Capital Deaccumulation and the Large Persistent Effects of Financial Crises

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Abstract

In a panel of OECD and emerging economies, I find that recessions are associated with larger initial drops in investment and more persistent drops in output if they occur simultaneously with banking crises. Furthermore, the banking crises that are followed by more persistent output slumps are associated with particularly large initial drops in investment. I show that these patterns can arise in a model where a financial shock temporarily increases the costs of external finance for investing entrepreneurs. This leads to a drop in investment and a very persistent slump in output and employment, provided wages are sufficiently rigid. Critical to the model is the distinction between different types of capital with different depreciation rates. Intangible capital and equipment have high depreciation rates, leading these stocks to drop substantially when investment falls after a financial shock. I find that this mechanism can account for almost a third of the persistent drop in output and employment in the US Great Recession (2007-2014).

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1 Introduction

This paper proposes a model to account for the long and protracted slumps in output that often follow financial crises. The length of the slump that followed the financial crisis of 2008 in many countries has been seen as puzzling in the literature, with many real and monetary models with financial shocks predicting much sharper recoveries (see, for instance, Kydland and Zarazaga (2016); Galí et al. (2012)).[1] However, the long slump after 2008 does not appear to have been particularly exceptional. In a panel of 40 OECD and emerging economies from 1970 onwards, I find that output typically shows no sign of returning to trend 6 years after a recession featuring a banking crisis. This finding is consistent with a large empirical literature.[2] By contrast, the majority of banking crises in the dataset last fewer than 4 years, suggesting that the slump in output persists substantially beyond the period of financial stress. Research on the behavior of credit spreads also supports this interpretation. For instance, Krishnamurthy and Muir (2017) find that increases in the spread predict output losses during banking crises, but output does not immediately return to trend once spreads have returned to normal levels.

I hypothesize that drops in investment during financial crises may be a major force contributing to long subsequent slumps. I find in my panel of countries that recessions marked by banking crises not only feature longer-lived output drops than other recessions, but are also characterized by larger drops in investment at the onset of the recession.[3] I also find that banking crises that are followed by more persistent output drops are associated with larger drops in the investment-output ratio around the time of the crisis. Neither of these two findings can be explained by the fact that recessions which are initially more severe are also more persistent. This is because recessions marked by financial crises do not feature much larger initial output drops than other recessions, even in cases where the output drop is ultimately very persistent.

I show that these patterns can arise in a model where financial shocks have persistent effects on output through their effects on investment and the capital stock. In the model I present, a financial shock leads to a particularly sharp drop in investment, which causes a persistent decrease in the stock of equipment and intangible capital and, therefore, output. The quantitative importance I find for this mechanism contrasts with the existing literature, which has found movements in the aggregate capital stock, as conventionally measured, to be of less importance in economic fluctuations. This is because the depreciation rate is sufficiently low that

[1]Galí et al. (2012) argue that their New Keynesian model can account for the slowness of the recovery from the US Great Recession, but only if the US economy continued to be hit by repeated negative shocks in the years after 2008. No shock individually has a persistent negative effect on output.

[2] Empirical studies have found financial crises to be followed by very persistent decreases in output relative to the pre-crisis trend in the US, in the world and in the OECD. Such empirical studies include include Cerra and Saxena (2008), Furceri and Mourougane (2012), Jalil (2015), Krishnamurthy and Muir (2017) and Abiad et al. (2014). The latter also find financial crises to be followed by persistent decreases in employment.

[3]That financial crises are followed by particularly large investment slumps has also been documented by Ottonello (2017).

the capital stock does not fluctuate much except at very low frequencies, both in models and in the data.[4] The key novel feature of my model that overturns this conclusion and generates larger and persistent output slumps is the important role of equipment and intangible capital in production. Unlike standard models, I disaggregate the capital stock into structures, on the one hand, and equipment and intangibles on the other. These are treated as separate inputs into a Cobb-Douglas production function. Equipment and structures represent the two types of business capital traditionally measured in the national accounts. Intangible capital refers to the assets firms have built up through past investment in product design, marketing and customer support, research and development, human capital and organizational development. Many of these assets are still not included in US national accounts. Crucially, I calibrate a relatively large stock of intangible capital (34% of the total US capital stock), based on the estimates of Corrado and Hulten (2010).[5] The calibration implies an even larger role for intangibles in production, so that the economic importance of equipment and intangibles is even greater than their share of the capital stock would imply.[6]

The propagation mechanism in the model results from movements in the stock of equipment and intangible capital, rather than from movements in structures. This is because equipment and intangible capital differ from structures in their depreciation rates. Equipment and intangible capital face estimated depreciation rates of around 20% per year, whereas the depreciation rate of structures is around 4% per year. I model a financial shock as a transitory increase in the riskiness of investment, which reduces the ability of entrepreneurs to borrow externally. This leads to a short-lived rise in the credit spread and a large drop in investment. The decrease in investment causes a substantial decrease in the stock of equipment and intangibles, due to the high depreciation rate of these types of capital. By contrast, the stock of structures does not decrease as much, due to the low depreciation rate of structures. Nevertheless, since equipment and intangibles contribute much more to production than do structures, the net effect is a decrease in output which persists until the stock of equipment and intangibles can be replenished. This takes many years. I find that the model produces a decrease in output after a financial shock that is around 50% larger than occurs in an almost identical setting with only one type of capital and a depreciation rate typical of the literature.

As the model would suggest, the US Great Recession after 2008 witnessed a persistent slump in output, a large decrease in the stock of equipment and intangibles relative to trend and a

[4]See Schwartzman (2012) for more discussion of this.

[5]These estimates are not atypical of the recent intangible capital literature. This is discussed in more depth in the discussion of calibration in the online appendix.

[6]For the model to match long-run average investment rates in structures, equipment and intangibles in the steady state, I find that the elasticity of output with respect to equipment and intangibles needs to be 0.38, versus an elasticity of 0.15 for structures. The calibration implies such a high elasticity of output with respect to equipment and intangibles is necessary to match the pattern of investment given the high depreciation rate of these assets. See the online appendix for details.

smaller decrease in the stock of structures. However, the crisis of 2008, like many other financial crises, was also followed by a large and persistent decrease in employment. I find that the model can replicate this feature of the data, provided that wages are rigid to an extent that appears consistent with US aggregate wage data. In order to allow for the possibility of wage rigidity, I model the labor market using search frictions, with wages set by an alternating offer bargaining protocol similar to Hall and Milgrom (2008). This bargaining procedure nests the traditional Nash Bargaining solution of the labor search literature as a special case, but can also lead to greater wage rigidity endogenously, depending on the model parameters. I estimate the parameters of this bargaining process using a Bayesian approach, based on NIPA and BLS data on wages, output and employment. I find that the data favors a parametrization that implies more rigid wages than Nash bargaining. At the estimated parameter values, a financial shock leads to a persistent slump in employment, with employment taking six years to recover half-way to the steady state after the shock. By contrast, if standard Nash bargaining is used instead, employment hardly falls after a financial shock and the fall in output is consequently much more muted.

I find that the calibrated model can account for approximately one third of the the persistent decrease in output and employment observed during the US Great Recession. In particular, I analyze how the model economy responds to a series of financial shocks that lead to the the same spikes in the credit spread as in occurred in the US over the 2007-2014 period. In the model economy, these shocks produce a decrease in output and employment around a third as large as the fall in these variables relative to trend during 2007-2010. Moreover, I find that the resultant behavior of aggregate variables in the model shows striking similarities to actual economic developments between 2007 and 2014 in three other key respects. First, output and employment show no tendency to return to their steady state levels over the seven years after the financial crisis. Second, the model economy experiences a significant decrease in the stock of equipment and intangible capital after the financial shocks. Third, it experiences a smaller decrease in the stock of structures. In each of these three respects, the behavior of model variables matches that of detrended variables in the data. In sum, these similarities suggest that the declining stock of equipment and intangibles may be the cause of close to a third of the persistent slump in output and employment during the Great Recession.

I show that the effects of non-financial shocks in the model are consistent with the more transitory decreases in output that appear to occur in non-financial recessions. Specifically, I consider TFP shocks as well as shocks to the level of government employment, as in Michaillat (2014). Due to the existence of financial and search frictions, private employment does not change rapidly in response to changes in government employment in the short run. Consequently, the government employment multiplier is close to 1. Moreover, the responses of output to TFP and government employment shocks is sizable and shows some persistence, but rather less persistence than for financial shocks. In addition, TFP shocks and government employment shocks have less effect on investment than financial shocks. As such, these shocks may account for the more transitory decreases in output and smaller decreases in investment seen in non-financial recessions.

The remainder of the paper is organized as follows. Section 2 discusses how the paper relates to the existing literature. Section 3 considers the empirical behavior of output and investment across a large sample of financial and non-financial recessions. Section 4 presents the model. Section 5 discusses model calibration. Section 6 presents quantitative results and compares movements of variables in the model to the US Great Recession. Section 7 concludes.

2 Relevant Literature

This paper contributes a mechanism through which financial crises have persistent effects on aggregate output through movements in the stock of equipment and intangible capital. As far as I am aware, this is the first paper in the literature to emphasize that the relatively high depreciation rates of these types of capital imply that their stocks can decrease substantially after a financial shock, contributing to persistently low output and employment.

The paper relates most closely to the recent literature on modeling the persistent economic effects of financial crises, but the mechanism is distinct from those discussed in this literature. Much of this literature is based on endogenous growth models, such as in Queralto (2020), Anzoategui et al. (2019), Garcia-Macia (2017) and the literature discussed therein. In these papers a financial crisis can reduce innovation and so have a permanent negative effect on total factor productivity and output. Critical to the endogenous growth approach is that large scale spillovers arise from technological innovation, leading to persistent negative effects on other firms when a financially constrained firm stops innovating. Unlike in these models, in my setting, the persistent effects of financial crises occur entirely within an exogenous growth context, without any spillover effects of intangible investment.

Other models of the persistent economic effects of financial crises include those of Ottonello (2017), Khan and Thomas (2013) and Schmitt-Grohé and Uribe (2017) and the literature discussed therein. Among these, the closest to this paper is Ottonello (2017) who explores the relationship between 'investment slumps' and slow recoveries after financial crises. However, the channel he focuses on is different, relying on capital becoming unemployed after a financial crisis rather than a deaccumulation of capital after the crisis. My approach differs from all these papers in drawing a connection between low investment in intangibles and the persistence of financial crises.

This paper also closely relates to the literature on the measurement of intangible investment and its behavior over the business cycle and during the Great Recession. I rely heavily on the work estimating the stock of intangible capital and intangible investment in the United States by Corrado and Hulten (2010). McGrattan and Prescott (2014) consider the role played by intangible capital in the Great Recession, and find that the importance of technology shocks in business cycles and the Great Recession is increased when allowance is made for intangible capital. Hall (2015) argues that low investment and the deaccumulation of capital following the financial crisis had a sizeable and persistent negative effect on the economy's aggregate capacity, but does not construct a general equilibrium model that gives rise to this mechanism.

The paper also relates to recent work by Perez-Orive (2016) and Lopez and Olivella (2018) on the role of intangible capital in the transmission of financial shocks. These models emphasize different channels of transmission from mine. Perez-Orive (2016) does not consider effects of financial shocks on employment and assumes that intangible capital depreciates slowly, contra the evidence discussed in Corrado and Hulten (2010). Lopez and Olivella (2018) focus on the non-pledgability of intangible capital as collateral. Compared to this paper, their model generates a much less persistent output drop after a financial shock.

Finally, this paper also relates to the literature on the interaction between financial frictions and search frictions in labor markets. Prominent examples include Garin (2015) and Petrosky-Nadeau (2014).

3 Motivating Evidence

A large literature has found that banking crises are associated with particularly persistent declines in output.[7] In this section I present some evidence from a sample of developed and emerging economies that the persistent declines in output after financial crises might be due to low levels of investment as a result of the crisis. I consider 'financial' and 'non-financial' recessions in a sample consisting of OECD countries and countries tracked by JP Morgan's Emerging Markets Outlook over the period 1970-2014.[8] I define a recession as 'financial' if it occurs within a year of a banking crisis, as measured by Reinhart and Rogoff (2009), and 'non-financial' otherwise.[9] I use World Bank annual data and drop countries for which GDP or investment data is not available for the whole sample period, leaving a sample of 21 OECD and 19 emerging economies.

Figure 1a, in the top left panel of Figure 1, shows the average decrease in GDP relative to

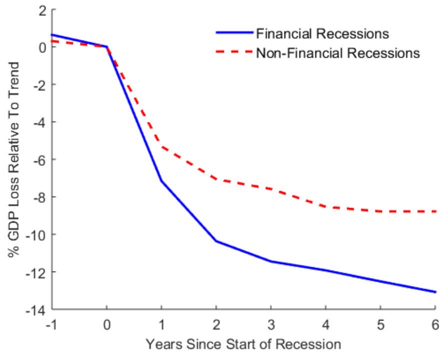
[7] See footnote 2 above for some examples from this literature.

[8] Calvo et al. (2006) argue that the Emerging Markets Outlook provides a useful list of emerging economies to study in the context of financial crises, since whether a country appears in the Emerging Markets Outlook can be viewed as a proxy for whether it has well-developed financial markets.

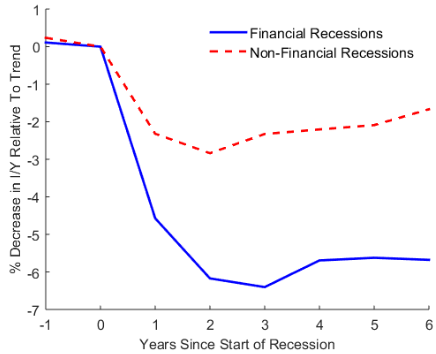
[9] Reinhart and Rogoff define a banking crisis as the closure, merger, takeover or large scale government assistance of one or more major financial institutions in a context marked either by bank runs or by a string of such outcomes for similar institutions. The total number of financial recessions in the sample is 33, including 16 in the OECD. The total number of non-financial recessions is 69, including 32 in the OECD.

Figure 1: Decreases in Output and the Investment-Output Ratio after Financial and Non-Financial Recessions

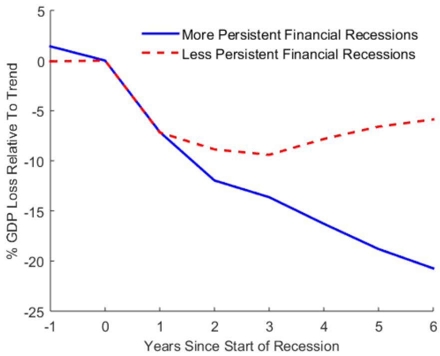
(a) Output Decrease in Financial vs. Non-Financial Recessions



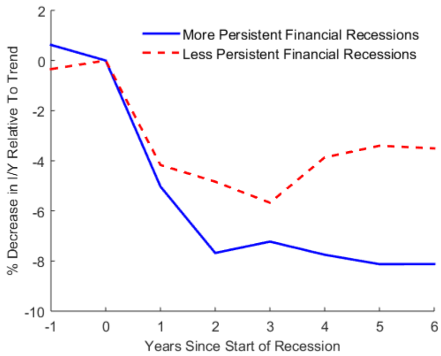
(b) I/Y Decreases in Financial vs. Non-Financial Recessions



(c) Output Decrease in Persistent vs. Non-Persistent Financial Recessions



(d) I/Y Decrease in Persistent vs. Non-Persistent Financial Recessions



Notes: GDP and investment data from the World Bank. Sample consists of the 21 OECD and 19 emerging market economies tracked by JP Morgan's Emerging Markets Outlook for which data is available over the period 1970-2014. Figure 1a shows the average decrease in GDP relative to the 8-year linear pre-recession trend, in the years during and after a financial and non-financial recession. The year of the recession is marked as '1' on the X axis. Financial recessions are recessions that occur within a year of a banking crisis, as measured by Reinhart and Rogoff (2009). Figure 1b shows the behavior of the investment-output ratio $\frac{I}{Y}$ relative to the 8-year pre-recession trend for financial and non-financial recessions. Figure 1c and 1d show the behavior of GDP and $\frac{I}{Y}$ under persistent and non-persistent financial recessions. Persistent and non-persistent financial recessions are defined according to equation (1).

the 8-year linear pre-recession trend, in the years during and after a financial and non-financial recession. The year of the recession is marked as '1' on the X axis. It is apparent from the figure that both financial and non-financial recessions are associated with decreases in GDP that are surprisingly persistent. There is no evidence of a return of GDP to trend 6 years after the last pre-recession year. Furthermore, Figure 1a is consistent with other findings in the literature that financial recessions are especially associated with persistently poor output performance. Indeed, while the output drop is slightly larger for financial recessions than others in the year of the recession, the gap between the two lines increases noticeably over the next 5 years.

However, the difference between financial and non-financial recessions is starker for the behavior of investment. Figure 1b shows the behavior of the investment-output ratio $\frac{I}{Y}$ relative to the pre-recession trend for financial and non-financial recessions. The drop in $\frac{I}{Y}$ in Figure 1b is more than twice as large for financial recessions in the first year of the recession and the following year. By contrast, the drop in GDP in Figure 1a was only moderately larger for financial recessions in these years. Thus, these two figures suggest that financial recessions may be associated with somewhat larger GDP drops than non-financial recessions in the years immediately after the recession. However, they suggest, that financial recessions may see substantially larger drops in $\frac{I}{Y}$ in these years.

This begs the question of whether the persistently poor performance of GDP after financial recessions may be related to the drop in investment during the recession. Some evidence suggesting that this may be the case is provided by Figures 1c and 1d. These figures show the behavior of GDP and investment in 'more persistent' as compared to 'less persistent' financial recessions. I define a financial recession as 'more persistent' if the output loss relative to trend satisfies the following condition:

$$\min[YLoss_5; YLoss_6] > \max[YLoss_1; YLoss_2; YLoss_3] \quad (1)$$

where $YLoss_i$ denotes the loss in GDP, relative to the linear pre-recession trend, i years after the last pre-recession year. This rough measure of persistence is designed to reduce the risk that recessions are marked as more persistent simply because another recession rapidly follows, or because the recession starts late in the year.[10] In the sample, approximately half of financial recessions are labelled as more persistent by this measure.

Figure 1c indicates that more persistent financial recessions do not immediately feature a much larger drop in output than less persistent financial recessions, but they do, unsurprisingly, feature a much more persistent drop in output. On the other hand, Figure 1d indicates that more persistent financial recessions see a significantly larger drop in investment within the first 2 years. This is consistent with the possibility that a large initial drop in investment foreshadows a slower recovery from the recession, and that the large initial drops in investment during financial recessions may play a role in the slow recoveries from these recessions.

Motivated by this evidence, the next section presents a model in which low investment in equipment and intangible capital during financial recessions contributes to the slow recoveries from these recessions. In Section 6.3, I show that the mode is consistent with dynamics of different types of investment and capital during the US Great Recession.

[10] The patterns are substantively very similar if recessions are instead measured as persistent when $\text{mean}[YLoss_5; YLoss_6] > \text{mean}[YLoss_1; YLoss_2; YLoss_3]$.

4 The Model

The model is built around a DSGE framework with two types of capital: structures and intangibles, which differ only in their depreciation rates. 'Intangibles' is intended to proxy for equipment and intangible capital in the data, both of which have relatively high depreciation rates in contrast to structures. There are two kinds of friction in the model: financial frictions, similar to Bernanke et al. (1999) and labor market frictions, along the lines of Mortensen and Pissarides (1994). The next section, 4.1, summarizes the agents in the model and their interactions. In the following sections I discuss the households and perfectly competitive firms first, before considering the financial frictions and wage bargaining.

4.1 Agents

Each household consists of a large family of workers and entrepreneurs who share consumption. In addition to workers and entrepreneurs, there are a large number of competitive banks, competitive capital-goods producing firms and competitive recruiting firms owned by households. Entrepreneurs own structures and intangibles, and employ workers. Entrepreneurs use their workers and capital to produce consumption goods. Capital-producing firms buy consumption goods, structures and intangible capital from entrepreneurs and transform them into new structures and intangibles, which they sell back to entrepreneurs. Recruiting firms carry out recruiting of workers on behalf of entrepreneurs. Banks exist in order to transfer funds from workers to entrepreneurs. Workers can hold risk-free deposits in banks and entrepreneurs can borrow from banks, in order to fund investment and hiring. In addition to workers, entrepreneurs, capital goods producers, recruiters and banks, there is a government, which taxes households and employs workers in order to produce public goods. The inclusion of the government sector allows for consideration of the effect of government employment shocks in the model but has little effect on the dynamic response to other shocks.

4.2 Preferences

All agents act to maximize the expected present discounted utility of their household. The preferences of the representative household are as follows.

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} \left(1 + \nu N_t + \nu N_{G,t}\right)^{\sigma} + \frac{Y_{G,t}^{1-\sigma}}{1-\sigma} \right] \tag{2}$$

Here C_t is aggregate consumption, N_t is the measure of workers who are employed in the private sector, $N_{G,t}$ workers are employed in the government sector and $Y_{G,t}$ is total production of public goods by the government sector. It is assumed that the total measure of workers in the economy is equal to 1. Hence, at any time t, there are $1 - N_t - N_{G,t}$ unemployed workers. σ denotes the coefficient of risk aversion and ν determines the disutility of working.[11]

4.3 Shocks

I consider three distinct shocks in the model: a shock to the riskiness of investment by entrepreneurs, denoted by ς_t, a government employment shock affecting $N_{G,t}$ and a shock to total factor productivity, denoted by Z_t. I assume that $\varsigma_t = \overline{\varsigma} e_{\varsigma,t}$, $N_{G,t} = \overline{N}_G e_{g,t}$ and $Z_t = \overline{Z} e_{z,t}$, where $e_{\varsigma,t}$, $e_{g,t}$ and $e_{z,t}$ follow AR(1) processes in logs:

$$\log(e_{\varsigma,t}) = \rho_\varsigma \log(e_{\varsigma,t-1}) + \epsilon_{\varsigma,t} \tag{3}$$

$$\log(e_{g,t}) = \rho_g \log(e_{g,t-1}) + \epsilon_{g,t} \tag{4}$$

$$\log(e_{z,t}) = \rho_z \log(e_{z,t-1}) + \epsilon_{z,t} \tag{5}$$

The values of all aggregate shocks are revealed at the start of the period. I assume ς_{t+1} is revealed one period in advance, at time t.

4.4 Capital Goods Producers

The consumption good is the numeraire in this economy. At the end of each period t, capital goods producers buy consumption goods, structures and intangible capital from entrepreneurs and transform these into new structures and intangible capital according to a constant returns technology. They sell these capital goods back to entrepreneurs immediately, who use them for production in the period $t + 1$.

The representative capital goods firm produces structures and intangibles according to the production functions:

$$K_{S,t+1} = (1 - \delta_S) K_{S,t} + I_{S,t} - \kappa_S \left(\frac{I_{S,t}}{I_{S,t-1}}, \frac{I_{S,t}}{K_{S,t}} \right) I_{S,t} \tag{6}$$

$$K_{I,t+1} = (1 - \delta_I) K_{I,t} + I_{I,t} - \kappa_I \left(\frac{I_{I,t}}{I_{I,t-1}}, \frac{I_{I,t}}{K_{I,t}} \right) I_{I,t} \tag{7}$$

where $I_{S,t}$ denotes the consumption goods used by the capital goods firm in the production of new structures and $I_{I,t}$ denotes the consumption goods used by the firm in the production

[11] Total household utility will take this form if the household allocates aggregate consumption C_t between its employed workers and unemployed workers in order to maximize the sum of its workers' utility, where an unemployed worker consuming $c_{u,t}$ gains utility $\frac{c_{ut}^{1-\sigma}}{1-\sigma} + \frac{Y_{G,t}^{1-\sigma}}{1-\sigma}$ and an employed worker consuming $c_{e,t}$ gains utility equal to $\left(1 - \frac{\nu}{1+\nu}\right)^{-\sigma} \frac{c_{et}^{1-\sigma}}{1-\sigma} + \frac{Y_{G,t}^{1-\sigma}}{1-\sigma}$. See Chodorow-Reich and Karabarbounis (2016) for a detailed derivation of the optimal allocation of household consumption between employed and unemployed agents under almost identical assumptions, but for the case of log rather than power utility.

of new intangibles. $K_{S,t}$ and $K_{I,t}$ denote the quantity of existing structures and intangibles, respectively, used by the firm in the production of new structures and intangibles. As will be seen shortly, $K_{S,t}$ and $K_{I,t}$ will be equal, in equilibrium, to the aggregate stock of structures and intangibles in the economy.

Here $\kappa_S(\cdot)$ and $\kappa_I(\cdot)$ are convex adjustment cost functions, given by:

$$\kappa_S\left(\frac{I_{S,t}}{I_{S,t-1}}, \frac{I_{S,t}}{K_{S,t}}\right) = \kappa_1\left(\frac{I_{S,t}}{I_{S,t-1}} - 1\right)^2 + \kappa_2\left(\frac{I_{S,t}}{K_{S,t}} - \frac{I_S^\star}{K_S^\star}\right)^2 \frac{K_S^\star}{I_S^\star}\frac{K_{S,t}}{I_{S,t}} \tag{8}$$

$$\kappa_I\left(\frac{I_{I,t}}{I_{I,t-1}}, \frac{I_{I,t}}{K_{I,t}}\right) = \kappa_1\left(\frac{I_{I,t}}{I_{I,t-1}} - 1\right)^2 + \kappa_2\left(\frac{I_{I,t}}{K_{I,t}} - \frac{I_I^\star}{K_I^\star}\right)^2 \frac{K_I^\star}{I_I^\star}\frac{K_{I,t}}{I_{I,t}} \tag{9}$$

Here variables with a star superscript denote the value of quantities in the steady state. Thus, there are adjustment costs to both the level of the capital stock and the level of investment.[12]

Capital goods cannot be stored from one period to the next without the aid of the capital goods producing firms. It may be assumed that structures and intangibles capital units $K_{S,t}$ and $K_{I,t}$ are damaged in period t as a result of being used to produce consumption goods and are in need of repair. In order to be used in period $t+1$, capital must be converted into new capital units $K_{S,t+1}$ and $K_{I,t+1}$ by a capital goods producer, through the production functions (6) and (7).

Consequently, at the end of period t, entrepreneurs willingly sell their entire stock of structures and intangibles to the capital goods producers, who use them to produce new capital units $K_{S,t+1}$, $K_{I,t+1}$, which they sell back to entrepreneurs, ready for use in the next period. Capital goods producers buy units of structures and intangibles from entrepreneurs at prices $q_{S,t}$ and $q_{I,t}$ and then sell structures and intangibles back to entrepreneurs at prices $\tilde{q}_{S,t}$ and $\tilde{q}_{I,t}$.[13]

Capital goods firms choose the quantity of new structures and intangibles they wish to produce in order to maximize the present discounted value of their profits, which they distribute to the households.

[12] These functional forms are chosen to coincide with capital and investment adjustment cost functional forms considered by Christiano and Davis (2006). In section 6.1, I discuss the consequences of removing adjustment costs to the level of capital or to the level of investment for the model.

[13] The assumption that entrepreneurs need to sell capital to capital goods producers who transform it into new capital is technically convenient because it makes adjustment costs to aggregate capital external to the entrepreneur. Very similar assumptions are also made in the related frameworks of Bernanke et al. (1999) and Christiano et al. (2014), for the same reason. Since entrepreneurs need the capital goods producers to transform capital from $K_{S,t}$ and $K_{I,t}$ into $K_{S,t+1}$ and $K_{I,t+1}$ in order to produce in the next period, entrepreneurs will be willing to sell their entire stock of capital to capital producers provided $\tilde{q}_{S,t}$ and $\tilde{q}_{I,t}$ are positive. Consequently, it is not in general the case that $q_{S,t} = \tilde{q}_{S,t}$ and $q_{I,t} = \tilde{q}_{I,t}$ in equilibrium.

4.5 Government

The public sector produces public goods according to the production function:

$$Y_{G,t} = Z_G N_{G,t} \tag{10}$$

Since $Y_{G,t}$ enters into the household's utility an additively separable way in (2), the value of Z_G does not affect the decisions of any private agent.

The government pays public sector employees the wage $w_{G,t}$. It is assumed to set this equal to the average private sector wage w_t. The government funds wage payments to its employees using lump-sum taxes on households. Ricardian equivalence holds in the model, so it is without loss of generality to assume that the government balances its budget every period. That is, the lump sum tax τ_t is equal to total government spending:

$$\tau_t = w_t N_{G,t} \tag{11}$$

4.6 Labor Markets

Recall that workers are either employed or unemployed. Workers who are employed in the private sector are matched with entrepreneurs and separate with probability δ_N at the end of each period. Workers employed by the government also separate from jobs with probability δ_N. Unemployed workers search for jobs randomly and at the end of each period they find a job in the private sector with probability f_t and find a job in the public sector with probability $f_{G,t}$.

The probability of finding a job in the private sector f_t is given by the ratio of aggregate private hiring to the number of unemployed workers, that is:

$$f_t = \frac{N_{t+1} - (1 - \delta_N) N_t}{1 - N_t - N_{G,t}} \tag{12}$$

The probability of finding a job in the public sector is determined in a similar way:

$$f_{G,t} = \frac{N_{G,t+1} - (1 - \delta_N) N_{G,t}}{1 - N_t - N_{G,t}} \tag{13}$$

Entrepreneurs cannot hire workers directly. They must pay a recruiting firm, which posts vacancies, screens and trains workers and then passes these workers on to entrepreneurs. The recruiting firm charges each entrepreneur a price of h_t for use of this service.[14] The government does not hire workers directly either: it also pays the recruiting firm the fee h_t to recruit workers

[14] This assumption can be interpreted as entrepreneurs outsourcing their recruiting and training activities to recruiting firms. That hiring is carried out by recruiting firms is technically convenient because it allows adjustment costs to the level of vacancies that are external to the entrepreneurs.

on its behalf.

The costs of the representative recruiting firm consist of three parts. First of all, the recruiting firm must post vacancies in order to recruit workers at a fixed cost of h_1 per vacancy. Let v_t denote the number of private sector vacancies and $v_{G,t}$ the number of public sector vacancies posted by the representative recruiting firm in period t. Second, the recruiting firm must pay a convex cost of adjusting the number of vacancies it posts given by $\frac{h_2}{2}\left(\frac{v_t+v_{G,t}}{v_{t-1}+v_{G,t-1}}-1\right)^2(v_t+v_{G,t})$.[15] Third, once a vacancy has matched with a worker, the recruiting firm must pay an additional cost of h_0 units of output in order for the worker to be hired. These represent training costs.

Let the total number of matches in the labor market in period t be given by the Cobb-Douglas function $M_t = A_M^{1-\psi}(v_t)^{1-\psi}(1-N_t-N_{G,t})^{\psi}$. This depends on the number of private and public sector vacancies and the number of unemployed workers, given by $1-N_t-N_{G,t}$. Then the probability that a vacancy matches with a worker is given by $\left(\frac{A_M(v_t+v_{G,t})}{1-N_t-N_{G,t}}\right)^{-\psi}$. It is assumed that a law of large numbers holds such that the recruiting firm posts exactly $\left(\frac{A_M(v_t+v_{G,t})}{1-N_t}\right)^{\psi}$ vacancies for each worker it recruits.

Therefore, the period t profit made by the representative recruiting firm is equal to:

$$\begin{aligned}&(h_t-h_0)[N_{t+1}-(1-\delta_N)N_t+N_{G,t+1}-(1-\delta_N)N_{G,t}]\\&-h_1(v_t+v_{G,t})\left[1+\frac{h_2}{h_1}\left(\frac{v_t+v_{G,t}}{v_{t-1}+v_{G,t-1}}-1\right)^2\right]\end{aligned} \tag{14}$$

The total quantity of private sector vacancies posted is equal to:

$$v_t = [N_{t+1}-(1-\delta_N)N_t]\left(\frac{A_M(v_t+v_{G,t})}{1-N_t-N_{G,t}}\right)^{\psi} \tag{15}$$

and the total quantity of public sector vacancies is given by:

$$v_{G,t} = [N_{G,t+1}-(1-\delta_N)N_{G,t}]\left(\frac{A_M(v_t+v_{G,t})}{1-N_t-N_{G,t}}\right)^{\psi} \tag{16}$$

The representative recruiting firm chooses the number of vacancies it posts so as to maximize the present discounted value of its profits, which it returns to the household.

[15]In the calibrated model, these adjustment costs to vacancies are quantitatively tiny. Less than 0.25% of the marginal costs of hiring an extra worker for the recruiting firm in each period are due to the marginal cost of adjusting the total number of vacancies, for realizations of the shocks of 5 or fewer standard deviations away from the mean. Consequently, I find that the model dynamics are hardly affected by variation in the level of these vacancy adjustment costs in the neighborhood of this small value. However, I find that very small adjustment costs to vacancy posting are necessary to prevent indeterminacy from occurring in the model at relevant values of the other parameters. In fact, in the benchmark calibration I consider, indeterminacy does not occur even with zero adjustment costs. Consequently, starting from the benchmark calibration, the model results are almost unchanged if recruiting firms are removed and entrepreneurs hire workers directly, with no convex adjustment costs.

4.7 Entrepreneurs

An entrepreneur holds three types of physical asset in a period t: structures $k_{S,t}$, intangibles $k_{I,t}$, and workers n_t, which she uses to produce output in the form of consumption goods. Production occurs at the beginning of the period. At the end of each period, the entrepreneur sells some of her output to the household, for consumption purposes, and sells the rest of her output to the capital goods producing firms, who use it to produce new capital, and to recruiting firms, who use it to recruit workers. As explained in section 4.4, the entrepreneur's period t structures and intangibles are damaged in the process of production and so she cannot use them to produce subsequently. Therefore, she willingly sells her structures and intangibles to capital producing firms at prices $q_{S,t}$ and $q_{I,t}$ after she has used them to produce in period t. The entrepreneur may also sell her workers to other entrepreneurs at price h_t if she so desires. Rather than entrepreneurs literally selling workers to one another, this can be viewed as entrepreneurs selling equity in their private enterprises to one another, thereby selling a fraction of the value of their workers.

After selling her capital and output, the entrepreneur chooses the stock of capital and workers she wants to hold in order to produce in the next period. Denote her choices by $\tilde{k}_{S,t}, \tilde{k}_{I,t}, n_{t+1}$. The entrepreneur buys her capital from capital-producing firms, at prices $\tilde{q}_{S,t}$, $\tilde{q}_{I,t}$. She buys her workers from recruiting firms or other entrepreneurs at price h_t – recall that h_t was the fee charged by a recruiting firm to an entrepreneur for each worker hired.

After acquiring capital and workers at time t, an entrepreneur draws a shock ξ at the start of period $t+1$ which affects the value of her assets. Specifically, if an entrepreneur buys $\tilde{k}_{S,t}$ and $\tilde{k}_{I,t}$ units of structures and intangibles at the end of period t and draws shock ξ_{t+1} at the start of $t+1$, then the entrepreneur starts period $t+1$ with structures and intangibles given by:

$$k_{S,t+1} = \xi_{t+1}\tilde{k}_{S,t}$$
$$k_{I,t+1} = \xi_{t+1}\tilde{k}_{I,t}$$

As in Bernanke et al. (1999), I assume that ξ has a unit mean lognormal distribution that is independently drawn across time and across entrepreneurs.[16] Let ς_{t+1} denote the standard deviation of $\log(\xi)$ at time $t+1$, which varies over time in response to aggregate shocks as discussed in section 4.3 above.

After observing the shock ξ entrepreneurs use their workers and capital to produce output. In addition to affecting the entrepreneur's stock of capital, the shock ξ affects the entrepreneur's total factor productivity for one period. Indeed, an entrepreneur with assets $k_{S,t+1}, k_{I,t+1}$ and

[16] The shock ξ can be viewed as capturing the many risks that businesses face. For instance, businesses whose products become obsolete as a result of technological change may find that their capital has fallen substantially in value.

n_{t+1} that draws shock ξ_{t+1} produces output y_{t+1} according to:

$$y_{t+1} = Z_{t+1} k_{S,t+1}^{\alpha_S} k_{I,t+1}^{\alpha_I} (\xi_{t+1} n_{t+1})^{1-\alpha_I-\alpha_S} \tag{17}$$

$$\equiv \xi_{t+1} Z_{t+1} \tilde{k}_{S,t}^{\alpha_S} \tilde{k}_{I,t}^{\alpha_I} n_{t+1}^{1-\alpha_I-\alpha_S} \tag{18}$$

where the second line uses that $k_{S,t+1}$ and $k_{I,t+1}$ themselves depend on ξ_{t+1}. Here Z_{t+1} is the aggregate level of total factor productivity, which varies in response to shocks as discussed in section 4.3. Define $\tilde{y}_{t+1} = Z_{t+1} \tilde{k}_{S,t}^{\alpha_S} \tilde{k}_{I,t}^{\alpha_I} n_{t+1}^{1-\alpha_I-\alpha_S}$, so that we may write $y_{t+1} = \xi_{t+1} \tilde{y}_{t+1}$.

Simultaneously with production, the entrepreneur pays wages $w_{t+1}(\xi_{t+1})$, which in general depend on ξ_{t+1}. These wages are set by bargaining, as discussed in section 4.13 below. After period $t+1$ production, the entrepreneur sells her structures and intangibles to the capital producing firms at prices $q_{S,t}$, $q_{I,t}$. Furthermore, fraction δ_N of her workers exogenously separate from their jobs.

The net effect of this is that the entrepreneur who bought assets $\tilde{k}_{S,t}, \tilde{k}_{I,t}$ and n_{t+1} at time t earns a return on these assets R_{t+1}^K equal to:

$$R_{t+1}^K = \left(\frac{\xi_{t+1}\tilde{y}_{t+1} - w_{t+1}(\xi_{t+1}) n_{t+1}}{\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}} \right) + \left(\frac{\xi_{t+1}(q_{S,t+1}\tilde{k}_{S,t} + q_{I,t+1}\tilde{k}_{I,t}) + h_{t+1}(1-\delta_N) n_{t+1}}{\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}} \right)$$

It is convenient to assume now, and verify later on, that the wage paid by the entrepreneur takes the following form in equilibrium:

$$w_{t+1}(\xi_{t+1}) n_{t+1} = w_{0,t+1} n_{t+1} + \vartheta(1-\alpha_I-\alpha_S)\xi_{t+1}\tilde{y}_{t+1} \tag{19}$$

where $w_{0,t+1}$ is an aggregate variable unaffected by ξ_{t+1} or the entrepreneur's decisions, and ϑ is a parameter representing the bargaining weight of labor. Then, the entrepreneur's return can be written as:

$$R_{t+1}^K = R_{0,t+1}^K + \xi_{t+1} R_{1,t+1}^K$$

where

$$R_{0,t+1}^K = \frac{-w_{0,t+1} n_{t+1} + h_{t+1}(1-\delta_N) n_{t+1}}{\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}} \tag{20}$$

$$R_{1,t+1}^K = \frac{[1-\vartheta(1-\alpha_I-\alpha_S)]\tilde{y}_{t+1} + q_{S,t+1}\tilde{k}_{S,t} + q_{I,t+1}\tilde{k}_{I,t}}{\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}} \tag{21}$$

Therefore, the entrepreneur's rate of return is homogeneous of degree zero with respect to the variables $(\tilde{k}_{S,t}, \tilde{k}_{I,t}, n_{t+1})$. As will become evident, the linearity of the entrepreneur's problem ensures that, in equilibrium, all entrepreneurs choose levels of capital and labor proportional to their net worth. Thus, the values of $R^K_{0,t+1}$ and $E_t[R^K_{1,t+1}]$ are invariant across entrepreneurs.

4.8 Financial Markets

Entrepreneurs may finance their purchases of capital and labor at the end of a period t by taking loans from the banks. The banks in turn raise funds by allowing workers to save in risk-free deposits. Thus, the banks transfer funds from the workers to the entrepreneurs.

The loan obtained by an entrepreneur from a bank is assumed to take the form a standard debt contract. That is, the entrepreneur who borrows b_t at time t, must repay $b_{t+1} = (1 + i_{t+1})b_t$ after producing in period $t + 1$, if she has the resources to repay, and will default on her debt otherwise. The interest rate on the debt i_{t+1} is assumed to depend on the aggregate state at time $t + 1$ as discussed below.[17]

Entrepreneurs who draw very low ξ_{t+1} may find themselves unable to afford to repay their debt. Let $\overline{\xi}_{t+1}$ denote the cutoff value, such that the entrepreneur will be able to repay her debt if and only if she draws $\xi_{t+1} \geq \overline{\xi}_{t+1}$. Then, in particular, it follows that:[18]

$$(R^K_{0,t+1} + R^K_{1,t+1}\overline{\xi}_{t+1})(\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}) = b_t(1 + i_{t+1}) \tag{22}$$

Given $\overline{\xi}_{t+1}$ this equation determines i_{t+1} in each state $t + 1$.

If the entrepreneur draws $\xi_{t+1} < \overline{\xi}_{t+1}$, she declares bankruptcy, in which case the bank confiscates all her assets and sells them to other entrepreneurs. However, the bank must also pay a monitoring cost in this case, in order to verify that the entrepreneur truly cannot repay her debt. The monitoring cost is assumed to equal fraction μ of the value of the entrepreneur's assets and output, that is $\mu[R^K_{0,t+1} + \xi_{t+1}R^K_{1,t+1}](\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1})$.[19]

It is assumed that each bank must pay a risk-free return on all deposits it issues and cannot borrow through any other means. Consequently, each bank must make non-negative profits in every period, in every state of the world. Furthermore, free entry and perfect competition among banks imply that banks cannot make positive expected profits on any loan. Then, an

[17] The details of the debt contract are imposed exogenously here. However, that the contract between the entrepreneur and the bank should take the form of a debt contract can be justified on grounds of costly state verification. See footnote 19.

[18] More precisely, the equation follows only if $\overline{\xi}_{t+1} \in (0, \infty)$. However, this will always be the case in equilibrium.

[19] As discussed in Bernanke et al. (1999), the standard debt contract assumed here can be viewed as an incentive compatible solution to a costly state verification problem, in which the entrepreneur privately observes her realization of ξ, and the bank must pay the monitoring cost in order to observe it. The debt contract is optimal given the constraint that the bank make non-negative expected profits on every loan in every aggregate state $t + 1$.

equilibrium will exist in which banks make exactly zero profits on every loan in expectation in every aggregate state.[20] That is, the following break-even condition for the bank will hold on each loan in each aggregate state $t+1$:

$$\begin{aligned} b_t(1+r_t) = &[1-F_t(\bar{\xi}_{t+1})](1+i_{t+1})b_t \\ &+ (1-\mu)(\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}) \int_0^{\bar{\xi}_{t+1}} R^K_{0,t+1} + \xi R^K_{1,t+1} dF_t(\xi) \end{aligned} \tag{23}$$

Here r_t is the risk free rate paid by the bank to its depositors, which is predetermined at time t and is identical across banks in a competitive equilibrium.

4.9 Evolution of Net Worth

At this point, it may be convenient to review the timing of events experienced by the entrepreneur. At the end of each period t, the entrepreneur sells her existing stock of capital to capital-producing firms and may sell some of her workers to other entrepreneurs. She then buys capital units $\tilde{k}_{S,t}, \tilde{k}_{I,t}$ from capital producing firms and buys workers from other entrepreneurs or recruiting firms. She may choose to finance some of her expenditure on capital and workers by borrowing from a bank. At the start of period $t+1$, she draws the idiosyncratic shock ξ_{t+1}, affecting the value of her stock of capital and her output. She then produces output and pays wages. After production, she either defaults or does not depending on whether or not she drew $\xi_{t+1} < \bar{\xi}_{t+1}$.

Let x_t denote the value of the entrepreneur's net worth at time t, just before she purchases new capital and hires new workers (by paying a recruiting firm to recruit them). Then her expected period $t+1$ net worth will be given by:

$$E_t[x_{t+1}] = E_t\left[\int_{\bar{\xi}_{t+1}}^{\infty} (R^K_{0,t+1} + \xi R^K_{1,t+1})(\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}) - b_t(1+i_t) dF(\xi, \varsigma_t)\right] \tag{24}$$

$$= (\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}) E_t\left[R^K_{1,t+1}\int_{\bar{\xi}_{t+1}}^{\infty} (\xi - \bar{\xi}_{t+1}) dF(\xi, \varsigma_t)\right] \tag{25}$$

$$= x_t L_t E_t\left[R^K_{1,t+1}(1 - \Gamma_t(\bar{\xi}_{t+1}))\right] \tag{26}$$

[20] I assume here that the equilibrium takes this form. Given the restrictions imposed on banks due the need to issue risk-free deposits and free entry, the only potential deviation from this equilibrium to consider would be a bank that makes positive expected profits on some loans in some aggregate states, and negative expected profits on other loans in those states, with zero expected profits on each loan overall. However, such a deviation would increase the proportion of entrepreneurs that default, by making the required repayment from entrepreneurs more stochastic, and so would increase monitoring costs with no benefits, which cannot be profitable.

where L_t and Γ_t are defined by:

$$L_t = \frac{\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}}{x_t} \tag{27}$$

$$\Gamma_t(\bar{\xi}_{t+1}) = [1 - F_t(\bar{\xi}_{t+1})]\bar{\xi}_{t+1} + G_t(\bar{\xi}_{t+1}) \tag{28}$$

and

$$G_t(\bar{\xi}_{t+1}) = \int_0^{\bar{\xi}_{t+1}} \xi dF_t(\xi) \tag{29}$$

That is, L_t denotes the leverage ratio and $[1 - \Gamma_t(\bar{\xi}_{t+1})]R^K_{1,t+1}$ denotes the fraction of the total return R^K_{t+1} retained by the entrepreneur in expectation. Note that (22) was used in the derivation of (25).

After entrepreneurs have produced, they sell assets to repay their debts, as well as taking on new debts in order to produce once more. Furthermore, after production and default have taken place, fraction χ of each entrepreneur's net worth is exogenously transferred to the workers in her household. In addition, fraction $1 - \chi$ of the workers' wealth is exogenously transferred to entrepreneurs in the same household. This assumption of sharing wealth within the household is a modelling device to prevent entrepreneurs from saving enough to outgrow their financial constraints, a common concern in the literature, as discussed by Quadrini (2011). It can be interpreted variously as representing entry and exit, or that firms are not free to vary their payments of dividends.[21]

Before writing down the entrepreneur's optimization problem, it is convenient to simplify the bank's break even constraint (23). Using (22) to substitute for i_{t+1} in (23) and then using (27), (28) and (29) to substitute for the integral and for b_t, we obtain:

$$[1 - \mu F_t(\bar{\xi}_{t+1})]R^K_{0,t+1} + [\Gamma_t(\bar{\xi}_{t+1}) - \mu G_t(\bar{\xi}_{t+1})]R^K_{1,t+1} = \frac{(L_t - 1)(1 + r_t)}{L_t} \tag{30}$$

4.10 Entrepreneur's Optimization Problem

The entrepreneur wishes to maximize the present value of her household's consumption. This amounts in her case to maximizing the expected present value of the wealth that flows from her

[21] One interpretation of this transfer is that the workers' wealth is held by inactive entrepreneurs. Each period fraction $1 - \chi$ of active entrepreneurs remain active, with the rest becoming inactive. In turn, fraction $1 - \chi$ inactive entrepreneurs become active, with the rest remaining inactive. The scales of transfers to and from active entrepreneurs that this implies are particularly large by the standards of the literature. I find that such rapid transfers are necessary in order to prevent entrepreneurs from rapidly outgrowing financing constraints after a negative financial shock, due to the collapse in interest rates induced by the shock. A more elaborate model would provide developed microfoundations for why firms do not outgrow financing constraints so rapidly in practice; this may be related to the fact that the risk-free rate is much less volatile than implied by the model in this paper.

to the workers. The entrepreneur's expected net worth after production at time $t+1$ is given by (26), of which fraction χ flows to the household.

Let $V(x_t, s_t)$ denote the continuation value of an entrepreneur at time t who has net worth x_t, after production and default have taken place. s_t denotes the aggregate state. At this time, the entrepreneur must choose her purchases of structures, intangibles and workers, these choices imply choices of leverage L_t, which, in combination with the bank's break-even condition imply choices of $\bar{\xi}_{t+1}$ for each state s_{t+1}. Then, the Bellman equation for the entrepreneur is:

$$V(x_t, s_t) = \max_{\tilde{k}_{S,t}, \tilde{k}_{I,t}, n_{t+1}, L_t, \{\bar{\xi}_{t+1}(s_{t+1})\}} E_t[\chi m_{t+1} E[x_{t+1}] + (1-\chi) V(x_{t+1}, s_{t+1})] \tag{31}$$

where the maximization is subject to the conditions that $E[x_{t+1}]$ evolves in accordance with (26), L_t satisfies the definition (27), $R^K_{0,t}$ and $R^K_{1,t}$ satisfy their definitions (20) and (21), and, for each aggregate state s_{t+1}, $\bar{\xi}_{t+1}(s_{t+1})$ satisfies the bank's break even condition (30). Here m_{t+1} denotes the household's stochastic discount factor, given by:

$$m_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)} \tag{32}$$

where $u'(C_t)$ is the derivative of the household's utility function, given in (2), with respect to C_t.

It is easy to verify that the linearity of the entrepreneur's problem ensures that entrepreneurs will choose $\tilde{k}_{S,t}$, $\tilde{k}_{I,t}$ and n_{t+1} proportional to x_t, and therefore that L_t and $\bar{\xi}_{t+1}$ will be invariant across entrepreneurs.[22]

4.11 Implications for Aggregates

In a slight abuse of notation, let $\tilde{k}_{S,t}(x)$ denote the total stock of structures chosen by an entrepreneur with net worth x at the end of period t. Define $\tilde{k}_{I,t}(x)$, $n_{t+1}(x)$ analogously. Since all entrepreneurs' demands for capital and labor are linear in their individual net worth, the aggregate demands for capital and labor among entrepreneurs are given by:

$$K_{S,t+1} = X_t \tilde{k}_{S,t}(1) \equiv \tilde{k}_{S,t}(X_t) \tag{33}$$

$$K_{I,t+1} = X_t \tilde{k}_{I,t}(1) \equiv \tilde{k}_{I,t}(X_t) \tag{34}$$

$$N_{t+1} = X_t n_{t+1}(1) \equiv n_{t+1}(X_t) \tag{35}$$

[22]To see this, note that it must be the case that $V(x, s_t) \geq xV(1, s_t)$ because an entrepreneur with net worth x can obtain $xV(1, s_t)$ simply by choosing x times as much capital and workers each period as an entrepreneur who has net worth 1 at time t. By an analogous argument, we have that $V(1, s_t) \geq \frac{1}{x} V(x, s_t)$. Then, it must be that an entrepreneur with net worth x can do no better, and no worse, than choosing x times as much capital and workers as an entrepreneur with net worth 1. The strict concavity of the entrepreneur's problem then implies that this must be the unique optimal decision.

Here X_t denotes the aggregate net worth of entrepreneurs. In deriving these expressions, I made use of the fact that the mean of ξ is 1, and that a law of large numbers holds so that the aggregate stock of capital evolves as if all entrepreneurs draw the shock $\xi = 1$.

Leverage of all entrepreneurs can therefore be obtained by aggregating equation (27):

$$L_t = \frac{\tilde{q}_{S,t} K_{S,t+1} + \tilde{q}_{I,t} K_{S,t} + h_t N_{t+1}}{X_t} \tag{36}$$

Entrepreneurs sell C_t units of output to households, which constitutes aggregate consumption, and sell $I_{S,t} + I_{I,t}$ units to capital goods producers, which constitutes aggregate investment. The total of these must equal the total output produced by entrepreneurs, net of monitoring and hiring costs:

$$\begin{aligned} C_t + I_{S,t} + I_{I,t} = & Z_t K_{S,t}^{\alpha_S} K_{I,t}^{\alpha_I} N_t^{1-\alpha_I-\alpha_S} - h_t(N_{t+1} - (1-\delta_N)N_t) \\ & - \mu(\tilde{q}_{S,t-1} K_{S,t} + \tilde{q}_{I,t-1} K_{I,t} + h_{t-1} N_t)[F_{t-1}(\bar{\xi}_t) R_{0,t}^K + G_{t-1}(\bar{\xi}_t) R_{1,t}^K] \end{aligned} \tag{37}$$

Aggregate net worth at time $t+1$ will equal fraction $1-\chi$ of the wealth earned by entrepreneurs, plus fraction $1-\chi$ of the savings of workers. Aggregating equation (26), we have that

$$X_{t+1} = (1-\chi) X_t L_t R_{1,t+1}^K (1 - \Gamma_t(\bar{\xi}_{t+1})) + (1-\chi)(1+r_t)(L_t - 1) X_t \tag{38}$$

where the second right hand side term reflects that the savings of workers are equal to bank lending to entrepreneurs, given by $(L_t - 1)X_t$, and worker savings earn interest rate r_t. Using (20), (21), (36) and the bank break-even condition (30) to substitute for r_t, L_t, $R_{0,t}^K$ and $R_{1,t}^K$, this can be rewritten as:

$$\begin{aligned} X_{t+1} = & (1-\chi)\{C_{t+1} + I_{S,t+1} + I_{I,t+1} - w_{t+1} N_{t+1} \\ & + q_{S,t+1} K_{S,t+1} + q_{I,t+1} K_{I,t+1} + h_{t+1} N_{t+2}\} \end{aligned}$$

That is, entrepreneurial net worth is always equal to fraction $1-\chi$ of profits gross of hiring costs, plus the total value of the stock of capital and next period labor. Here, in an abuse of notation, I use w_t to refer to $E_\xi w_t(\xi)$.

Since the total leverage L_t and default cutoff $\bar{\xi}_{t+1}$ are the same for all entrepreneurs, it follows from (22) and (27) that all entrepreneurs pay the same interest rate on their debt, given by:

$$(R_{0,t+1}^K + R_{1,t+1}^K \bar{\xi}_{t+1}) = \frac{L_t - 1}{L_t}(1 + i_{t+1}) \tag{39}$$

The gap between the expected borrowing interest rate $E_t[i_{t+1}]$ and the rate r_t that workers save at is of particular significance. I shall refer to it as the credit spread in the model. Thus, the spread is defined as:

$$1 + \text{Spread}_t = \frac{E_t[1 + i_{t+1}]}{1 + r_t}$$

4.12 Worker Optimization and Value Functions

Workers can save in risk-free deposits in banks, earning interest rate r_t. The household may use its workers' savings to consume. Therefore, household consumption will satisfy the following Euler equation:

$$u'(C_t) = \beta(1 + r_t)u'(C_{t+1}) \tag{40}$$

The wage earned by a privately employed worker each period will depend on the productivity shock ξ of the entrepreneur he is matched with.[23] Let $\mathscr{W}_t(\xi_t)$ represent the continuation value of a privately employed worker *at the beginning of period t*, conditional on being matched with an entrepreneur who has drawn shock ξ_t.

Then $\mathscr{W}(\xi_t, s_t)$ satisfies the following Bellman equation:

$$\mathscr{W}_t(\xi_t) = w_t(\xi_t) + (1 - \delta_N)E_t[m_{t+1}\mathscr{W}_{t+1}(\xi_{t+1})] + \delta_N E_t[m_{t+1}\mathscr{U}_{t+1}] \tag{41}$$

That is, the worker's continuation value depends on his wage and the probability of him remaining employed in the next period in the usual way. $\mathscr{U}_t$ is the continuation value of an unemployed worker at the start of t.

The continuation value of a worker employed in the public sector is similarly given by:

$$\mathscr{W}_{G,t} = w_{G,t} + (1 - \delta_N)E_t[m_{t+1}\mathscr{W}_{G,t+1}] + \delta_N E_t[m_{t+1}\mathscr{U}_{t+1}] \tag{42}$$

where $w_{G,t}$ is the public sector wage at time t.

The value of an unemployed worker $\mathscr{U}_t$ satisfies:

$$\mathscr{U}_t = \overline{u}_t + f_t E_t[m_{t+1}\mathscr{W}_{t+1}(\xi_{t+1})] + (1 - f_t - f_{G,t})E_t[m_{t+1}\mathscr{U}_{t+1}] + f_{G,t}E_t[m_{t+1}\mathscr{W}_{G,t+1}] \tag{43}$$

where the flow value of unemployment, $\overline{u}_t$ is given by:

$$\overline{u}_t = \frac{-\left(\frac{\partial u}{\partial N_t}\right)}{\left(\frac{\partial u}{\partial C_t}\right)} = \frac{\nu\sigma C_t}{(\sigma - 1)(1 + \nu N_t + \nu N_{G,t})} \tag{44}$$

[23] Since all entrepreneurs are identical and make identical decisions in proportion to their net worth, wages across entrepreneurs will only vary due to variation in ξ.

The flow value of being unemployed in (44) represents the household's marginal disutility of an extra person working, evaluated in units of consumption. Hence, it equals $\frac{-\left(\frac{\partial u}{\partial N_t}\right)}{\left(\frac{\partial u}{\partial C_t}\right)}$. Using the utility function in equation (2), this is equal to the right hand side of (44). Note that there are no unemployment benefits.[24]

4.13 Wage Bargaining

Wages are set immediately before production takes place, at the beginning of the period, but after aggregate and idiosyncratic shocks have been revealed. Wages are set by bargaining, according to a protocol in which the entrepreneur and worker alternate in making wage offers to one another, as in the closely related frameworks of Hall and Milgrom (2008) and Christiano et al. (2016). The details of the bargaining procedure and the derivation of the bargaining solution are slightly complex and so are left to the online appendix. Here, I describe them only briefly.

Bargaining occurs in continuous time within the period – a period is divided into measure 1 of subperiods over which production and bargaining may take place.[25] The entrepreneur gets the opportunity to make an offer immediately at the beginning of the period. If the worker rejects this offer then agents must spend time developing new wage offers to propose. Opportunities for the worker to propose a wage offer to the entrepreneur arrive at a Poisson rate proportional to ϑ throughout the period and opportunities for the entrepreneur to propose a wage offer to the worker arrive at a Poisson rate proportional to $1-\vartheta$. No production takes place while agents bargain and so bargaining is a costly activity. Bargaining ends and production begins as soon as an offer is accepted. While agents bargain, the bargaining process may completely break down, an event which arrives at Poisson rate ρ. In this case the match between the entrepreneur and worker is terminated, and the worker becomes unemployed. While bargaining, entrepreneurs must pay a flow cost γ to keep producing new wage offers. Workers do not face the disutility of working $\overline{u}_t$ while bargaining. In equilibrium, the entrepreneur's initial wage offer at the beginning of the period is immediately accepted and production begins immediately.

[24] The absence of unemployment benefits is consistent with the findings of Chodorow-Reich and Karabarbounis (2016) that effective benefits represent a very small fraction of the wage, once eligibility and cost of takeup are considered.

[25] The main contrast between the alternating offer bargaining procedure assumed here and the approach of Christiano et al. (2016) is that Christiano et. al. (and Hall and Milgrom (2008)) assume that the worker and employer take turns to make an offer – first the employer makes an offer, then the worker, then the employer and so on. I instead assume that the entrepreneur and worker make offers at a Poisson rate. The advantage of this is that is it allows the entrepreneur and worker to have different bargaining strengths, depending on the speed at which they can make wage offers. This yields a solution that nests the standard Nash Bargaining solution used by, e.g. Mortensen and Pissarides (1994), as a special case.

This bargaining procedure leads to an equilibrium wage that satisfies:

$$(1-e^{-\rho})\left[\frac{h_t+\pi_t(\xi)-w_t(\xi)}{1-\vartheta}-\frac{\mathscr{W}_t(\xi)-\mathscr{U}_t}{\vartheta}\right]+e^{-\rho}\left[\frac{\gamma+\pi_t(\xi)-w_t(\xi)}{1-\vartheta}-\frac{w_t(\xi)-\overline{u}_t}{\vartheta}\right]=0 \quad (45)$$

where $\pi_t(\xi) = \frac{(1-\alpha_I-\alpha_S)y_t(\xi)}{n_t}$ is the marginal revenue product of the worker. The derivation of (45) and a discussion of the interpretation of the terms in this equation are provided in the online appendix.

4.14 Equilibrium Prices

The capital goods producer chooses how many units of structures, $K_{S,t}$, and how many units of output, $I_{S,t}$, to use in the production of new structures, according to the production function in (6). Likewise the capital goods producer chooses how many units of intangibles, $K_{I,t}$, and output, $I_{I,t}$, to use to produce new intangibles, according to the production function in 7).

Maximization of present discounted profits by the capital goods producer yields the following two first-order conditions for $K_{j,t}$ and $I_{j,t}$, for $j \in \{S, I\}$.

$$1=\tilde{q}_{j,t}\left[1-\frac{\kappa_1}{I_{t-1}}\left(\frac{I_t}{I_{t-1}}-1\right)I_t-\frac{\kappa_1}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2-\frac{\kappa_2}{K_t}\left(\frac{I_t}{K_t}-\frac{I^\star}{K^\star}\right)\frac{K^\star}{I^\star}K_t\right] \quad (46)$$

$$q_t=\tilde{q}_{j,t}\left[1-\delta_j-\kappa_2\left(\frac{I_t}{K_t}-\frac{I^\star}{K^\star}\right)\frac{K^\star}{I^\star}K_t\left(-\frac{I_t}{K_t^2}\right)-\frac{\kappa_2}{2}\left(\frac{I_t}{K_t}-\frac{I^\star}{K^\star}\right)^2\frac{K^\star}{I^\star}\right] \quad (47)$$

These determine the equilibrium prices $q_{S,t}, \tilde{q}_{S,t}, q_{I,t}$ and $\tilde{q}_{I,t}$. In the steady state, these conditions imply that $q_{S,t} = 1-\delta_S$; $q_{I,t} = 1-\delta_I$; $\tilde{q}_{S,t} = 1$, and $\tilde{q}_{I,t} = 1$.

The recruiting firm chooses the number of vacancies it posts in order to maximize its present discounted profits. This yields the first order condition:

$$h_t = h_0 + h_1 c_{V,t}\left(\frac{A_M(v_t+v_{G,t})}{1-N_t-N_{G,t}}\right)^{\psi} \quad (48)$$

where $c_{V,t}$ is the marginal cost (in terms of present discounted profit) to the recruiting firm per vacancy posted, given by:

$$c_{V,t}=h_1+h_2\left(\frac{v_t}{v_{t-1}}-1\right)\left(\frac{1}{v_{t-1}}\right)-E_t\left[m_{t+1}c_{V,t+1}h_2\left(\frac{v_{t+1}}{v_t}-1\right)\left(\frac{v_{t+1}}{v_t^2}\right)\right] \quad (49)$$

Equation (48) determines h_t in equilibrium.

4.15 Definition of Equilibrium

At any time t, the aggregate state of the economy can be described by the variables $K_{S,t}, K_{I,t}, N_t, N_{G,t}, Z_t, \varsigma_t$. An equilibrium of the model consists of: a law of motion of these aggregate variables, depending on the realization of the shocks $\epsilon_\varsigma, \epsilon_z, \epsilon_G$; a decision rule for the households determining consumption C_t as a function of the aggregate state; decision rules for the entrepreneur determining capital and labor demand $\tilde{k}_{S,t+1}$, $\tilde{k}_{I,t+1}$, n_{t+1}, default cutoff $\overline{\omega}_{t+1}$ and leverage L_t as functions of the aggregate state and the state of the entrepreneur (ξ, x); a function describing wages w_t paid by an entrepreneur as functions of the aggregate state and the state of the entrepreneur; value functions $V(\cdot)$ and $\mathscr{W}(\cdot)$ describing the entrepreneur and privately employed workers' continuation values as functions of the aggregate state and state of the entrepreneur; value functions $\mathscr{W}_G(\cdot)$ and $\mathscr{U}(\cdot)$ describing the continuation values of the publicly employed worker and unemployed worker as functions of the aggregate state, and functions of the aggregate state determining the variables $m_t, I_{S,t}, I_{I,t}, w_{G,t}, r_t, X_t, f_t, f_{G,t}, v_t, v_{G,t}, Y_{G,t}, \tau_t, R^K_{0,t}, R^K_{1,t}, q_{S,t}, q_{I,t}, \tilde{q}_{S,t}, \tilde{q}_{I,t}, h_t, \overline{u}_t$.

The functions describing these variables must satisfy equations governing: the evolution of stochastic shocks (3)-(5); aggregate capital demand, labor demand, consumption and investment (33)-(35) and (37); the laws of motion for structures and intangibles (6) and (7); the entrepreneur's optimization problem (31); the alternating offer wage bargain (45); the workers' Bellman equations (41)-(43); the flow value of unemployment (44); the public goods production function (10); the government's budget balance (11); the public sector wage equation $w_{G,t} = w_t$; the consumption Euler equation (40); the stochastic discount factor (32); the law of motion for entrepreneurial net worth (38); accounting identities for job finding rates and vacancies (12), (13), (15) and (16); the returns $R^K_{0,t}$ and $R^K_{1,t}$ (20) and (21); capital producing firms' first order conditions (46) and (47), and the recruiting firm's first order condition (48).

5 Model Calibration

I outline the calibration of the model here. Full details including the data sources on which the calibration is based are provided in the online appendix. I set the time period of the model to be one quarter. Most of the model parameters are calibrated jointly so that the steady state of the model matches various data moments. Table 1 shows the values of parameters chosen and outlines the moments targeted in the calibration of these parameters.

Of crucial importance to the mechanism of the model are the factor shares and depreciation rates for intangibles and structures. I calibrate the values of β, α_I, α_S, δ_I and δ_S so that the steady state of the model matches empirical targets for the stock of and investment in structures, equipment and intangibles, and for the labor share, relative to GDP. I take the average investment and stock of business structures and equipment from the NIPA and fixed

Table 1: Calibrated Parameters

Parameter	Value used	Target moment
β	0.99	Private Capital/Output
δ_I	0.049	Private Equipment & Intangible I/Y
δ_S	0.011	Private Structures I/Y
α_I	0.38	Private Labor Share
α_S	0.15	Structures/Total Capital
σ	2	Standard
h_0	0.019	Hiring Costs/Wage
h_1	0.31	Training Costs/Wage
h_2	0.002	Negligible, avoids indeterminacy
ν	0.18	40% of Wage
κ_1	1.2	Based on Christiano & Davis (2006)
κ_2	0.7	Based on Christiano & Davis (2006)
μ	0.51	Average Credit Spread
σ_σ	0.28	St. Dev. Spread
ρ_σ	0.84	Autocorrelation, Spread
$\bar{\varsigma}$	0.37	Business Failure Rate
μ	0.51	Annual Average Credit Spread
χ	0.61	Equity-to-Debt Ratio
$\overline{N}_G$	0.16	Public/Total Employment
δ_N	0.075	Unemployment Inflow Rate
A_M	1	Normalization
ψ	0.5	Shimer (2005)
ϑ	0.34	Unemployment Rate
σ_G	0.023	St. Dev. of Gov. Employment
ρ_G	0.94	Autocorrelation of Gov. Spending
σ_Z	0.072	St. Dev. of Cyclical TFP
ρ_Z	0.90	Autocorrelation of Cyclical TFP
ρ	0.02	Bayesian est. of equation (45)
γ_0	0.66	Bayesian est. of equation (45)

asset tables, averaged over the period 1995-2007. Since, as Corrado and Hulten (2010) discuss, many categories of intangible capital are not measured in the NIPA, I infer the level of investment in intangible capital and its stock using their estimates for the fraction of investment and capital stock represented by intangibles over the 1995-2007 period.

I set $\sigma = 2$, as is standard in the literature. I calibrate the financial parameters $\bar{\varsigma}$, μ and χ following a similar strategy to Bernanke et al. (1999): I set these to match a business failure rate of 3% annually, a credit spread of 1.7% annually and an entrepreneurial net-worth to debt ratio of 1.65. As discussed in the online appendix, I set the parameters of the adjustment cost function roughly based on what Christiano and Davis (2006) find best matches aggregate dynamics of investment and equity prices. I set the share of government employment at 16.7% of total employment in the steady state following Michaillat (2014). I set the parameters governing shocks to match estimates in the literature of the volatility and autocorrelation of credit spreads, the cyclical component of TFP and government spending and government employment.

I calibrate the values of training and vacancy posting costs, h_0 and h_1 respectively, to match estimates by Silva and Toledo (2009) for the fraction of the average wage spent by firms on hiring and training costs in the first quarter of employment. I find that the model exhibits indeterminacy in some cases unless the labor adjustment cost satisfies $h_2 > 0.001$. To minimize the importance of these adjustment costs, I set h_2 to the negligible level of 0.002 (less than 0.5% of total hiring costs). I set the separation rate δ_N and labor bargaining weight ϑ so that the steady state roughly matches the average inflow rate of unemployment and unemployment rate during 1995-2005. I set ν so that the flow value of unemployment evaluated in consumption units $\overline{u}_t$ is equal to 40% of the wage in the steady state, as in Shimer (2005). I set $\psi = 0.5$, following Shimer (2005), and normalize $A_M = 1$.

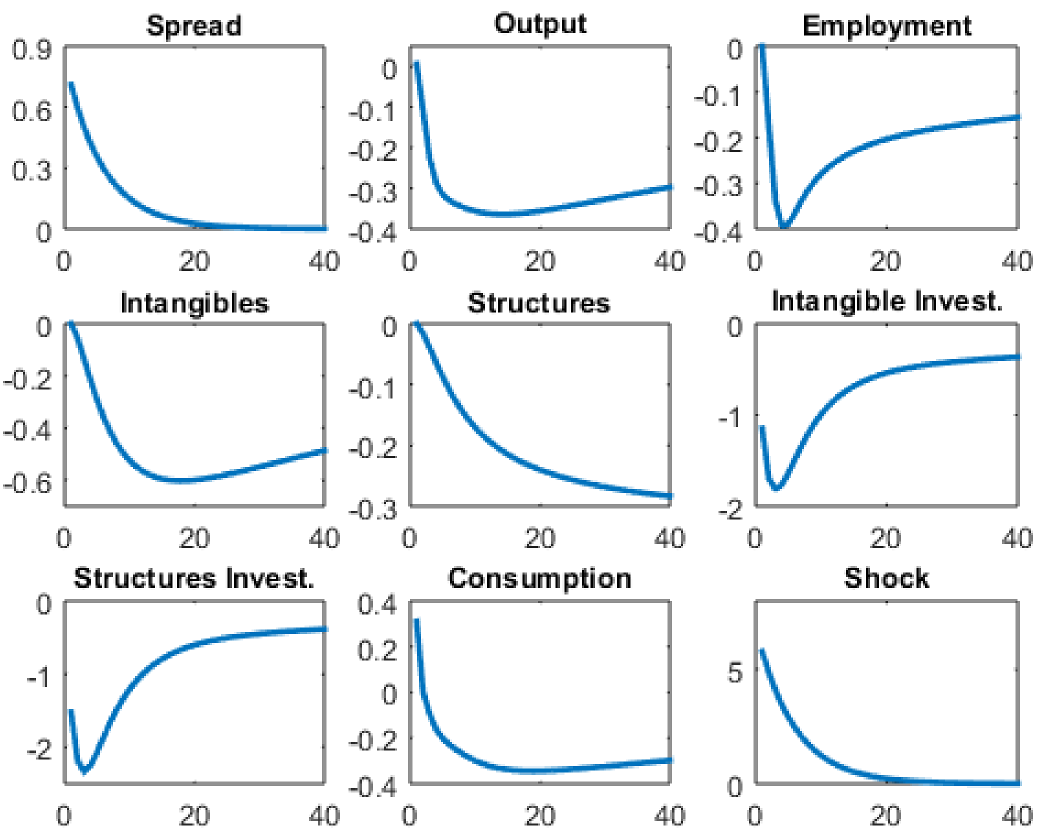
In addition to these parameters, the behavior of wages in the model depends on the cost to the entrepreneur of proposing wage offers γ and the rate at which wage bargaining breaks down ρ. I estimate these parameters by estimating the bargaining equation (45) via a Bayesian approach. The details are given in the online appendix. Essentially, I assume that wages follow (45) but are measured with iid error. For the empirical measure of wages, I use the new hire wage series of Haefke et al. (2013). I take the time series for productivity, consumption and employment directly from aggregate data and assume these variables follow a VAR process with 2 lags. Based on the current and forecasted future values of these variables, the worker Bellman equations, and the equations governing $\overline{u}_t$ and hiring costs, it is possible to calculate the value of wages implied by (45). I jointly Bayesian estimate the parameters ρ and γ in (45) and the parameters of the VAR process against the data on wages, productivity, consumption and employment. Using very loose priors, I find posterior mode estimates of $\rho = 0.02$ and $\gamma = 0.66$.

I interpret the entrepreneurial risk shock as representing a financial shock, and a severe increase in entrepreneurial risk as representing a financial crisis. As discussed further in the online appendix, this is because this shock induces a strong rise in the credit spread and only affects the real economy insofar as it affects the costs of borrowing for entrepreneurs. Krishnamurthy and Muir (2017) have shown that movements in credit spreads are powerful predictors of the severity of financial crises, and Christiano et al. (2014) have found in a estimated DSGE model featuring a similar entrepreneurial risk shock, that the risk shock accounts for 95% of the volatility in the credit spread at business cycle frequencies in the US.

6 Results of Model Simulations

Figure 2 shows the impulse responses of model variables to a one standard deviation innovation in the financial shock.[26] All impulse responses are shown as percentage deviation from steady state values. The credit spread itself does not respond particularly persistently to the shock. This is unsurprising, since the shock process was calibrated to match the empirical behavior of spreads, which display little persistence. The bottom right panel shows the deviation of ς from its steady state value; this closely matches the behavior of the spread, indicating that the shock itself is no more persistent than the spread.

Figure 2: Impulse Response to a Financial Shock: Baseline Model



Notes: This figure shows the impulse response of aggregate variables to a one standard deviation financial (i.e. entrepreneurial risk) shock, according to the calibrated model.

By contrast, the model generates a highly persistent response of output and employment. Output continues to decrease for the first four years after the shock hits and has recovered less than a quarter of its only way back to trend ten years after the shock hits. Moreover, the

[26]In the model simulations, I define 'Output' to be the sum of the output of entrepreneurs plus government spending.

decrease in output is quantitatively non-trivial: 0.38% for a one standard deviation innovation in the shock. Employment likewise decreases by 0.39% in the first year after the shock hits, and has only recovered half-way to the steady state six years after the shock hits. Therefore, the model appears to provide a mechanism that could account for some of the large and persistent output decreases following financial crises. Note that the shape of the response of output in the model closely matches the shape of the response of intangibles. This is because much of the decrease in output is caused by the decrease in intangible capital and the decrease in labor demand that results from this. While investment in structures actually drops much more than investment in intangibles after the financial shock, the stock of intangibles drops further and faster, due to the higher depreciation rate of intangibles. Since α_I is relatively large, 0.39, this induces a drop in output. The persistent decrease in the stock of intangible capital also induces a persistent decrease in employment, through the contribution of intangible capital to labor demand. The decrease in intangible capital and the resultant decrease in employment together account for most of the persistent slump in output. However, employment and, to some extent, output drop more rapidly than the stock of intangible capital initially. Employment reaches its lowest level 4 quarters after the shock hits, whereas the stock of intangible capital reaches its lowest level 18 quarters after the shock hits. The rapid drop in employment occurs because the tightening of financial conditions makes it more costly for entrepreneurs to borrow and therefore to fund their hiring costs. This immediately leads entrepreneurs to cut back on hiring.[27]

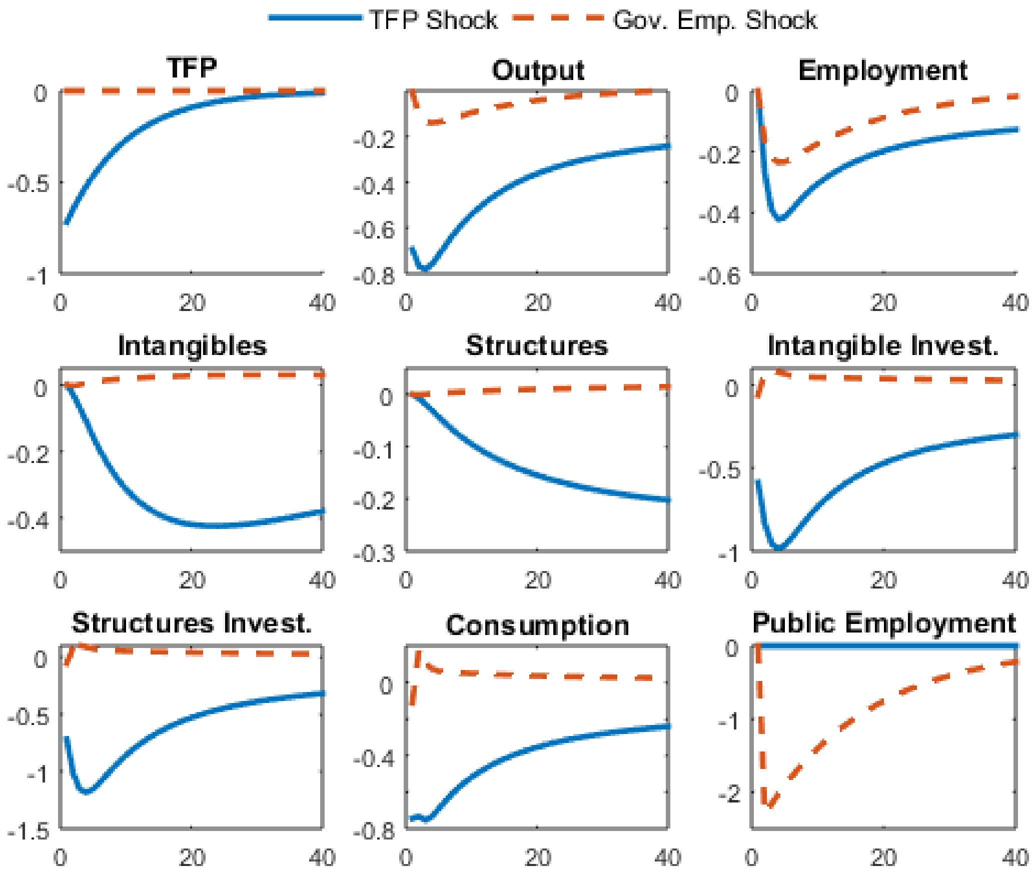
A counterfactual prediction of the model is that the level of consumption rises for one period after the shock and then falls. The initial rise in consumption after the shock is primarily an artifact of the assumption that time is discrete, with capital and labor chosen one period in advance. Therefore, output is for the most part set one period in advance.[28] When the shock hits, investment decreases immediately, which forces consumption to increase for one period, since output is largely predetermined. However, after the first period, output falls and so consumption falls along with investment. Indeed, for a one-standard deviation financial shock, consumption is only 0.01% above its steady state value in period 2, and falls below its steady state value in period 3.[29]

[27]This mechanism also occurs in other models combining financial and labor market frictions. See, for instance, Petrosky-Nadeau (2014).

[28]Output is 'for the most part' set one period in advance, because government wage payments and monitoring costs are not set one period in advance.

[29]If the model were adapted in some way so that the shock could affect output contemporaneously, rather than with a one-period delay, this would presumably resolve much of the anomalous behavior of consumption in the first period. One possible way to adapt the model to this effect would be for workers to produce output in the same period in which they are hired, so that a drop in hiring decreases output in the same period.

Figure 3: Impulse Response to Other Shocks



Notes: This figure shows the impulse response of aggregate variables to one standard deviation TFP and government employment shocks, according to the calibrated model.

6.1 Sensitivity to Model Assumptions

In the online appendix, I discuss how the effects of financial shocks in the model differ as various modeling assumptions are changed. I consider an alternative model with only one type of capital, an alternative model with Nash bargained wages, and alternative models with lower levels of adjustment costs.

I find that, if the model is amended so that there is only one kind of capital and a depreciation rate of 6.7%, output decreases by only 2/3, since the decline in the effective capital stock is smaller. While the decline in output is still relatively persistent in this case, employment recovers significantly more quickly than in the baseline model.

When the model is amended to feature Nash bargaining in the labor market, the response of output to a financial shock is only 2/3 as large as in the baseline model (although still persistent) and the response of employment is negligible. This is because wages fall enough so that labor demand hardly decreases after the shock.

I find that eliminating adjustment costs to the level of investment has little effect on most properties of the model, provided adjustment costs to the level of capital are retained. I find that eliminating adjustment costs to the level of capital, but retaining adjustment costs to the level of investment yields a larger but less persistent drop in output and employment compared to the baseline model.

6.2 Effects of Other Shocks

Figure 3 shows the response of output to contractionary public employment and TFP shocks in the model. These shocks have less persistent effects on output than the financial shock. Neither the TFP nor the public employment shock generates much of a hump-shaped response of output, unlike the financial shock. Moreover, output and employment have largely returned to steady state levels 40 quarters after these shocks hit, in contrast to after a financial shock. Notice that this is true in spite of the fact that the shock process for both of these shocks was itself calibrated to be more persistent than the financial shock, as shown in Table 1. The reason for the lower persistence is that these shocks affect investment less than the financial shock, and so the stock of intangibles decreases less. Consequently, the mechanism leading to a persistent decrease in output following a financial shock operates much less strongly for the other shocks.

That financial shocks in the model generate more persistent responses than non-financial shocks may account for why financial recessions appear to have more persistent output drops than other recessions, as discussed in section 3 above.

The public employment shock has a much larger effect on aggregate output and employment, than do government spending shocks in standard real business cycle models. Michaillat (2014) discusses how the public employment multiplier can be large in search models with wage rigidity. Intuitively, wage rigidity halts the usual mechanism whereby rising government spending raises wages and so suppresses private production and so crowding out is limited.

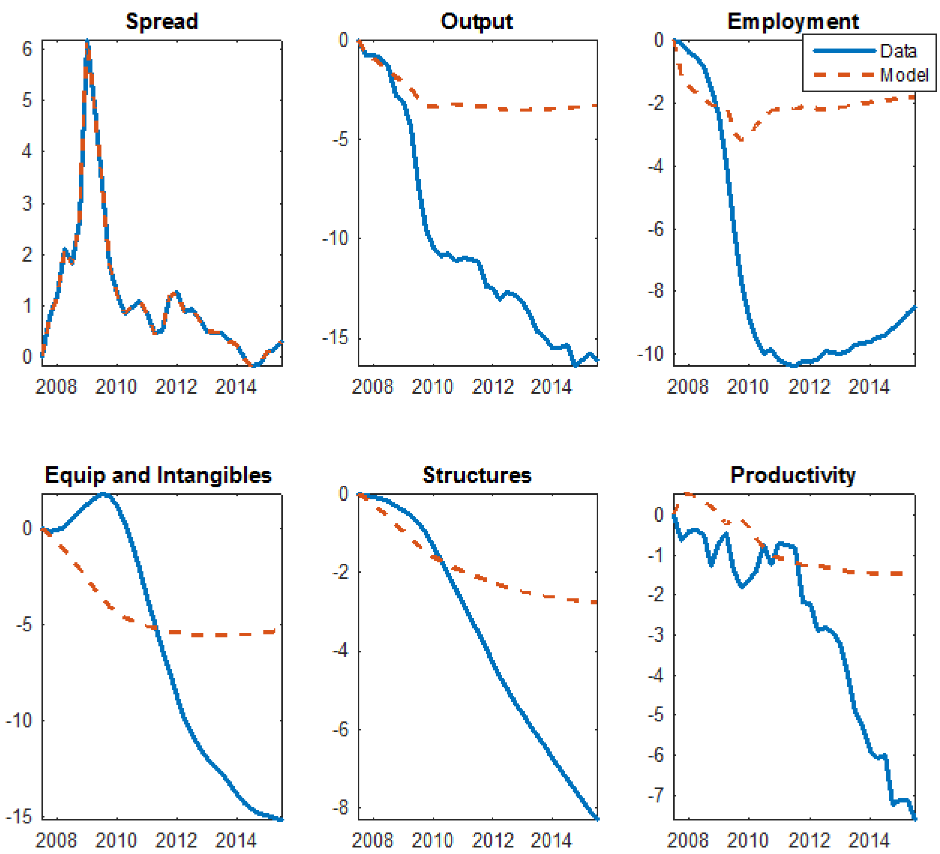
6.3 Comparison to the Great Recession

In order to evaluate the empirical relevance of the model, I compare the model predictions to the movements of aggregate variables in the US over the period during and after the Great Recession, that is over 2007-2014. In order to assess how far the poor performance of the US economy in this period can be attributed to the 2008 financial crisis, I feed into the model the sequence of financial shocks that would lead the model to exactly match the behavior of the credit spread in the US over this period.[30] I assume that the economy was in the steady state

[30]Data on the Credit Spread measured by Gilchrist and Zakrajšek (2012) is unfortunately only available to the end of 2012. For the last two years, I measure the credit spread using the different between the interest rate on BAA rated corporate bonds and the federal funds rate, which is highly correlated with the Gilchrist-Zakrajsek spread and has a similar variance. I subtract a constant from this, so that the two measures generate the same

in the first quarter of 2007. Given this sequence of shocks, I compare the implied behavior of other variables in the model to those in the data over this period. The results are shown in Figure 4. Data variables used in the figure are shown as percentage changes from their 2007 levels after eliminating a linear trend calculated over the period 1995-2006. Capital stocks at quarterly frequency were inferred using the perpetual inventory method, based on NIPA data on investment.

Figure 4: Great Recession: Data vs. Model



Notes: This figure shows the movement of aggregate variables in the data in the 2008-2015 period, relative to prior trends, and compares this to the predictions of the model when hit by a sequence of financial shocks leading to the same movement in credit spreads as in the data.

The model can replicate almost one third of the collapse in employment and output over the period 2008-2010. It is surprisingly capable of replicating the failure of these variables to return to trend by 2015. One area in which the model fails is accounting for the poor performance of productivity after 2011. For this reason, the model does not replicate the continuing decrease

value of the credit spread in the last quarter of 2012.

of output relative to its prior trend after 2011. It is possible that there has been a decrease in trend economic growth since 2008 in the US, which is absent from the model.

The model also accounts for around one third of the decrease in the stocks of structures and equipment and intangibles that is observed in the data. Here, it predicts an earlier decrease in these stocks than occurs in the data. That said, the NIPA measures used in constructing the figure may significantly understate the size and rapidity of the decrease in the stock of intangible capital in recent years. The measurement of equipment and intangibles I used for Figure 4 comes from the NIPA, due to limited data availability for recent years. This excludes marketing expenditures, which appear to be one of the most cyclical types of intangible investment. The limited available data on advertising expenditures in recent years appear to suggest that these expenditures fell strongly and rapidly in the Great Recession, suggesting that the decrease in equipment and intangibles was larger and faster than shown in Figure 4.

Nevertheless, the fact that the stock of equipment and intangible capital ultimately decreases quite a lot more in the data than in the model suggests that the model may be understating the true impact of capital deaccumulation on output and employment in the Great Recession. Accounting for the full fall in equipment and intangible capital would imply a larger drop in output and labor demand than in the model as it stands. This suggests that capital deaccumulation could potentially account for more than a third of the loss in output in the Great Recession.

7 Conclusion

In this paper, I suggest that movements in the stock of equipment and intangible capital may be a powerful force in the propagation of shocks in the economy and may have a major role to play in the persistent output drops following financial crises.

Based on a panel of OECD and emerging economies, I discuss several patterns which suggest that drops in investment during financial crises might be related to the persistent output drops following these crises. Financial recessions seem to be followed by more persistent drops in output and larger initial decreases in the investment-output ratio than non-financial recessions. Furthermore, the crises which saw more persistent output drops also saw larger drops in the investment-output ratio.

I construct a model in which a financial shock decreases investment, particularly hitting the stock of equipment and intangibles, due to their especially high depreciation rates. I show that this can lead to a large and persistent slump in output and employment, if wages are sufficiently rigid. I compare the behavior of aggregate variables in the US Great Recession to what would have occurred in the model, if movements in the credit spread in these years were caused by a sequence of financial shocks. I find that the model is capable of replicating around one third of the large and persistent decline in output and employment that occurred in the Great Recession.

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