - z_t^\star; 0\} + \sum_{j=1}^{\infty} \left(A_{t+j}(1 - M_{t+j}\rho_z)\mathbb{1}(z > z_t^\star)\left(\max\{z - z_{t+1,j-1}^\star; 0\} - \max\{z_t^\star - z_{t+1,j-1}^\star; 0\}\right)\Pi_{\tau=t}^{\tau=t+j-1}(M_\tau \rho_Z)\right),$$

which is

$$\begin{aligned}\hat{W}_S(z, \mathbf{X}_t) = {} & A_t(1 - M_t\rho_z)\max\{z - z_t^\star; 0\} \\ & + \sum_{j=1}^{\infty} A_{t+j}(1 - M_{t+j}\rho_z)\left(\Pi_{\tau=t}^{\tau=t+j-1}(M_\tau \rho_Z)\right) \\ & \times \left[\mathbb{1}(z_{t+1,j-1}^\star > z_t^\star)\mathbb{1}(z > z_{t+1,j-1}^\star)(z - z_{t+1,j-1}^\star) + \mathbb{1}(z_t^\star > z_{t+1,j-1}^\star)\mathbb{1}(z > z_t^\star)(z - z_t^\star)\right],\end{aligned}$$

which is

$$\begin{aligned}\hat{W}_S(z, \mathbf{X}_t) = {} & A_t(1 - M_t\rho_z)\max\{z - z_t^\star; 0\} \\ & + \sum_{j=1}^{\infty} A_{t+j}(1 - M_{t+j}\rho_z)\max\left\{z - \max\{z_t^\star; z_{t+1,j-1}^\star\}; 0\right\}\Pi_{\tau=t}^{\tau=t+j-1}(M_\tau \rho_Z)\end{aligned}$$

which is

$$\begin{aligned}\hat{W}_S(z, \mathbf{X}_t) = {} & A_t(1 - M_t\rho_z)\max\{z - z_t^\star; 0\} \\ & + \sum_{j=1}^{\infty} A_{t+j}(1 - M_{t+j}\rho_z)\max\left\{z - z_{t,j}^\star; 0\right\}\Pi_{\tau=t}^{\tau=t+j-1}(M_\tau \rho_Z)\end{aligned}$$

which is

$$\hat{W}_S(z,\mathbf{X}_t) = \sum_{j=0}^{\infty} A_{t+j}(1-M_{t+j}\rho_z)\max\left\{z - z^\star_{t,j};0\right\}\frac{\Pi_{k=0}^{k=j}(M_{t+k}\rho_z)}{M_{t+j}\rho_z}$$

and so the induction is proved.

It is straightforward to show that in the steady state (A.17) becomes:

$$\mathcal{A}_{t,j} = (1-M\rho_z)M^j\rho_z^j A^*$$

and so

$$\sum_{j=0}^{\infty}\mathcal{A}_{t,j} = A^*$$

Then, it is immediate that (A.13) and (A.14) become:

$$\begin{aligned}
\nu\left(\frac{M^e}{M^W}\right)^{\Theta} &= A^*\left(\frac{z^{\star-(\xi-1)}}{\xi-1}\right)\\
z^\star &= \frac{c_F - M(1-\rho_z)\nu\left(\frac{M^e}{M^W}\right)^{\Theta}}{(1-\rho_Z M)A^*}
\end{aligned}$$

□

Now, we use the Lemma to prove Proposition 2. Recall the Proposition: *$z_t^\star$ and M_t^E satisfy:*

$$\nu\left(\frac{M^e_{S,t}}{M^W_{S,t-1}}\right)^{\Theta} = \sum_{j=0}^{\infty}\mathcal{A}_{t,j}\left(\frac{z_{t,j}^{\star-(\xi-1)}}{\xi-1}\right) \tag{A.67}$$

and

$$z_t^\star = \frac{c_F - M_t(1-\rho_z)\nu\left(\frac{M^e_{S,t}}{M^W_{S,t-1}}\right)^{\Theta}}{(1-\rho_Z M_t)A_t} - \left(\frac{1}{(1-\rho_z M_t)A_t}\right)\sum_{j=1}^{\infty}\mathcal{A}_{t,j}\left(z^\star_{t,j} - z^\star_{t+1,j-1}\right) \tag{A.68}$$

where

$$\mathcal{A}_{t,j} = (1-\rho_Z M_{t+j})A_{t+j}\frac{\Pi_{k=0}^{k=j}(M_{t+k}\rho_z)}{M_{t+j}\rho_z} \tag{A.69}$$

Proof. First we derive (A.67). Recall that, when z is Pareto, $\mathbb{E}\left[\max\{z - z^\star;0\}\right] = \frac{z^{\star-(\xi-1)}}{\xi-1}$, which can be verified by integration.

Using this to take the expectation of the equation for $\hat{W}(z, \mathbf{X}_t)$ in (A.61) above, we have:

$$\nu \left(\frac{M^e_{S,t}}{M^W_{S,t-1}} \right)^\Theta = \mathbb{E}\left[\hat{W}(z, \mathbf{X}_t) \right] = \sum_{j=0}^{\infty} (1 - \rho_Z M_{t+j}) A_{t+j} \left(\frac{z_{t,j}^{\star -(\xi-1)}}{\xi - 1} \right) \frac{\Pi_{k=0}^{k=j} (M_{t+k}\rho_z)}{M_{t+j}\rho_z}$$

Using the definition of $\mathcal{A}$ in (A.69), we obtain (A.67).

Now, we derive (A.68). We use the Bellman equation:

$$0 = \hat{W}(z_t^\star, \mathbf{X}_t) = 0 = S_t(z^\star) + M_t \rho_z \hat{W}(z_t^\star, \mathbf{X}_{t+1})$$

where, as above,

$$S_t(z) = z \overline{\pi}^W_{S,t} - c_F + M_t (1 - \rho_z) \nu \left(\frac{M^e_{S,t+1}}{M^W_{S,t}} \right)^\Theta$$

Then, since $(1 - M_t \rho_z) A_t = S_t'(z)$, it follows that:

$$(1 - M_t \rho_z) A_t = \overline{\pi}^W_{S,t}$$

Then, $z_t^\star$ satisfies:

$$(1 - M_t \rho_z) A_t z_t^\star = c_F - M_t (1 - \rho_z) \nu \left(\frac{M^e_{S,t+1}}{M^W_{S,t}} \right)^\Theta - \sum_{j=0}^{\infty} (1 - M_{t+1+j}\rho_z) A_{t+1+j} \max \left\{ z_t^\star - z_{t+1,j}^\star ; 0 \right\} \frac{M_t \Pi_{k=0}^{k=j} (M_{t+1+k}\rho_z)}{M_{t+1+j}}$$

which is:

$$(1 - M_t \rho_z) A_t z_t^\star = c_F - M_t (1 - \rho_z) \nu \left(\frac{M^e_{S,t+1}}{M^W_{S,t}} \right)^\Theta - \sum_{j=0}^{\infty} A_{t+1+j} (1 - M_{t+1+j}\rho_z) \left(\max \left\{ z_t^\star ; z_{t+1,j}^\star \right\} - z_{t+1,j}^\star \right) \frac{M_t \Pi_{k=0}^{k=j} (M_{t+1+k}\rho_z)}{M_{t+1+j}}$$

Or

$$(1 - M_t \rho_z) A_t z_t^\star = c_F - M_t (1 - \rho_z) \nu \left(\frac{M^e_{S,t+1}}{M^W_{S,t}} \right)^\Theta - \sum_{j=0}^{\infty} (1 - M_{t+1+j}\rho_z) A_{t+1+j} \left(z_{H,j+1}^\star - z_{t+1,j}^\star \right) \frac{M_t \Pi_{k=0}^{k=j} (M_{t+1+k}\rho_z)}{M_{t+1+j}}$$

Using the definition of $\mathcal{A}$ in (A.69), this is:

$$z_t^\star = \frac{c_F - M_t(1-\rho_z)\nu\left(\frac{M^e_{S,t+1}}{M^W_{S,t}}\right)^\Theta}{(1-\rho_Z M_t)A_t} - \left(\frac{M_t\rho_z}{(1-\rho_z M_t)A_t}\right)\sum_{j=0}^\infty \mathcal{A}_{t+1,j}\left(z^\star_{H,j+1} - z^\star_{t+1,j}\right)$$

Now, note that

$$\mathcal{A}_{H,j+1} = \mathcal{A}_{t+1,j}M_t\rho_z$$

So we can write the equation for $z_t^\star$ as:

$$z_t^\star = \frac{c_F - M_t(1-\rho_z)\nu\left(\frac{M^e_{S,t+1}}{M^W_{S,t}}\right)^\Theta}{(1-\rho_Z M_t)A_t} - \left(\frac{1}{(1-\rho_z M_t)A_t}\right)\sum_{j=0}^\infty \mathcal{A}_{H,j+1}\left(z^\star_{H,j+1} - z^\star_{t+1,j}\right)$$

which is the same as (A.68). □

The intuition for the proposition is as follows. The left hand side of equation (A.13) is the cost of entry at time t. The right hand side is the value of entry, which is proportional to a weighted average of future $\frac{z^{\star xi-1}}{\xi-1}$, because, since the distribution of new z draws is Pareto, it is straightforward to show that $E[z|z>z^\star]\text{Prob}(z>z^\star) = \frac{z^{\star xi-1}}{\xi-1}$. The term $\mathcal{A}_{t,j}$ in equation (A.13) is simply a present discounted sum of future $\overline{\pi}^F_{S,t}$, which determines intermediate goods producer profits.

The optimal value of $z_t^\star$ is given by equation (A.14). The first term on the right hand side of (A.14) is $\frac{c_F}{(1-\rho_z M_t)A_t} \equiv \frac{c_F}{\overline{\pi}^F_{S,t}}$. This first term is the cutoff value of z needed to break even in the present period, since the intermediate goods producer's period profits are $\overline{\pi}^F_{S,t}z - c_F$. The remaining terms on the right hand side of (A.14) reflect the option value to a intermediate goods producer of remaining in operation in the hope of receiving a better draw of z in future. As such, these terms depend on a similar weighted sum of future $z^\star$ terms to the right hand side of equation (A.13).

It is straightforward to show that in the steady state (A.17) becomes:

$$\mathcal{A}_{t,j} = (1-M\rho_z)M^j\rho_z^j A^*.$$

and so

$$\sum_{j=0}^\infty \mathcal{A}_{t,j} = A^*$$

Then, it is immediate that (A.16), (A.13) and (A.14) become:

$$
\begin{aligned}
A_S^* &= \tfrac{1}{1-\rho_z M}\overline{\pi}_S^W, \\
\nu\left(\frac{M^e}{M_F}\right)^{\Theta} &= A^*\left(\frac{z^{\star-(\xi-1)}}{\xi-1}\right), \\
z^\star &= \frac{c_F - M(1-\rho_z)\nu\left(\frac{M^e}{M_F}\right)^{\Theta}}{(1-\rho_Z M)A^*}.
\end{aligned}
$$

A.5 Microfounding Interest Rate Decrease After Deregulation

Here, we provide a simple microfoundation for the idea that ending restrictions on the formation of new branches increased the supply of credit and reduced interest rates on loans, as suggested in the main text. Suppose that there are a large number of banks in the domestic economy (i.e. the US state) and, for simplicity, assume that there is perfect competition between banks and that banks are all owned by households outside of the domestic economy. In period t, each bank branch j receives deposits $\text{dep}_{j,t}$ from outside the domestic economy which earn the common interest rate r and it lends $-B_{j,t}$ to households and firms in the domestic economy at the market lending rate $r+\psi_t$. The bank takes r and $r+\psi_t$ as given.

The bank chooses its level of lending and deposits in branch j to maximize the profits of the branch, given by:

$$\text{Profit}_{j,t} = -(r+\psi_t)B_{j,t} - r\cdot \text{dep}_{j,t} - \text{Cost}(-B_{j,t}),$$

subject to the constraint $-B_{j,t} \le \text{dep}_{j,t}$. Here, $\text{Cost}(-B_{j,t})$ is a convex intermediation cost, and the function $\text{Cost}(\cdot)$ is assumed to be identical across bank branches and banks.

The bank's first order condition implies that $\psi_t = \text{Cost}'(B_{j,t})$. Since all bank branches will choose the same $B_{j,t}$, we have that $B_{j,t} = \frac{B_t}{\text{Num}_t}$ where Num_t is the total measure of bank branches in the domestic economy. Then, $\psi_t = \text{Cost}'(\frac{B_t}{\text{Num}_t})$. Since $\text{Cost}'(\cdot)$ is increasing in its argument, this is an upward sloping supply curve for aggregate loans, $-B_t$. It is immediate that an increase in the number of bank branches, Num_t, shifts the supply curve to the right and, conditional on total lending, $-B_t$, reduces ψ_t. Given the growth of $-B_t$ after deregulation in the data, a large enough increase in Num_t would induce the decrease in lending rates, $r+\psi_t$, that we observe. This, then, provides a possible microfoundation for the reduction in lending rates following bank deregulation.

B Data Appendix

In this section we investigate the effect of bank deregulation on economic activity by studying the relaxation of intrastate branching restrictions in the United States. Until the late 1970s most US states restricted intrastate branching. Between mid 1970s and mid 1990s states lifted these restrictions in cohorts, such that by 1993 almost all states had deregulated their banking sector. The restrictions to banking were completely repealed with the passage of the Reigle-Neal Interstate Banking and Branching Efficiency Act of 1994.

B.1 Specification

We are interested in examining the dynamic impact of bank deregulation on various economic outcomes. To that end, we estimate the impulse response function of inequality to the deregulation of the banking sector by using the local projections estimator proposed by Jorda (2005) and extended by Teulings and Zubanov (2014) to correct for the bias that arises when controlling for state fixed effects.[19] The specification that we estimate is in equation (15). However, in Figures 7 and 8 below we show that our results are robust to estimating (15) in levels rather than growth rates and to not including lags of the deregulation variable in the specification.

B.2 Data

To assess the effect of bank deregulation on economic activity, we gather data on the timing of bank deregulation and state level economic characteristics. There are two bank reforms that occurred in the US between mid 1970s and mid 1990s: the interstate banking deregulation and the intrastate branching deregulation. In what follows, we provide some historical context for these reforms and highlight how we construct the bank deregulation indicators D_{it}^{intra} and D_{it}^{inter} used in estimating equation (15). We draw similar conclusions from the two reforms, so in estimating the parameters of the model we use the impulse response functions to the intrastate branching deregulation. This also isolates the mechanism we explore from consideration of geographic expansion that firms might have and that we abstract from.

After the United States Constitution prohibited the states from issuing fiat money and from taxing interstate trade, states used their regulatory authority over banks to generate a substantial part of their revenues (Sylla, Legler, and Wallis, 1987). Since states could not exert charter fees from banks incorporated in other states, they prohibited out-of-state banks from operating

[19] Teulings and Zubanov (2014) show that, when controlling for group fixed effects (state fixed effects in our application), the local projection impulse response function is subject to a downward bias for $k \geq 1$. The bias increases with the forecast horizon k. Their suggested solution is to include in the local projection equations dummy variables for the event of interest (bank deregulation in our application) occurring between t and $t+k$. This not only improves efficiency, but also takes away the bias in the local projection estimator. In our specification, the effect of these dummy variables in captured by the coefficients $\delta_{j,k}$.

Figure 7: Effects of Deregulation on Interest Rates, Output, Employment and Wages, Robustness

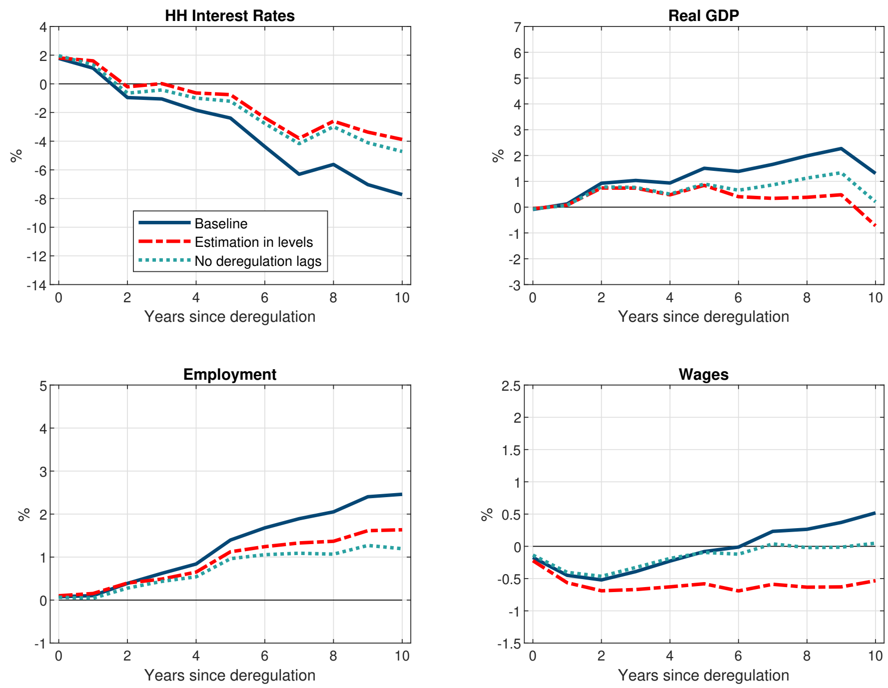
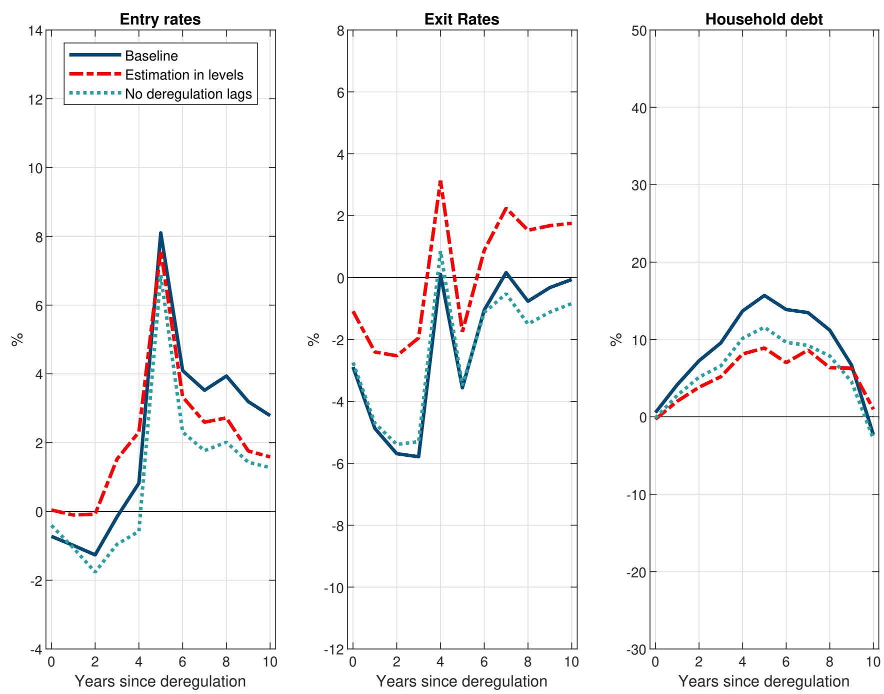


Figure 8: Effects of Deregulation on Firm Entry, Firm Exit and Household Debt, Robustness



in their territories – hence the origin of the prohibition on *interstate banking*. In addition to excluding banks from other states, the legislatures often restricted intrastate bank expansion – restriction on *intrastate branching*. The 1927 McFadden Act formalized the authority of the states over the regulation of national banks branching activities within their borders (Kroszner and Strahan, 2014). After the passage of the McFadden Act, banks tried to circumvent state branching restrictions by building multi-bank holding companies, but The Douglas Amendment to the 1956 Bank Holding Company Act ended this practice. According to the Douglas Amendment, banking was a state right and, therefore, states had the option to exclude out-of-state banks or holding companies from buying or building a bank or branch in their state. All states exercised this option, effectively prohibiting interstate banking.

Interstate banking deregulation. In 1978 Maine took the first step towards interstate banking deregulation by passing a law that allowed entry by bank holding companies from any state that allowed entry by Maine banks. The next to follow were Alaska and New York, who passed similar laws in 1982. Other states followed gradually, so that by 1992 all states but Hawaii had lifted interstate banking restrictions. These state changes, however, did not permit banks to open branches across state lines (Rice and Strahan, 2010). The restrictions to interstate banking were completely repealed with the passage of the Reigle-Neal Interstate Banking and Branching Efficiency Act of 1994. Reigle-Neal made interstate banking a bank right, not a state right, so that banks or holding companies could now enter another state without permission. Table 3 compiles the interstate banking deregulation dates and illustrates how states deregulated in cohorts rather than at the same time to reform, which we exploit in our empirical analysis. Based on this table we construct a dummy indicator D_{it}^{inter} that we set equal to 1 if state i lifted its restrictions to interstate banking in year t and zero otherwise.

Intrastate branching deregulation. Although there was some deregulation of branching restrictions in the 1930s, most states continued to enforce intrastate branching restrictions well into the 1970s. Only 11 states allowed unrestricted statewide branching in 1970 and a total of 16 states prohibited branching entirely. Between 1970 and 1994, however, 38 states eased their restrictions on branching (Rice and Strahan, 2010). First, states permitted branching via mergers and acquisitions through the holding company structure. Second, states began allowing banks to open new branches anywhere within state borders. Table 4 compiles the dates at which states started lifting intrastate branching restrictions. These dates refer to the year in which a state allowed branching via mergers and acquisitions. As before, we construct a dummy indicator D_{it}^{intra} that we set equal to 1 if state i lifted its restrictions to intrastate branching in year t and zero otherwise. Since the states listed in the first line of Table 4 lifted these restrictions before 1970 and the exact date is not known with certainty, the dummy variable is not defined before 1970 for these states.[20]

[20]This is one of the reasons we choose to estimate equations for each type of reform. The obvious alternative,

Indicators of economic activity. We consider various indicators of economic activity. To assess the immediate effect of the bank deregulation on the banking sector we examine the impulse responses of branch density, interest rate on loans, quantity of loans to households and businesses. We obtain these from the Call Report data of institutions regulated by the Federal Deposit Insurance Corporation (FDIC).

To asses the effect of bank deregulation on the activity of firms we measure the response of firm entry and exit, wages and employment. We use data on the number of establishments and establishment entry and exit from the Business Dynamics Statistics database. We use wage and employment data from the Bureau of Economic Analysis (BEA).

Table 3: Timing of Interstate Banking Deregulation

Year	States	Total
1978	Maine	1
1982	Alaska, New York	2
1983	Connecticut, Massachusetts	2
1984	Kentucky, Rhode Island, Utah	3
1985	District of Columbia, Florida, Georgia, Idaho, Maryland, Nevada, North Carolina, Ohio, Tennessee, Virginia	10
1986	Arizona, Illinois, Indiana, Michigan, Minnesota, Missouri, New Jersey, Oregon, Pennsylvania, South Carolina	10
1987	Alabama, California, Louisiana, New Hampshire, Oklahoma, Texas, Washington, Wisconsin, Wyoming	9
1988	Colorado, Delaware, Mississippi, South Dakota, Vermont, West Virginia	6
1989	Arkansas, New Mexico	2
1990	Nebraska	1
1991	Iowa, North Dakota	2
1992	Kansas	1
1993	Montana	1
1999	Hawaii*	1

Source: Morgan, Rime, and Strahan (2004); *Park (2012)

which we implement as a robustness exercise, is to simultaneously control for both. However, in this case we lose all observations related to interstate banking deregulation for all the states which lifted intrastate branching restrictions prior to 1970.

Table 4: Timing of Intrastate Branching Deregulation

Year	States	Total
< 1970	Alaska, Arizona, California, District of Columbia, Idaho, Maryland, Nevada, North Carolina, Rhode Island, South Carolina	10
1970	Vermont	1
1975	Maine	1
1976	New York	1
1977	New Jersey	1
1978	Virginia	1
1979	Ohio	1
1980	Connecticut	1
1981	Alabama, Utah	2
1982	Pennsylvania	1
1983	Georgia	1
1984	Massachusetts	1
1985	Nebraska, Oregon, Tennessee, Washington	4
1986	Hawaii, Mississippi	2
1987	Michigan, New Hampshire, North Dakota, West Virginia, Kansas	5
1988	Oklahoma, Florida, Illinois, Louisiana, Texas, Wyoming	6
1989	Indiana	1
1990	Kentucky, Missouri, Montana, Wisconsin	4
1991	Colorado, New Mexico	2
1993	Minnesota	1
1994	Arkansas	1
1999	Iowa	1

Source: Morgan, Rime, and Strahan (2004)

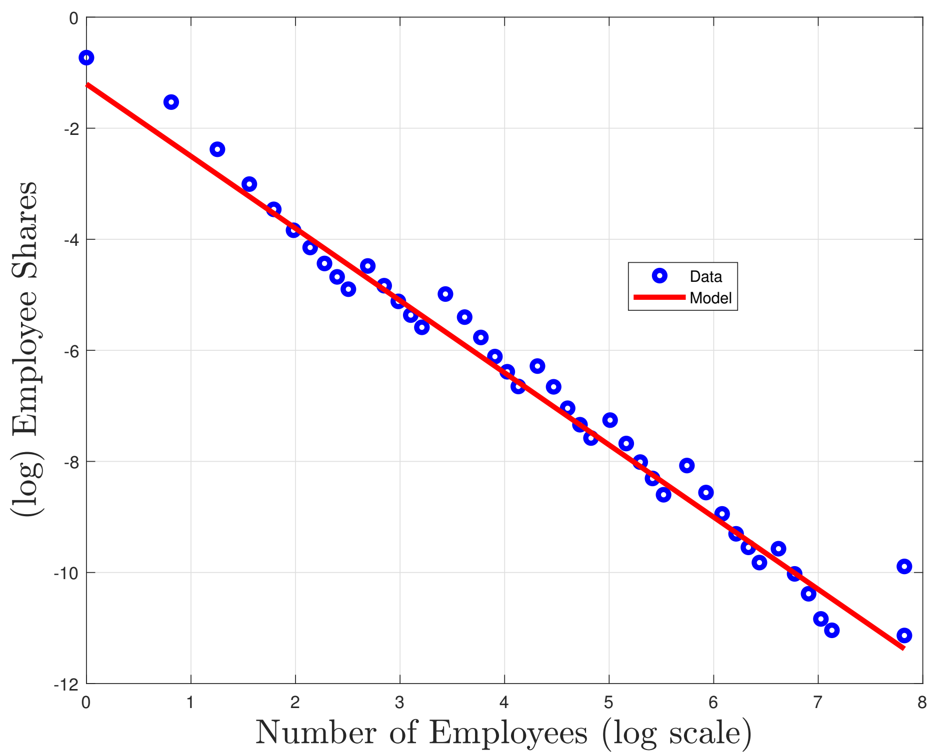
C Appendix for Quantitative Exercises

In this section we report additional results referenced in the main text.

C.1 The Firm Size Distribution

Figure 9 plots the distribution of firm size.

Figure 9: Distribution of firms by employment: model vs. data



C.2 Model Fit Under Alternative Parameterizations

In this section we compare the impulse responses of our benchmark calibration with those obtained under lower and higher values of the parameters $\eta, \upsilon_L, \zeta, \kappa, c^F, \rho_z, \nu$ and Θ. We generally consider cases of parameters that are 20% higher and 20% lower than the benchmark values (for elasticities, we consider 5 and 2 for υ_L, and 5 and 1.2 for ζ).

Figure 10: Model responses with different values in η, demand elasticity for intermediate goods

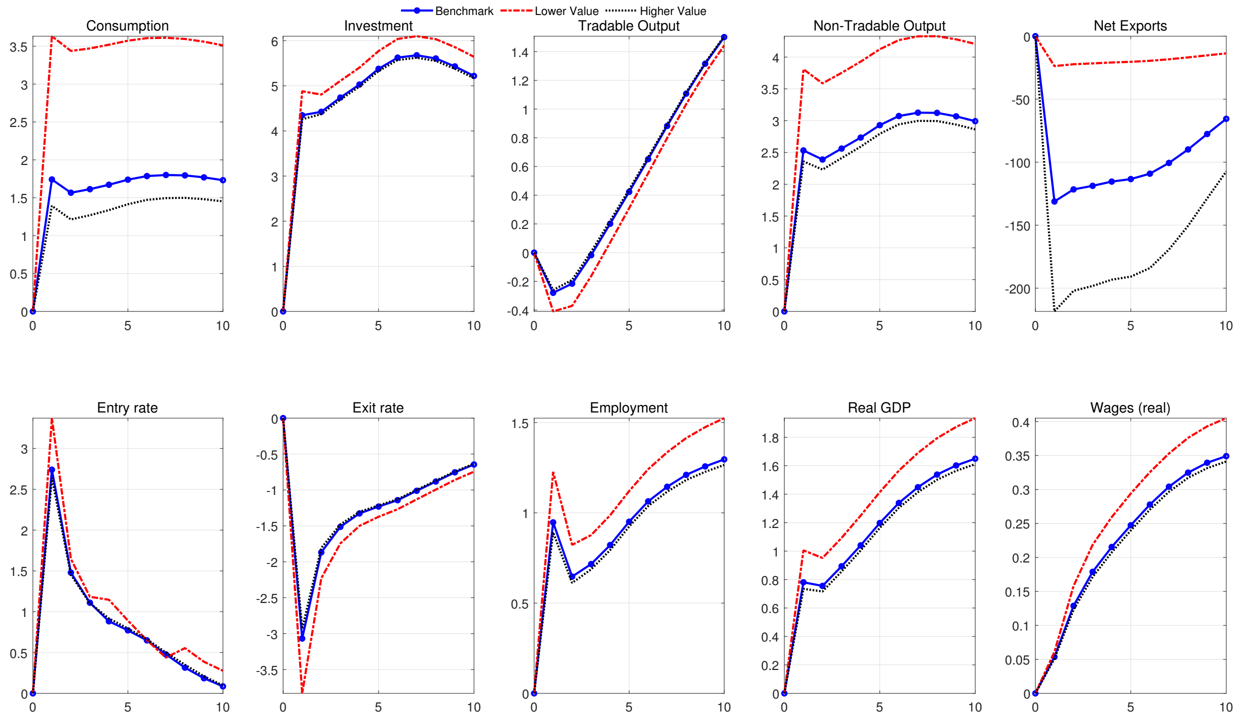


Figure 11: Model responses with different values in labor supply elasticity, υ_L

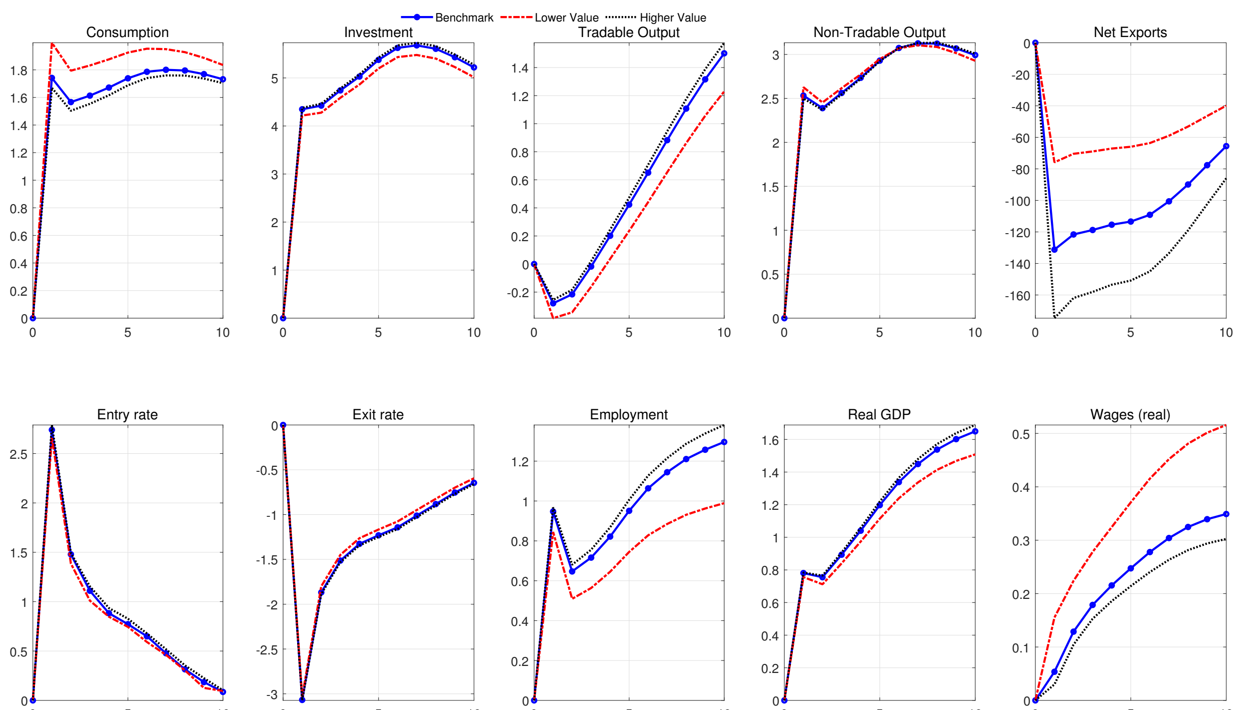


Figure 12: Model responses with different values in ζ, elasticity between home and foreign tradable goods)

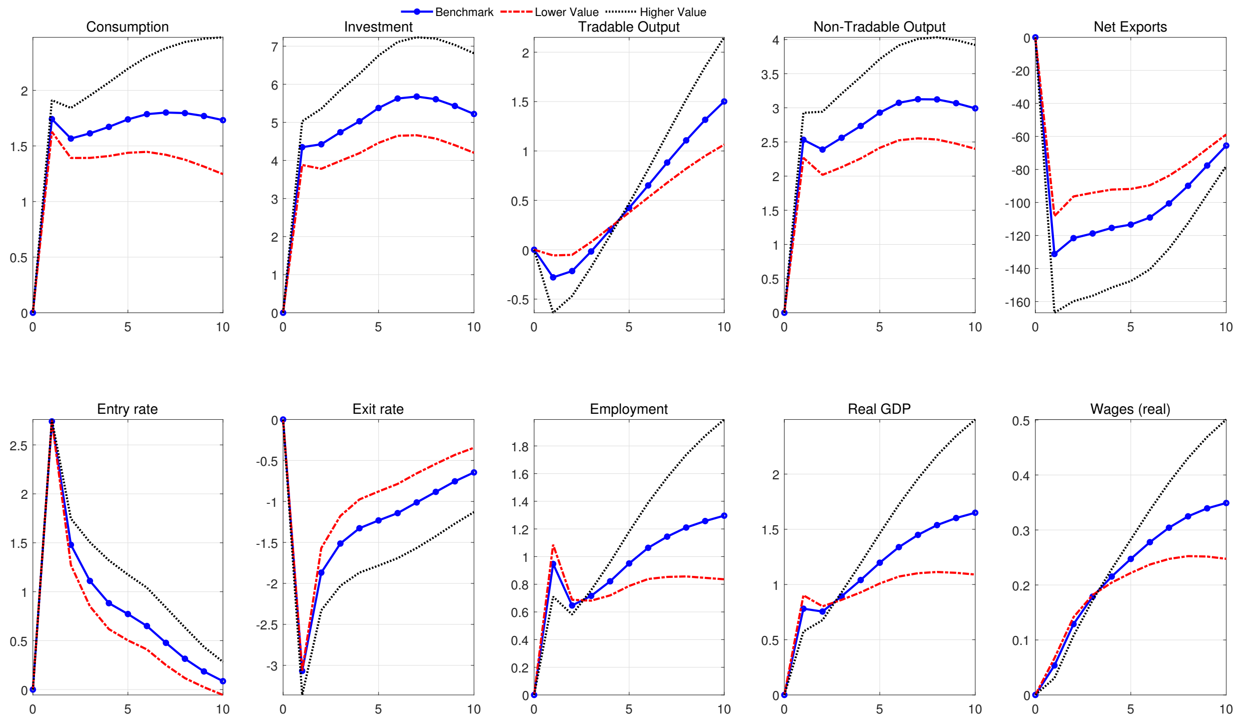


Figure 13: Model responses with different values in capital adjustment costs κ

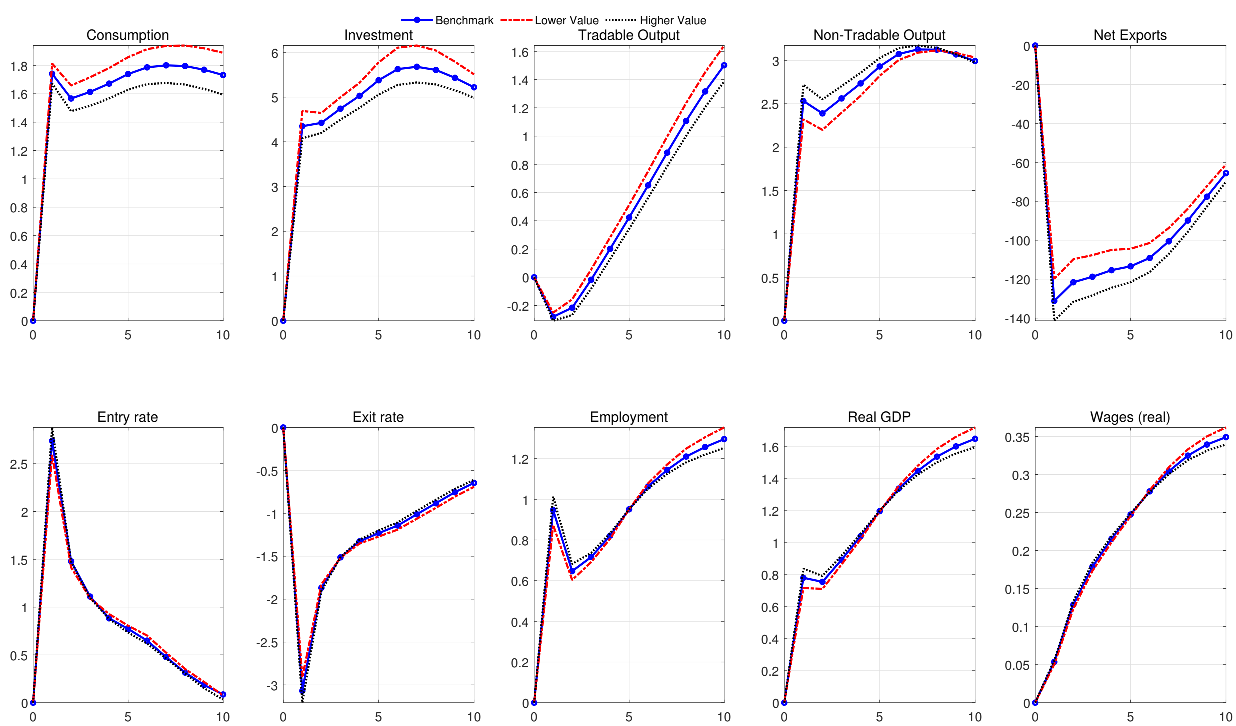


Figure 14: Model responses with different values in flow operating costs c^F

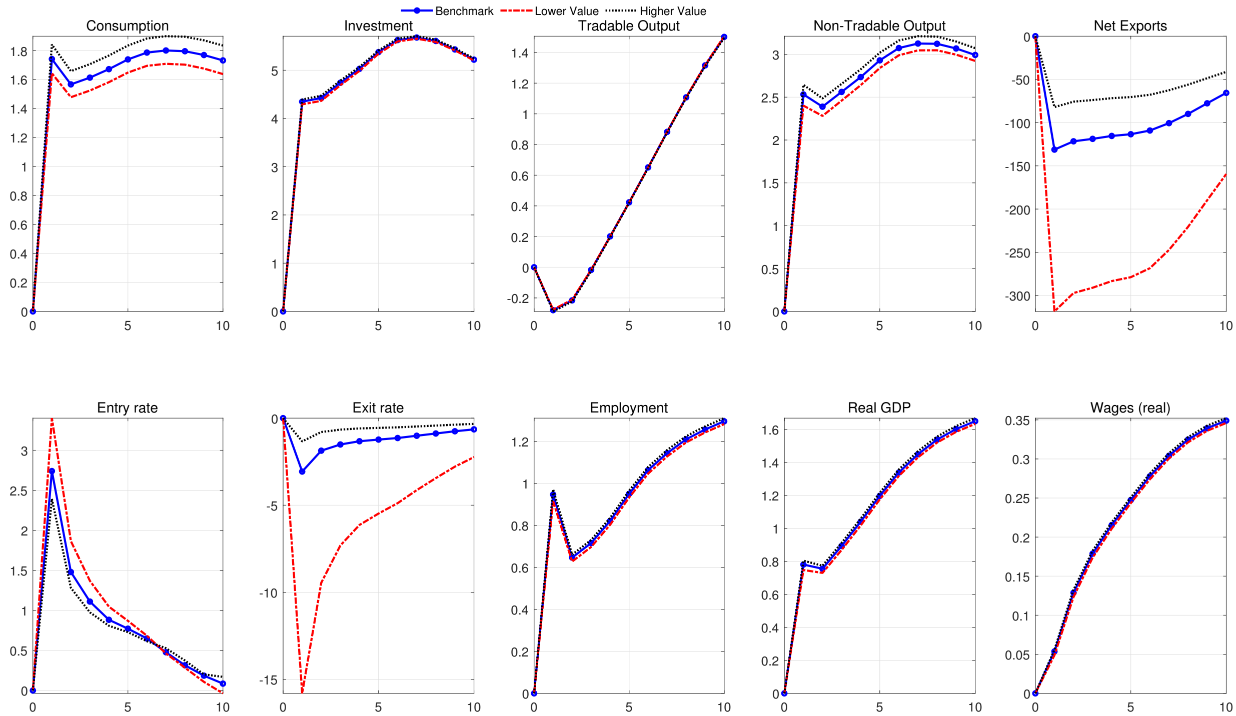


Figure 15: Model responses with different values in persistence for firm-level productivity, ρ_z

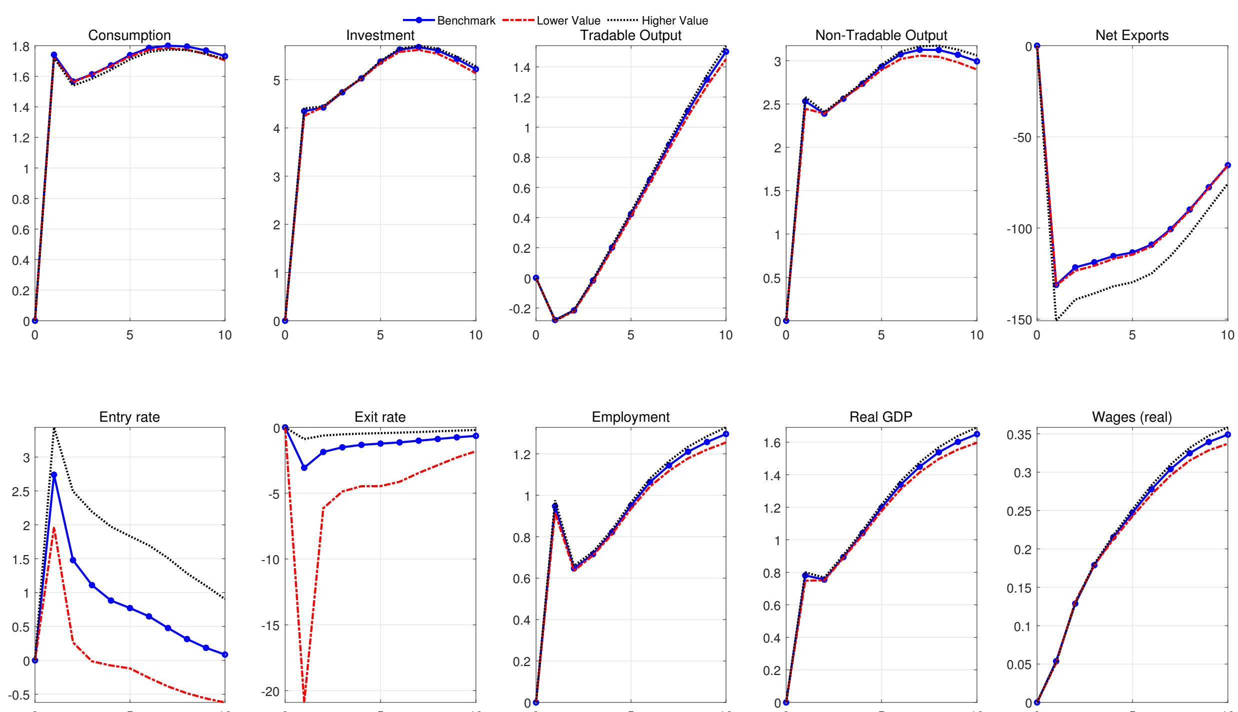


Figure 16: Model responses with different values in entry cost level parameter ν

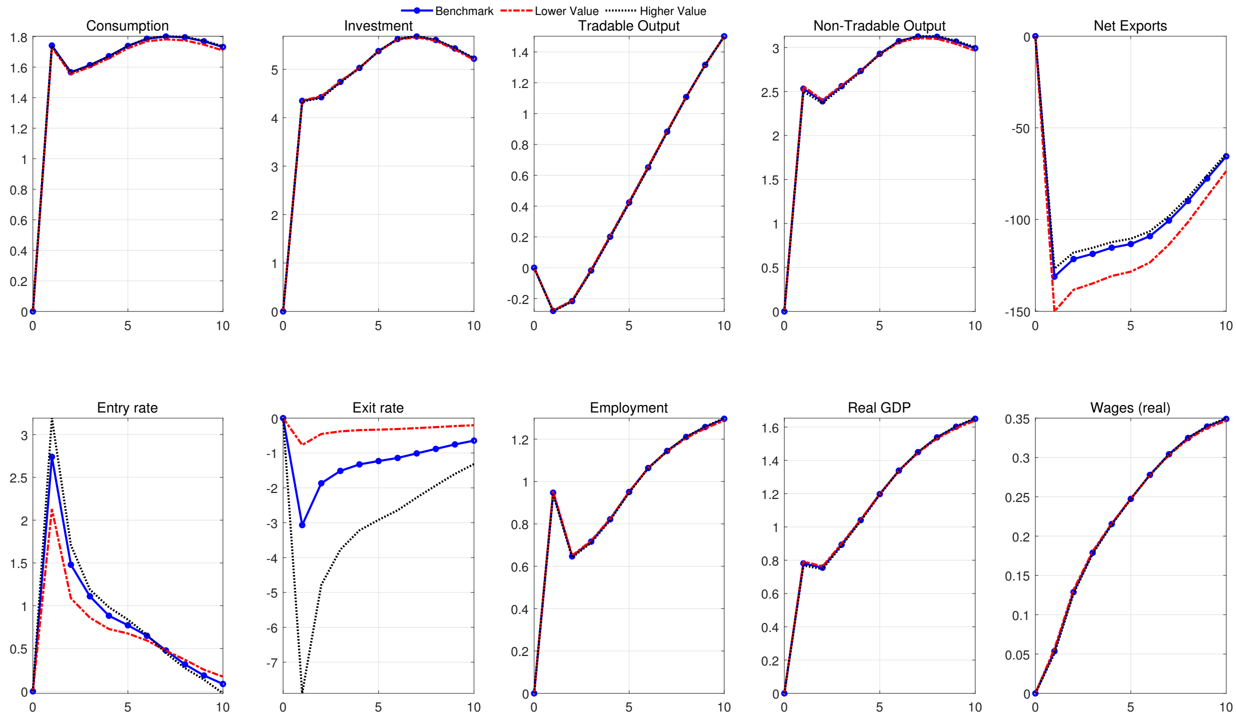


Figure 17: Model responses with different values in the elasticity of entry costs, Θ

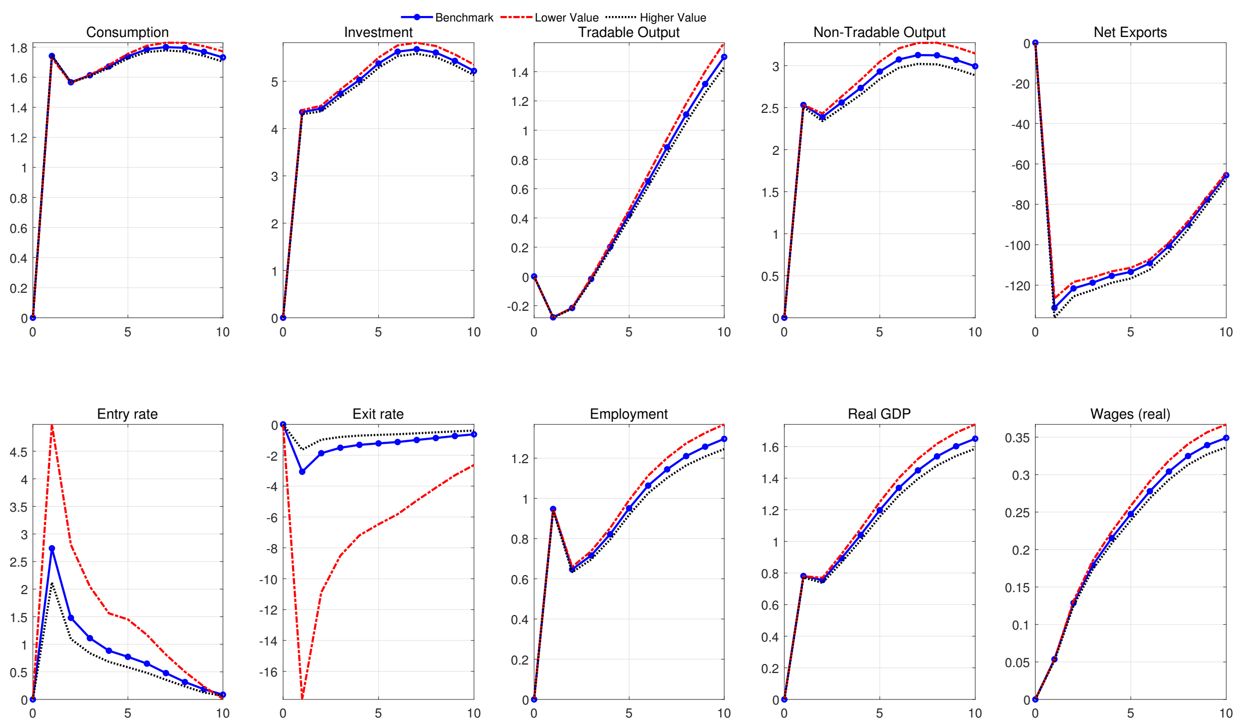
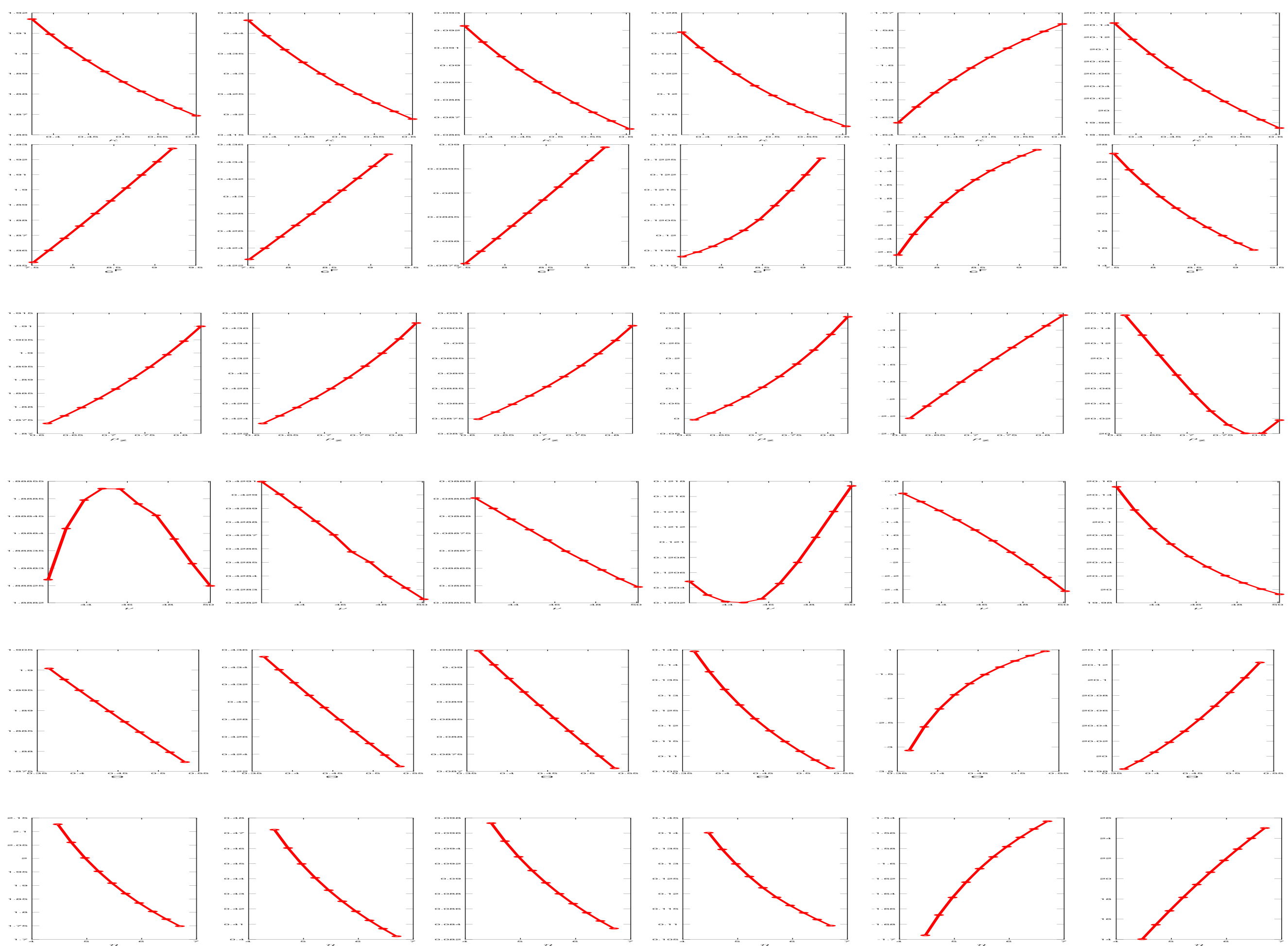


Figure 18 summarizes the sensitivity of key model moments to changes in parameter. For a given parameter, we compute the model's responses – the percent deviations from corresponding steady state values – averaged from periods 2 to periods 10, as summary moments. We observe that almost all of the model responses are monotonic in parameter values.

Figure 18: Model moments change with parameters (κ, c^F, ρ_z, ν, Θ, η, ζ)

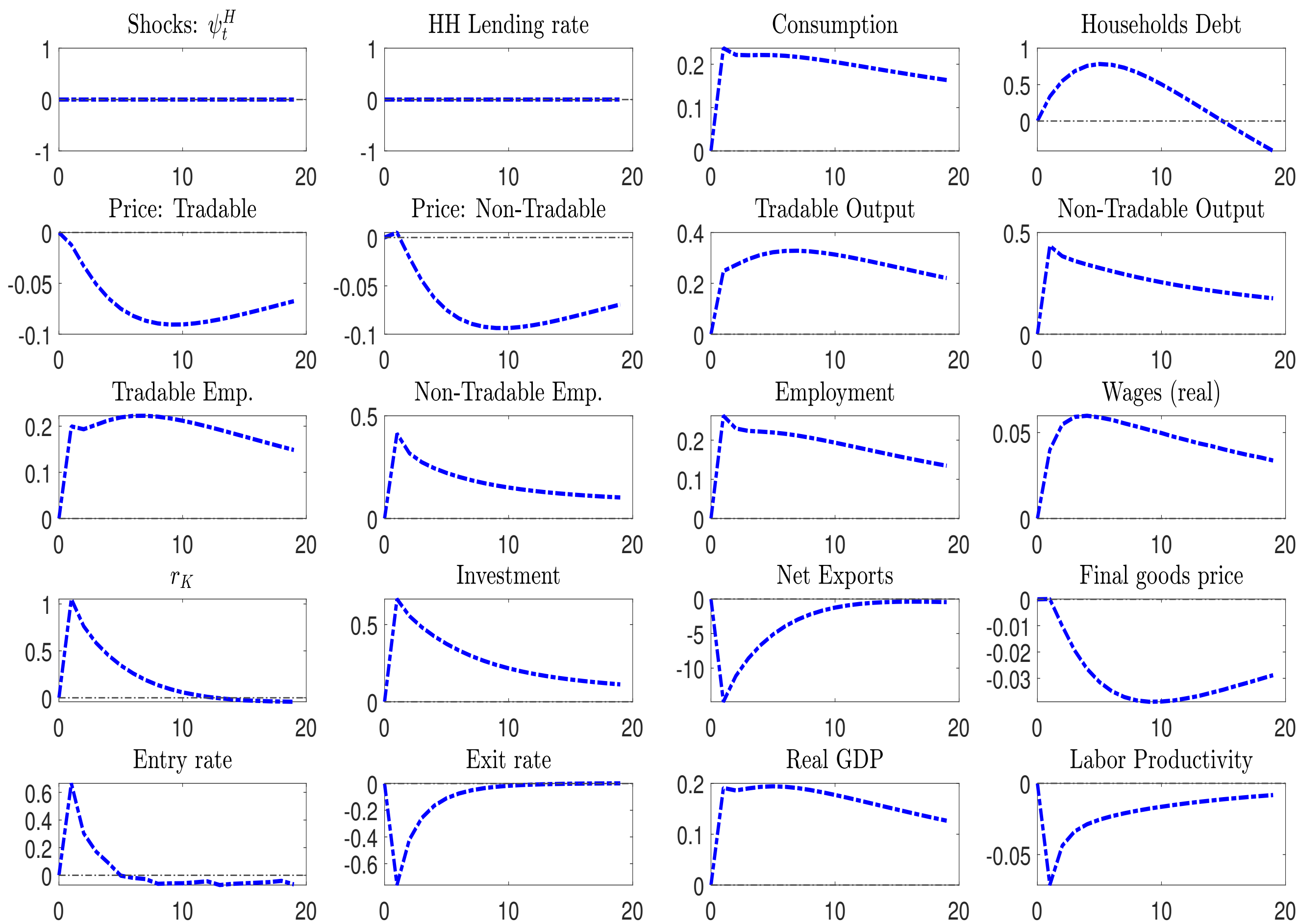


Notes: his figure plots the model moment changes as one of the model parameter changes. In the first row, we change κ as in the x-axis, and plot the model's responses for 6 variables in the y-axis (from left to right, Real GDP, Total Employment, Real Wages, Entry rates, Exit rates, Household Debt). The model's responses are computed as the percent deviations from corresponding steady state values averaged from periods 2 to periods 10. Rows from 1 to 7 are for κ, c^F, ρ_z, ν, Θ, η, ζ, respectively.

C.3 AR(1) Shocks to Aggregate Productivity

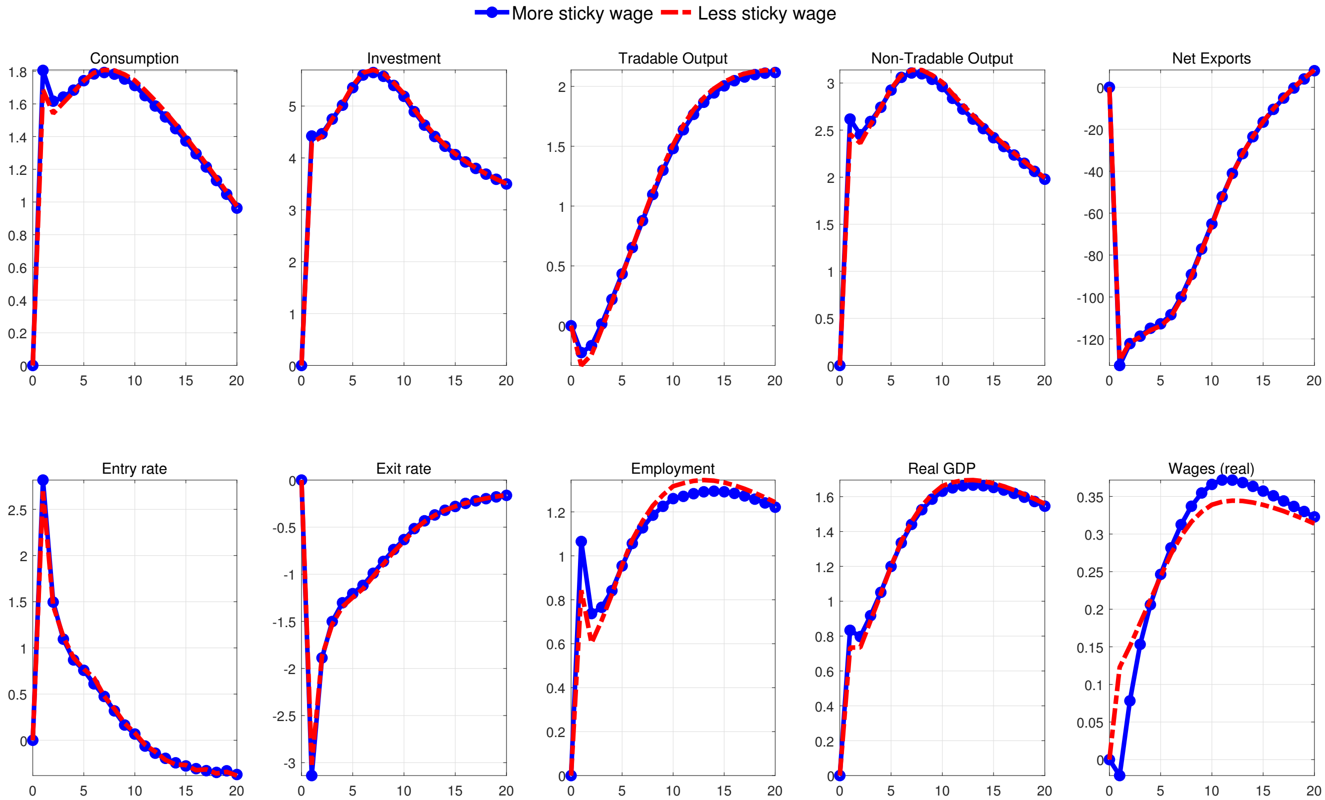
Shocks to Aggregate Productivity. We then study the model's responses with an aggregate productivity shock. This is a quite standard exercise in the literature for business cycles, and we can further understand how the model works. Specifically, we do it as follows:we slightly modify the wholesalers' production function to $y_i = (A_t z_i)^{1-\theta}(k_i^{1-\alpha} n_i^{\alpha})^{\theta}$, where $A_t = 1$ in steady states. Similarly, we assume there is an "MIT" shock to A_t in the beginning of period 1 so that A_1 is increased by 1%; over time, the shocks decay with a persistence parameter of 0.95. The results are now collected in Figure 19 and we only display the first 20 years. Like in a standard New Keynesian Model with productivity shocks, the nominal prices for the two types of goods both decline on impact and gradually return back, simply because with increased productivity, firms can produce more with the same amount of inputs. Since prices are with nominal frictions, firms still find it profitable to hire more labor and capital inputs and produce more. Thus, we see factor prices are all driven up (e.g., real wages and the real, capital rental cost), employment and output in the two sectors are also both increasing. Since the persistence of productivity shock is relatively high, labor income and profits increases are persistent and households increase their consumption in the first few periods and gradually return to the steady state. With GHH preferences in labor supply, households will supply more labor even if their wealth also increases, and thus the equilibrium employment overall increases. At the same time, households save more and accumulate more assets. The entry rate increases on impact and the exit rate decreases. Real GDP increases for about 0.2%, and total employment increases for about 0.25% on impact (note that using back of envelop calculation, $1-\theta$ is about 0.15, and the effective firm TFP, $(A_t)^{1-\theta}$, would increase for about 0.15%).

Figure 19: Model Responses to Aggregate Productivity AR(1) shocks



C.4 Sensitivity of Results to Changes in Model Parameters

Figure 20: The Role of Sticky Wages



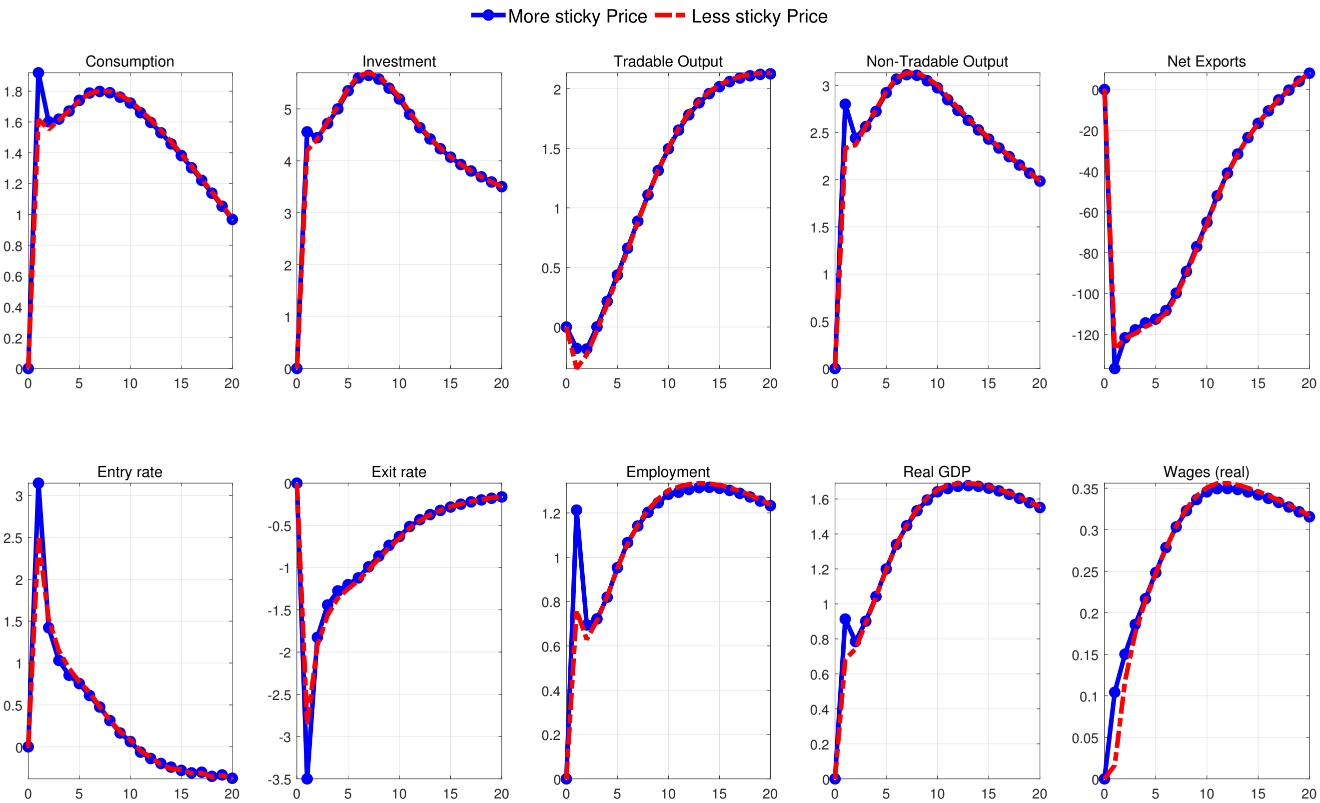
Notes: We vary the Calvo parameter for not adjusting nominal wages within one period, θ_w. We experiment with values of θ_w 50% higher and 50% lower than the benchmark value.

Table 6: Relative Importance of Household vs. Firm Channels: the Role of Firm Side Parameters

	Higher adj. cost		Lower adj. cost		Higher ν		Lower ν		Higher Θ		Lower Θ	
	(1) HHs	(2) Firms	(3) HHs	(4) Firms	(5) HHs	(6) Firms	(7) HHs	(8) Firms	(9) HHs	(10) Firms	(11) HHs	(12) Firms
GDP	27.9	70.8	28.9	70.1	28.2	70.6	28.3	70.5	28.4	70.4	28.0	70.8
Employment	28.8	69.9	29.8	69.1	29.1	69.6	29.2	69.6	29.4	69.4	28.8	69.9
Consumption	101.1	-5.4	88.3	8.5	94.6	1.6	94.3	2.0	94.4	1.8	94.4	1.9
Investment	7.1	92.7	8.0	91.9	7.4	92.4	7.4	92.3	7.2	92.5	7.6	92.1
Tradable output	-42.7	143.6	-23.8	124.2	-33.3	133.9	-31.3	132.0	-35.3	136.1	-29.7	130.2
Non-Tradable output	38.5	60.4	41.0	57.9	39.6	59.3	39.5	59.5	39.8	59.1	39.2	59.7
Entry rates	18.6	80.6	18.7	81.6	18.0	78.1	19.8	81.7	20.1	81.0	18.6	79.0
Exit rates	-20.6	-79.1	-20.6	-79.1	-20.1	-79.5	-25.0	-74.5	-21.9	-77.9	-19.3	-79.3

Notes: The numbers represent the percent of the benchmark response that can be explained by shocks to the household interest rate only and shocks to the firm interest rate only.

Figure 21: The Role of Sticky Prices



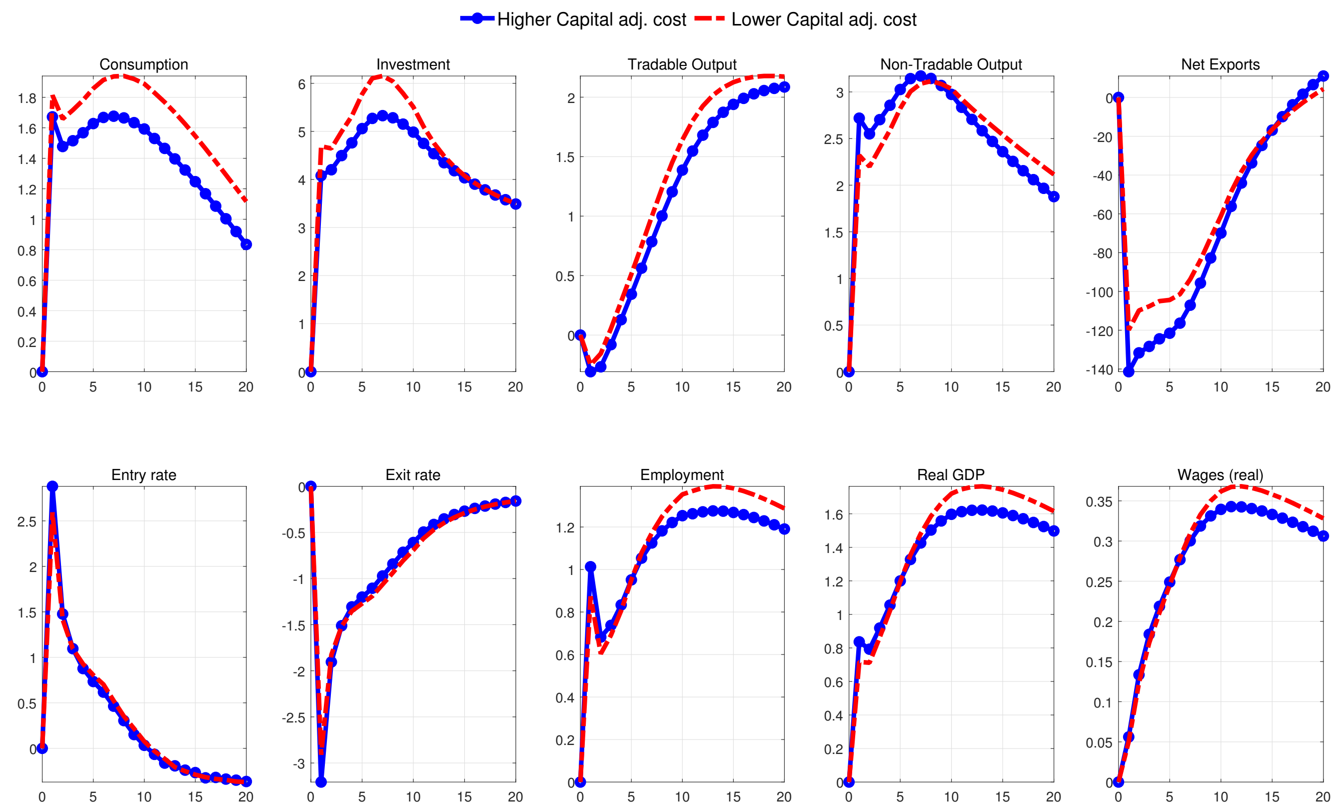
Notes: We vary the value of ϑ_P, the Calvo parameter for not adjusting nominal goods prices within one period, to be 50% higher or 50% lower than the benchmark value.

Table 5: Relative Importance of Household vs. Firm Channel: the Role of rigid Wages and Prices

	More rigid wage		Less rigid wage		More rigid Price		Less rigid Price	
	(1) HHs	(2) Firms	(3) HHs	(4) Firms	(5) HHs	(6) Firms	(7) HHs	(8) Firms
GDP	28.6	70.2	28.1	70.7	28.7	70.1	28.0	70.8
Employment	30.0	68.8	28.8	70.0	30.0	68.8	28.6	70.2
Consumption	94.2	2.1	94.5	1.7	93.9	2.4	94.8	1.4
Investment	7.5	92.3	7.5	92.3	7.6	92.2	7.4	92.4
Tradable output	-29.4	130.0	-34.0	134.7	-28.6	129.2	-35.2	135.9
Non-Tradable output	39.7	59.2	39.5	59.4	39.8	59.1	39.5	59.5
Entry rates	18.5	81.0	19.5	81.6	19.4	79.2	19.8	82.2
Exit rates	-20.5	-79.1	-20.7	-79.1	-21.2	-78.4	-20.8	-79.2

Notes: The numbers represent the percent of the benchmark response that can be explained by shocks to the household interest rate only and shocks to the firm interest rate only. We vary the value of ϑ_P and θ_w to be 50% higher or 50% lower than the benchmark value.

Figure 22: Impulse Responses with Different Capital Adjustment Costs



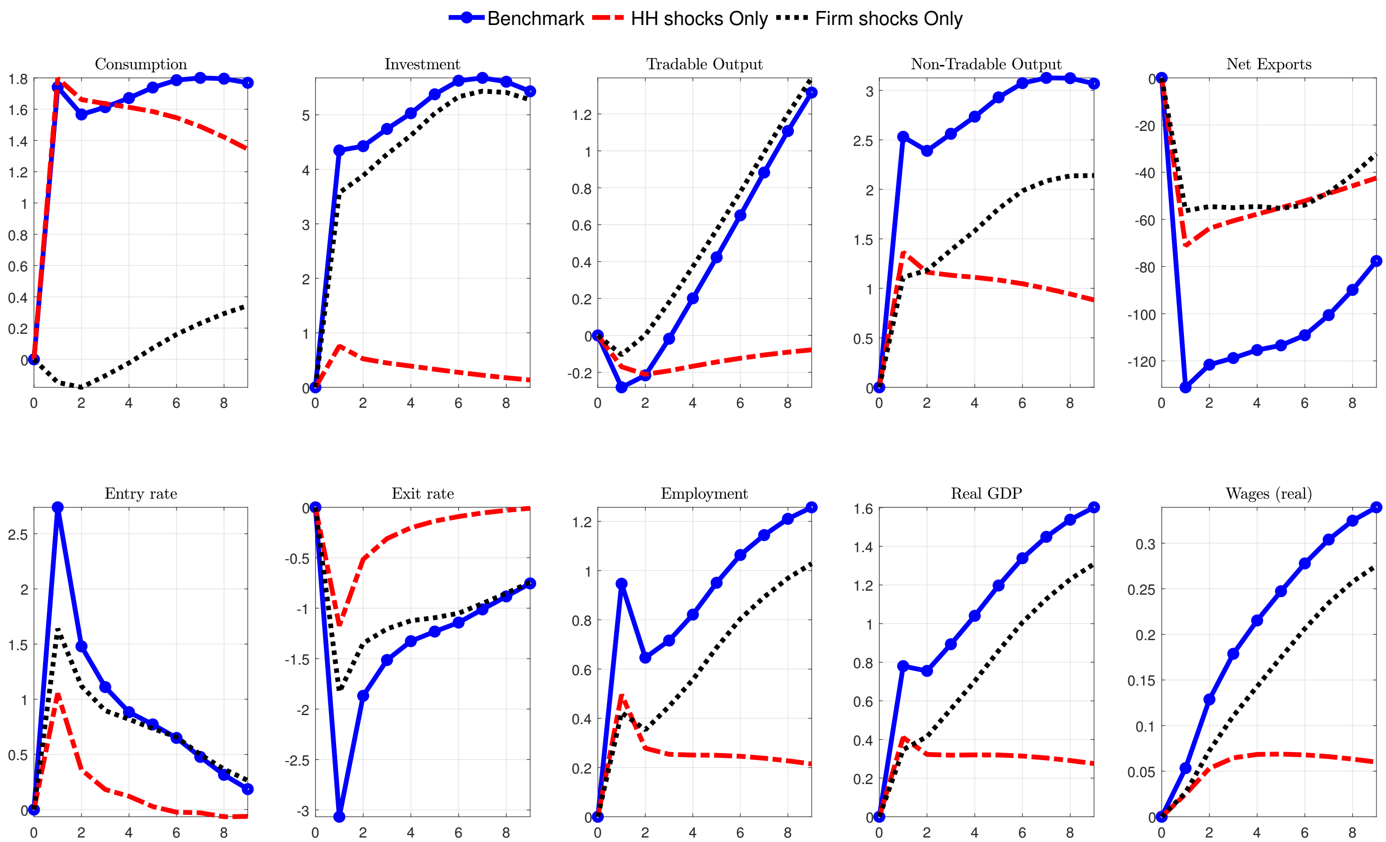
Notes: We compare responses with different values of the capital adjustment cost parameter κ, 50% higher or 50% lower than the benchmark value.

Table 7: Relative Importance of Household vs. Firm Channel: the Role of Household Side Parameters

	Higher χ		Lower χ		Higher $1/\upsilon_L$		Lower $1/\upsilon_L$		Higher σ		Lower σ	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	HHs	Firms	HHs	Firms	HHs	Firms	HHs	Firms	HHs	Firms	HHs	Firms
GDP	27.6	71.2	29.0	69.9	27.3	71.5	31.8	67.1	17.1	81.4	34.2	64.8
Employment	28.5	70.3	29.9	68.9	28.1	70.6	32.8	66.1	17.7	80.7	35.2	63.8
Consumption	94.5	1.7	94.4	2.0	92.8	3.4	99.7	-3.0	86.9	5.9	96.6	0.6
Investment	7.2	92.5	7.7	92.1	7.2	92.5	8.1	91.8	4.1	95.5	9.5	90.4
Tradable output	-31.0	131.6	-34.1	134.8	-25.6	126.1	-84.5	186.1	-14.9	115.7	-47.2	147.6
Non-Tradable output	38.8	60.1	40.4	58.6	38.1	60.8	44.8	54.3	25.4	72.8	46.5	52.8
Entry rates	19.0	82.2	19.4	81.5	20.6	81.8	23.9	80.1	10.5	88.1	23.8	76.7
Exit rates	-20.0	-79.7	-21.2	-78.6	-19.7	-79.9	-24.0	-76.3	-12.0	-87.3	-25.5	-74.5

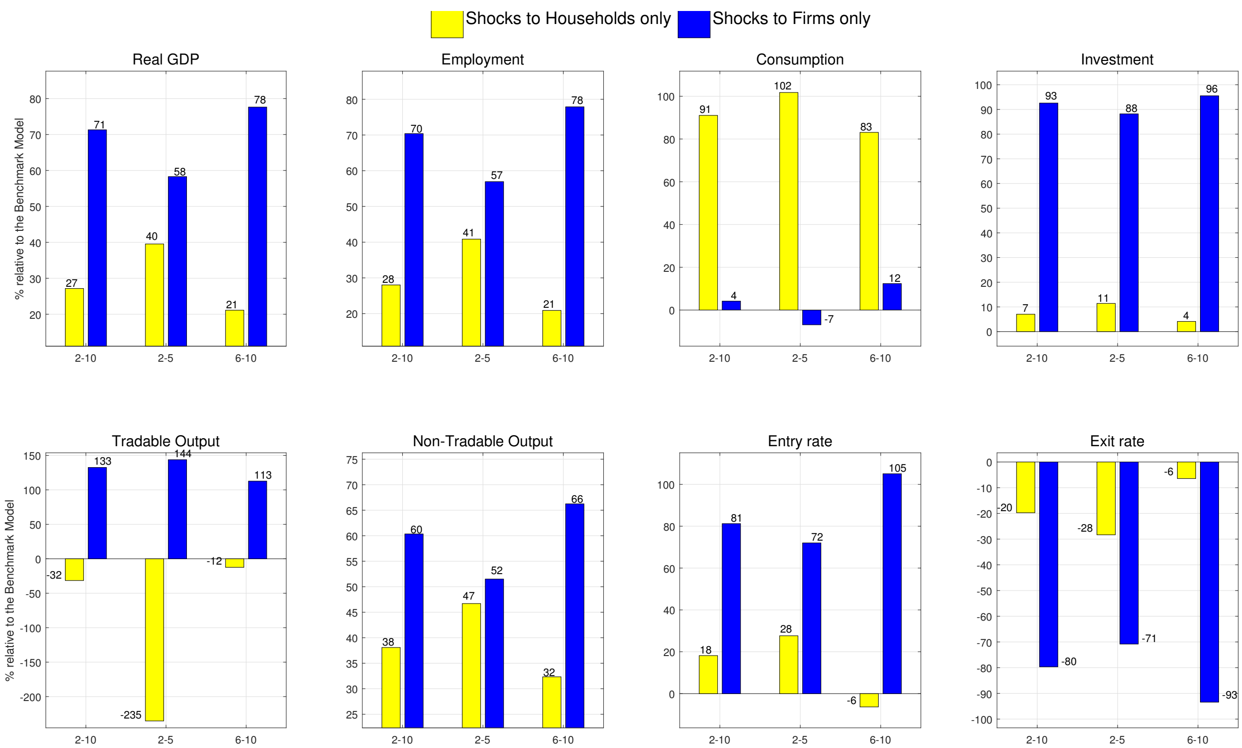
Notes: The numbers represent the percent of the benchmark response that can be explained by shocks to the household interest rate only and shocks to the firm interest rate only. We vary the share of hand-to-mouth households (i.e. χ) from 50% below our benchmark calibration (i.e. close to 10%) to 50% above the benchmark (close to 40%). We vary the labor supply elasticity parameter $1/\upsilon_L$ (with different values of 5 and 1.25), and the elasticity of intertemporal substitution.

Figure 23: Larger Firm Debt



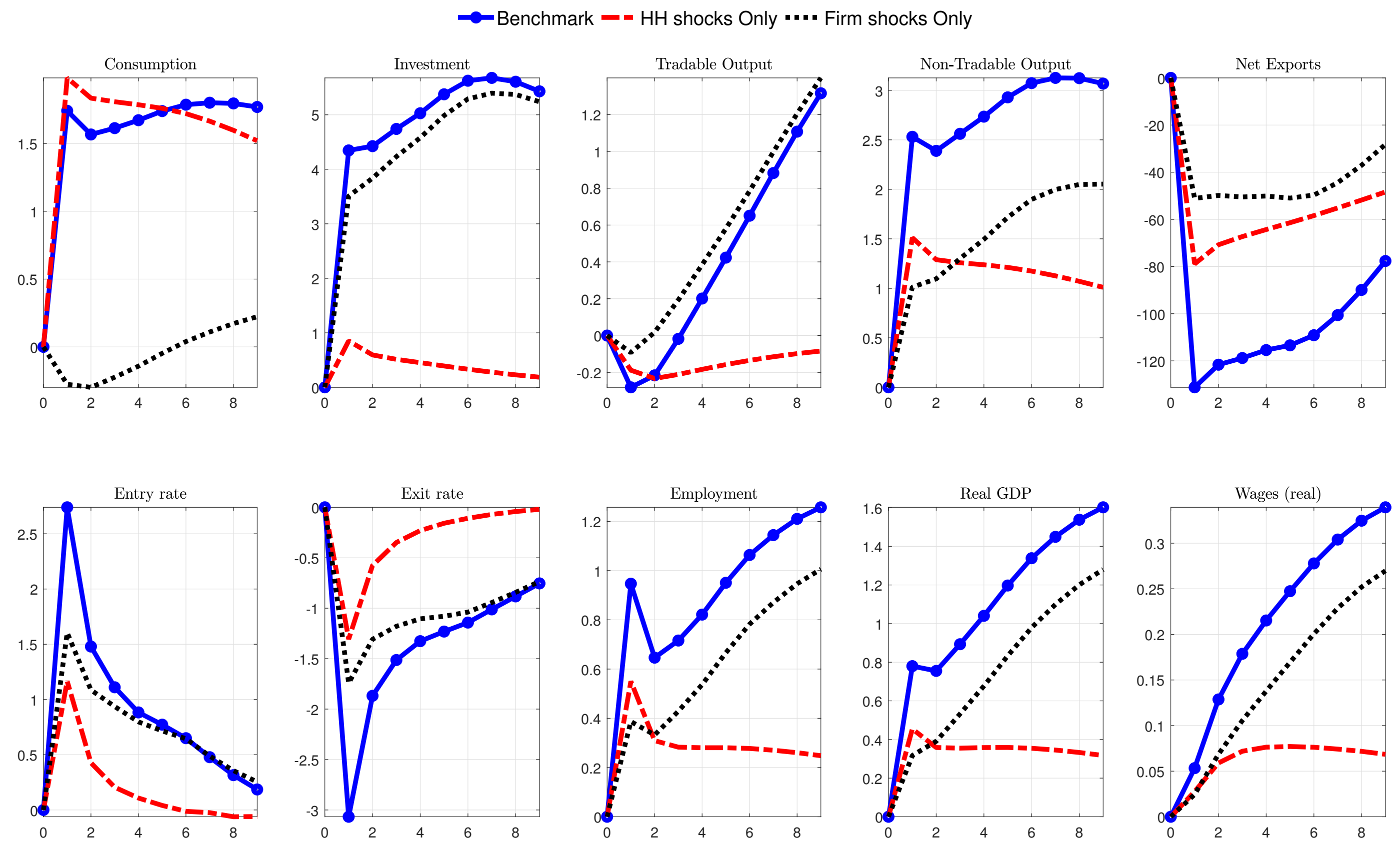
Notes: We consider a case where firm debt is 50% larger than in the baseline steady state. Household debt also adjusts to the change in dividend policy so that the total indebtedness of the domestic economy at the initial steady state is the same as in the baseline calibration.

Figure 24: Larger Firm Debt: Household vs. Firm Channels



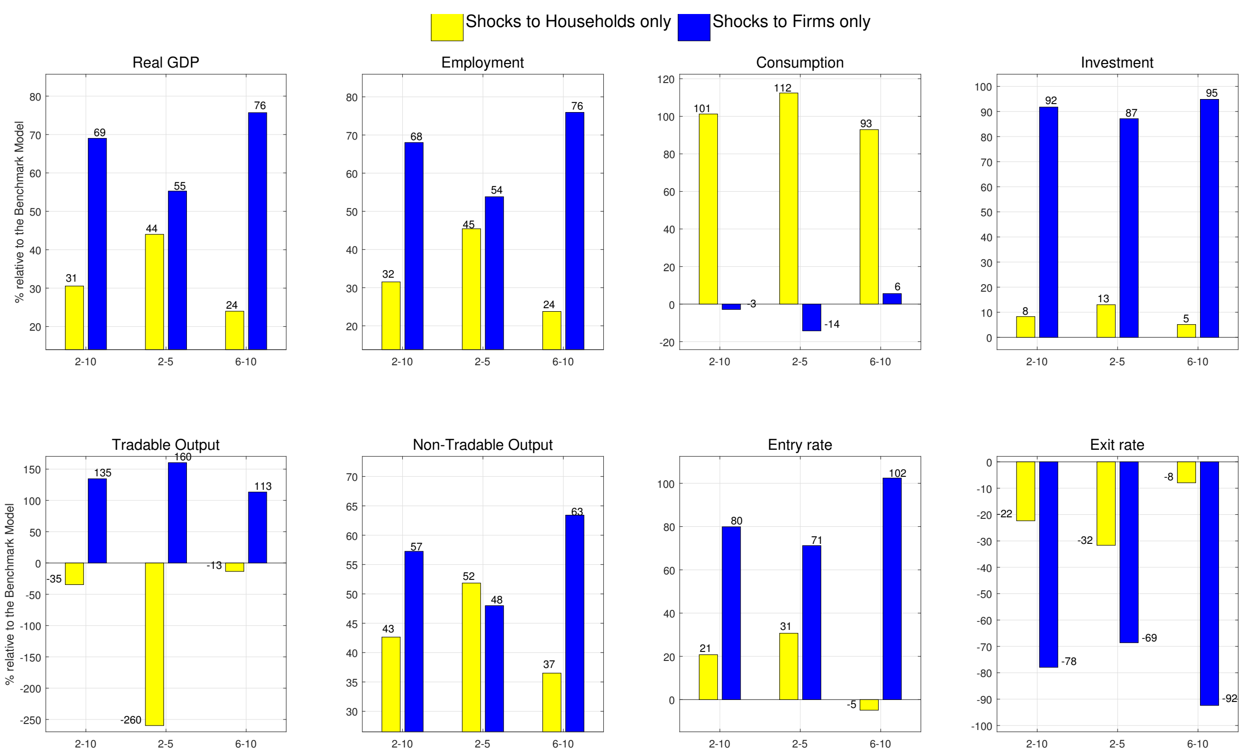
Notes: We consider a case where firm debt is 50% larger than in the baseline steady state. Household debt also adjusts to the change in dividend policy so that the total indebtedness of the domestic economy at the initial steady state is the same as in the baseline calibration. The numbers represent the percent of the benchmark response that can be explained by shocks to the household interest rate only and shocks to the firm interest rate only.

Figure 25: No Firm Debt



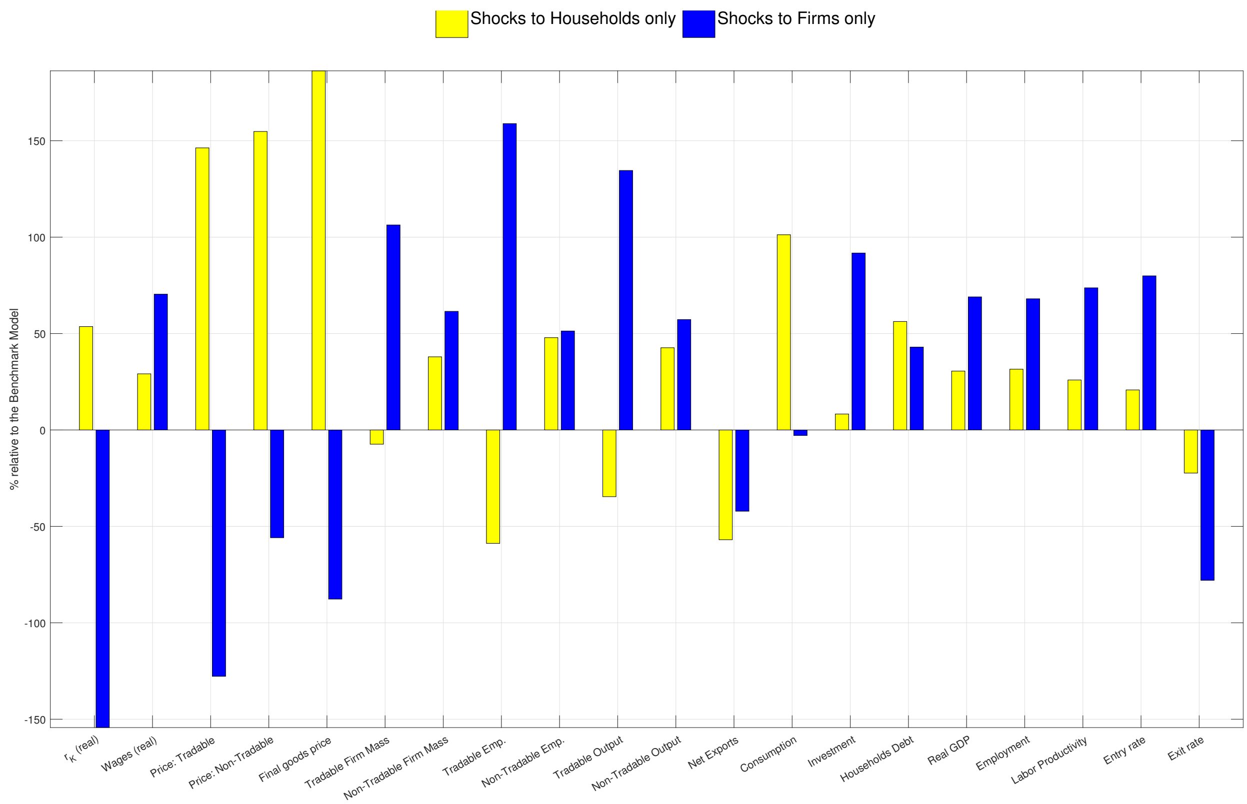
Notes: We consider a case where initial firm debt is equal to zero. Household debt also adjusts to the change in dividend policy so that the total indebtedness of the domestic economy at the initial steady state is the same as in the baseline calibration.

Figure 26: No Firm Debt: Household vs. Firm Channels



Notes: We consider a case where initial firm debt is equal to zero. Household debt also adjusts to the change in dividend policy so that the total indebtedness of the domestic economy at the initial steady state is the same as in the baseline calibration. The numbers represent the percent of the benchmark response that can be explained by shocks to the household interest rate only and shocks to the firm interest rate only.

Figure 27: No Firm Debt: Household vs. Firm Channels; (con't) for Other Variables



Notes: We consider a case where initial firm debt is equal to zero. Household debt also adjusts to the change in dividend policy so that the total indebtedness of the domestic economy at the initial steady state is the same as in the baseline calibration. The numbers represent the percent of the benchmark response that can be explained by shocks to the household interest rate only and shocks to the firm interest rate only.