Online Appendix

This online appendix discusses additional material not included in the paper "Capital Deaccumulation and the Large Persistent Effects of Financial Crises". Appendix A discusses the details of the derivation of the alternating offer wage bargain. Appendix B discusses the calibration strategy for the model in further detail. Appendix C discusses how the effects of financial shocks in the model vary when the modeling assumptions are changed.

A Alternating-Offer Bargaining

A.1 Bargaining Protocol

This section provides a precise microfoundation for the wage bargaining equation in (45). Bargaining occurs over time, simultaneously with production. Assume that a period t can itself be divided into a continuum of measure 1 subperiods. Therefore, within a period, time is continuous. I index time within the period by $j \in [0, 1]$ to distinguish from t, which is discrete. All shocks are revealed before $j = 0$. Production can occur over $j \in [0, 1]$. Immediately after $j = 1$, investment, hiring, default and consumption all take place. The entrepreneur makes an initial wage offer to the worker at the instant $j = 0$. The offer will be a wage for the whole period. The worker can either accept or reject. If the worker accepts the offer, then production begins immediately and the offered wage is paid immediately at $j = 1$. If the worker rejects the offer, then the entrepreneur and worker start preparing new offers. This takes time. It is assumed that the entrepreneur becomes ready to make a new offer to the worker at Poisson rate $\lambda(1 - \vartheta)$. Similarly, the worker becomes ready to make an offer to the entrepreneur at Poisson rate $\lambda\vartheta$. Therefore, it is stochastic as to who will have a chance to make the next wage offer, the worker or the entrepreneur. The parameter λ governs the speed at which any agent can make an offer, and the parameter ϑ governs the relative speed at which workers and entrepreneurs can make offers.

Whenever the entrepreneur has the chance to make a new offer to the worker, then once again the worker can accept or reject. If the worker accepts then the worker commences work and production takes place for whatever is left of the period; total output produced will be lower than if an agreement had been reached at the beginning of the period because some time has passed. The agreed wage is paid at the end of the period. If the worker rejects again, then both entrepreneur and worker go back to preparing new offers and become ready to make a new offer at, respectively Poisson rates $\lambda(1 - \vartheta)$ and $\lambda\vartheta$.

Whenever the worker has a chance to make an offer, things proceed similarly. The entrepreneur can accept or reject. If she accepts then the worker starts work, output is produced in whatever is left of the period and the worker is paid at the end of the period. If the entrepreneur rejects, then both entrepreneur and worker go back to preparing new offers and can make another offer at Poisson rates $\lambda(1-\vartheta)$ and $\lambda\vartheta$.

The entrepreneur and worker continue to make new offers in this way until an offer is accepted. However, the entrepreneur and worker would both prefer, all else equal, to reach an agreement as soon as possible because bargaining is costly in two ways. First, the longer time is spent bargaining the less time there is to produce output, and the less output is produced. The worker's full marginal product of labor $\pi_t(\xi_t) = (1-\alpha_I-\alpha_S)\frac{y_t(\xi_t)}{n_t}$ is produced if a deal is made at the beginning of the period but output is only $(1-j)\pi_t$ if a deal is instead reached at point $j \in (0,1]$.

Second, while the entrepreneur and worker are continuing to bargain – until they reach an agreement – there is a flow probability ρ that bargaining breaks down and the match separates. On the other hand, the worker benefits a little from bargaining in that he does not have to work while he is bargaining, and so does not face the disutility of working ν.

If the end of the period is reached, i.e. $j = 1$, and no offer has yet been accepted but at the same time bargaining has not broken down, then no production takes place this period and the worker is not paid. At this point, the worker and entrepreneur separate with probability δ_N, just as they would if they had reached agreement.

A.2 Continuation Values

Let $\hat{\mathscr{W}}_t(\xi, j)$ denote the continuation value of an employed worker in sub-period j who is matched with an entrepreneur of productivity ξ and *who has not yet reached agreement with his employer over wages this period*. Similarly, let $V_{N,t}(\xi, j)$ denote the marginal value of the worker to the entrepreneur in sub-period j if the entrepreneur is still matched with the worker but has not yet reached agreement with the worker over wages this period.

Recall that at the beginning of the period the entrepreneur makes an offer to the worker. As bargaining is totally wasteful, the entrepreneur maximizes her profits by offering the worker the wage $w_t(\xi)$ that makes the worker indifferent between accepting the offer and rejecting, and the worker will accept this offer.[31] The value of an employed worker $\mathscr{W}_t(\xi)$ at the beginning of the period is therefore the worker's value of accepting this initial offer.

Worker indifference between accepting and rejecting the offer implies that:

$$\mathscr{W}_t(\xi) = \hat{\mathscr{W}}_t(\xi, 0) \tag{A.1}$$

[31] As is standard in the contract literature, it is assumed that the worker breaks indifference in favor of accepting the offer, since the resultant equilibrium can always be approached by a wage offer which the worker is strictly better off accepting.

Here, the left hand side denotes the value of accepting the offer and the right hand side denotes the value of rejecting.

Now suppose, hypothetically, that in some sub-period $j \in (0, 1)$ is reached and the worker and entrepreneur are still matched but no agreement has yet been reached. Suppose that the entrepreneur gets an opportunity to make an offer to the worker at j. Then the entrepreneur will make an offer that makes the worker indifferent between accepting and rejecting and the worker will accept. Let $\hat{w}_t(\xi, j)$ be the offer the entrepreneur would make in sub-period j. Then $\hat{w}_t(\xi, j)$ satisfies:

$$\hat{w}_t(\xi, j) + \hat{\mathscr{W}}_t(\xi, 1) = \mathscr{W}_t(\xi, j) \tag{A.2}$$

where the left hand side denotes the worker's value if he accepts, and the right hand side denotes his value if he rejects. The value to the worker of acceptance here is the value of the wage paid for the remainder of the period, plus the continuation value that the worker would have at the end of the period even if no agreement had been reached, i.e. $\hat{\mathscr{W}}_t(\xi, 1)$.

Since the worker will accept, then the entrepreneur correspondingly receives a value of:

$$(1 - j)\pi_t - \hat{w}_t(\xi, j) + \hat{V}_{N,t}(\xi, 1)$$

That is, the entrepreneur gains a continuation value equal the value of the worker's remaining marginal product that will be produced in the period, i.e. $(1 - j)\pi_t$ (because the worker will immediately start work at j), minus the wage, plus the marginal value the entrepreneur would have at the end of the period from the worker even if no agreement had been reached.

Suppose that some $j \in (0, 1)$ is reached, the pair are still bargaining, and the worker gets an opportunity to make an offer to the entrepreneur. Then the worker will likewise make an offer that makes the entrepreneur indifferent between accepting and rejecting and the entrepreneur will accept. Let $w_t'(\xi, j)$ denote the offer the worker makes at x. Then $w_t'(\xi, j)$ satisfies:

$$(1 - j)\pi_t - w_t'(\xi, j) + \hat{V}_{N,t}(\xi, 1) = \hat{V}_{N,t}(\xi, j) \tag{A.3}$$

where the left hand side is the entrepreneur's continuation value if she accepts the offer and the right hand side is her value if she rejects. Since the entrepreneur will accept the offer, the worker will then get the value:

$$w_t'(\xi, j) + \hat{\mathscr{W}}_t(\xi, 1)$$

A.3 HJB Equations

The bargaining protocol therefore implies that $\hat{V}_{N,t}(\xi, j)$ and $\hat{\mathscr{W}}_t(\xi, j)$ satisfy the following HJB equations:

$$\begin{aligned} 0 = -\gamma - \rho\hat{V}_{N,t}(\xi, j) + \lambda(1-\vartheta)[(1-j)\pi_t - \hat{w}_t(\xi, j) + \hat{V}_{N,t}(\xi, 1) - \hat{V}_{N,t}(\xi, j)] \\ +\lambda\vartheta[(1-j)\pi_t - w'_t(\xi, j) + \hat{V}_t(\xi, 1) - V_t(\xi, j)] + \frac{\partial \hat{V}_{N,t}(\xi, j)}{\partial j} \end{aligned} \tag{A.4}$$

$$\begin{aligned} 0 = \nu C_t + \rho[\mathscr{U}_t - \nu j C_t - \mathscr{W}_t(\xi, j)] + \lambda(1-\vartheta)[\hat{w}_t(\xi, j) + \hat{\mathscr{W}}_t(\xi, 1) - \mathscr{W}_t(\xi, j)] \\ +\lambda\vartheta[w'_t(\xi, j) + \hat{\mathscr{W}}_t(\xi, 1) - \mathscr{W}_t(\xi, j)] + \frac{\partial \mathscr{W}_t(\xi, j)}{\partial j} \end{aligned} \tag{A.5}$$

Equation $(A.4)$ is the HJB equation of the entrepreneur while bargaining.[32] During bargaining, the entrepreneur receives a flow value of - γ, that is, the cost of continuing to produce wage offers. With probability ρ the match terminates and the entrepreneur loses the marginal value of the match, $\hat{V}_{N,t}(\xi, j)$. At rate $\lambda(1-\vartheta)$ the entrepreneur gets to make an offer to the worker and the worker accepts, in which case the wage will be $\hat{w}_t(\xi, j)$ and the entrepreneur will get value $[(1-j)\pi_t - \hat{w}_t(\xi, j) + \hat{V}_{N,t}(\xi, 1) - \hat{V}_{N,t}(\xi, j)]$, that is, the value of the deal minus the value of continuing to bargain. At rate $\lambda\vartheta$ the worker makes an offer, the wage is $w'_t(\xi, j)$ and the entrepreneur gets $[(1-j)\pi_t - w'_t(\xi, j) + \hat{V}_t(\xi, 1) - V_t(\xi, j)]$. Finally, even if bargaining continues without resolution, the entrepreneur's value changes at rate $\frac{\partial V_{N,t}(\xi,j)}{\partial j}$.

The worker's HJB equation $(A.5)$ is very similar. The flow value of bargaining is given by νC_t, that is, the marginal value to the household of not working, evaluated in consumption units. If the match terminates — which it does at rate ρ — the worker gets to enjoy being unemployed for the remainder of the period – i.e. gets $\nu C_t(1-x)$ from this and gets the value of being unemployed at the start of the recruitment phase $\mathscr{U}_t - \nu j C_t$ but loses the value of being employed $\mathscr{W}_t(\xi, j)$. If the entrepreneur gets to make an offer, the worker gets $[\hat{w}_t(\xi, j) + \hat{\mathscr{W}}_t(\xi, 1) - \mathscr{W}_t(\xi, j)]$, and if the worker gets to make an offer, he gets $[w'_t(\xi, j) + \hat{\mathscr{W}}_t(\xi, 1) - \mathscr{W}_t(\xi, j)]$. This gives rise to equation $(A.5)$.

At the end of the period, if a deal has still not been reached, the worker and entrepreneur separate remain matched with probability $(1-\delta_N)$. In that case, the entrepreneur will be able to 'sell' the worker at price h_t. Therefore

[32] Macroeconomists are usually accustomed to seeing the discount rate multiplied by the continuation value on the left hand side of HJB equations. Here, this does not appear because I assume no discounting *within the period* – discounting occurs between periods not within them. So the left hand side of each HJB equation is just zero.The discount rate is so low at the quarterly level that the quantitative impact of this assumption is small.

$$\hat{V}_{N,t}(\xi, 1) = (1 - \delta_N) h_t \tag{A.6}$$

Substituting $(A.2)$ and $(A.3)$ into equations $(A.4)$ and $(A.5)$ gives rise to two linear differential equations that can be solved for $\hat{V}_{N,t}(\xi, j)$ and $\hat{\mathscr{W}}_t(\xi, j)$ given boundary conditions $(A.6)$ and $(A.1)$.[33] Using $w_t(\xi) = \hat{w}_t(\xi, 0)$, and taking the limit as $\lambda \to \infty$, the solution gives

$$(1 - e^{-\rho}) \left[\frac{h_t + \pi_t(\xi) - w_t(\xi)}{1 - \vartheta} - \frac{\mathscr{W}_t(\xi) - \mathscr{U}_t}{\vartheta} \right] + e^{-\rho} \left[\frac{\gamma + \pi_t(\xi) - w_t(\xi)}{1 - \vartheta} - \frac{w_t(\xi) - \nu C_t}{\vartheta} \right] = 0 \tag{A.7}$$

which was the wage bargaining solution considered in (45).

Taking the limit $\lambda \to \infty$ implies that agents can make wage offers infinitely quickly, which is obviously implausible. However, if agents can each make offers on average at least once per week, and there are 13 weeks in a quarter, this implies that $\lambda > 26$. At levels of λ this high or higher, the approximation error in assuming $\lambda \to \infty$ is incredibly small, provided $\rho < 4$ and $\vartheta \in (0.05, 0.95)$. Thus I let $\lambda \to \infty$ in order to produce a bargaining solution that is easily interpretable.

I now briefly discuss the interpretation of the terms in the wage bargaining equation (A.7). The term $h_t + \pi_t(\xi) - w_t(\xi)$ is the surplus the entrepreneur gains from being matched with a worker, relative to the alternative in which the worker and entrepreneur separate. Similarly $\mathscr{W}_t(\xi) - \mathscr{U}_t$ is the match surplus of the worker relative to separating. In the limit as $\rho \to \infty$ the second square bracketed term in (45) approaches zero. Therefore, the bargained wage that solves (45) is the one in which the worker's match surplus is proportional to the entrepreneur's, as in the traditional Nash Bargain used by the search and matching literature, including Mortensen and Pissarides (1994). The bargaining weight of the worker is then given by ϑ.[34]

On the other hand, as $\rho \to 0$ the alternating-offer wage simplifies to:

$$w_t(\xi) = (1 - \vartheta)\overline{u}_t + \vartheta(\gamma + \pi_t(\xi))$$

In that case, the bargained wage does not depend on the continuation value of the unemployed worker, and so does not depend on the job finding rate f_t. This leads to a wage that is much more insensitive to cyclical fluctuations – that is, wages are endogenously more rigid. It may be verified that the alternating offer bargaining solution (45) implies that the wage satisfies (19)

[33] Details of the solution are available upon request.

[34] In a number of recent contributions to the literature, such as Monacelli et al. (2011), the possibility that an entrepreneur may default affects wage bargaining. This mechanism is ruled out here, because whether an entrepreneur defaults is perfectly predictable from the shock ξ, which is known before wage bargaining takes place. Moreover, since the number of workers per entrepreneur is large, each worker acts in the knowledge that his own bargaining behavior will matter too little to affect whether or not the entrepreneur defaults.

for any value of ρ, provided $w_{0,t}$ is defined accordingly.

B Model Calibration

This section discusses the model calibration in more detail. Table 2 shows the moments used and the main parameter values chosen for the baseline model. The parameters governing the wage bargaining process are shown in Table 3 and the parameters governing the shock processes are shown in Table 4. I set the time period of the model to be one quarter. Most of the model parameters are calibrated so that the steady state of the model matches various data moments.

Particularly important are the factor shares and depreciation rates for intangibles and structures, α_I, α_S, δ_I, δ_S. I calibrate these so that the steady state of the model produces empirically reasonable values for the stock of and investment in structures, equipment and intangibles, and for the labor share. I take the average investment and stock of business structures and equipment from the NIPA and fixed asset tables, averaged over the period 1995-2007. I infer the level of investment in intangible capital and its stock using Corrado and Hulten's (2010) estimate that, over the 1995-2007 period, intangible investment accounted for 55% of total business investment and intangible capital accounted for 34% of total business capital.[35] I assume that private output is equal to the level of private consumption plus business investment in structures, equipment and intangibles and I calculate the implied values of the investment/private output ratio and the capital/private output ratio for equipment, structures and intangibles. Finally, using the NIPA measure of total worker compensation averaged over the 1995-2007 period, I also calculate private labor compensation as a share of private output.

I calibrate the values of β, α_I, α_S, δ_I and δ_S so that the steady state of the model matches the following five moments from these calculations: private total capital/private output; private equipment plus intangible investment/private output; private structures investment/private output, and private labor compensation/private output. The values of the moments used and the corresponding parameter values are shown in Table 2. The implied value of α_I is rather high: 0.38. This arises for three reasons. Firstly, the inclusion of intangible investment increases the value of output above the standard measure. This significantly depresses the fraction of output

[35] This can be calculated from Table 1 in Corrado and Hulten (2010). Corrado and Hulten calculate the level of investment in intangibles by summing over measurements of investment in a wide variety of assets including software, research and development, marketing activities and training. They infer the stock of intangibles based on estimates of the depreciation rates of these various assets. It is worth noting that the level of intangible capital calculated by Corrado and Hulten (2010) is actually smaller than many others in the literature. For instance, Eisfeldt and Papanikolaou (2014) estimate that the stock of 'organization capital' alone is greater than the stock of property plant and equipment. Their approach implies that more than 70% of business capital and an even larger fraction of business investment is intangible. See also Hulten and Hao (2008) and McGrattan and Prescott (2014) for methods that imply a somewhat larger fraction of the capital stock that is intangible than the approach used here. Calibrating to larger stocks and investment rates in intangibles would naturally imply a larger response of output to financial shocks in this paper.

Table 2: Calibrated Parameters

Parameter	Value Used	Target Moment	Moment Value	Source
β	0.99	Private Capital/ Annual Output Ratio	1.89	NIPA and Corrado and Hulten (2010)
δ_I	0.049	Private Equipment & Intangible Investment/Output	0.21	
δ_S	0.011	Private Structural Investment/Output	0.036	
α_I	0.38	Private Labor's Share of Business Output	0.47	
α_S	0.15	Ratio of Structural Capital to Total Capital	0.44	
σ	2	Standard	2	
$\bar{\varsigma}$	0.37	Business Failure Rate	0.03	Bernanke, Gertler, Gilchrist (1999)
μ	0.51	Annual Average Credit Spread	0.017	Gilchrist and Zakrajsek (2012)
χ	0.61	Equity-to-Debt Ratio	1.65	Masulis (1983)
κ_1	1.2	Investment Adjustment Costs	1.2	Based on Christiano and Davis (2006) discussion.
κ_2	0.7	Capital Adjustment Costs	0.7	Based on Christiano and Davis (2006) discussion.
$\overline{N}_G$	0.16	Public Employment/ Total Employment	0.167	Michaillat (2014)
δ_N	0.075	Inflow Rate into Unemployment	0.075	Davis et. al. (2010)/ CPS
h_0	0.37	Training Costs/Quarterly Wage	0.04	Silva and Toledo (2009)
h_1	0.019	Hiring Costs/Quarterly Wage	0.55	
h_2	0.002	Vacancy Adjustment Costs	0.002	Tiny
A_M	1	Normalization		
ψ	0.5	Hiring Cost Elasticity	0.5	Shimer (2005)
ν	0.18	Disutility of Working =40% of Wage		Shimer (2005)
ϑ	0.34	Unemployment Rate	0.06	

accruing to labor to 0.47, implying a higher α_S and α_I. Furthermore, wage bargaining implies that some of the returns to an entrepreneur's investment will transfer to her workers, who will be able to bargain for a higher wage if their employer invests more.[36] This tends to distort the level of profits below the social return to capital, implying that a higher level of α_I and α_S is required to match the labor share in the data. Thirdly, the share of intangible capital α_I is more than twice as large as the share of structures, α_S, in spite of the fact that the stock of structures and stock of intangibles are relatively similar in the steady state. The reason for this is that the high depreciation rate of intangibles requires that this type of capital receive a high gross rate of return in equilibrium, since otherwise entrepreneurs would not wish to invest in an asset that depreciates quickly. For intangibles to receive a very high gross rate of return relative to structures, their share in production must be very large relative to structures.

I set $\sigma = 2$, as is standard in the literature. In order to calibrate the financial parameters $\bar{\varsigma}$, μ and χ, I follow a strategy similar to Bernanke et al. (1999). I match a business failure rate of 3% annually, a credit spread of 1.7% annually[37] and an entrepreneurial net-worth to debt ratio of 1.65.[38]

I set the parameters of the adjustment cost function based on Christiano and Davis (2006) and Röhe (2012). However, these papers do not estimate the parameters of an adjustment cost function containing adjustment costs to the level of both capital and investment. Instead, they separately adjustment costs functions consisting of adjustment costs only to the level of capital (i.e. $\kappa_1 = 0, \kappa_2 > 0$) and adjustment costs only to the level of investment ($\kappa_1 > 0, \kappa_2 = 0$).[39] Röhe (2012) compares the results of an estimated medium-sized DSGE model using adjustment costs to either the level of investment or the level of capital (but not both). He finds that each kind of adjustment costs provides a better fit for particular aspects of the aggregate data, but that the overall fit is better with capital adjustment costs. Christiano and Davis (2006) find, using a simple neoclassical model, that to produce a volatility of aggregate investment relative to the volatility of the return to capital (based on the stock market) that is vaguely in line with the data it is necessary to have at least $\kappa_2 \geq 1$ or $\kappa_1 \geq 3.5$. Adjustment costs smaller than this cannot account for the low volatility of investment relative to the rate of return in the data.[40]

[36] See Acemoglu and Shimer (1999) for a discussion of this issue.

[37] This is the average value of the credit spread measured by Gilchrist and Zakrajšek (2012) over the period 1973-2012.

[38] Masulis (1983) finds that the equity value/debt varies between 1.3 and 2 for US corporations over the period 1937-1984.

[39] There is not, to my knowledge, any work that estimates the parameters of an adjustment cost function containing adjustment costs to the level of both capital and investment.

[40] Even at these parameter values, Christiano and Davis (2006) still find excessive volatility of investment relative to the return to capital and suggest that adjustment costs would need to be around twice as large as these numbers in order for the volatility of investment relative to returns to be close to the data. Since my calibration of the adjustment cost parameters follows Christiano and Davis (2006), it arguably represent the lower end of what the aggregate date suggests for the level of adjustment costs. Models which estimate the elasticity of investment with respect to marginal Q at the firm level frequently find evidence of much larger

Based on Röhe (2012) and Christiano and Davis (2006), I infer that adjustment costs both to the level of capital and to the level of investment likely play a role in explaining the low volatility of investment relative to the return to capital in the data, with the former type of adjustment cost likely playing a more important and the latter a less important role. As a first pass, I conjecture that adjustment costs to the level of capital may account for two thirds of the low volatility of investment relative to returns in the data (compared to a model with no adjustment costs) and adjustment costs to the level of investment account for one third of the low volatility of investment relative to returns. This suggests that it is relevant to consider adjustment costs to the level of investment of at least one third of the Christiano and Davis figure of 3.5, and adjustment costs to the level of capital of at least two thirds of the Christiano and Davis figure of 1. Therefore, I set κ_1 to $1.2 \simeq \frac{1}{3} \times 3.5$ and κ_2 to $0.7 \simeq \frac{2}{3}$. Naturally, these figures for adjustment costs represent little more than guesswork. For this reason, I discuss the effect of varying the level of adjustment costs below.

I set the share of government employment to be 16.7% of total employment in the steady state, as found by Michaillat (2014) to be true in CES data.

B.1 Calibration of Labor Market Parameters

I calibrate the values of training and vacancy posting costs, h_0 and h_1 respectively, to match estimates by Silva and Toledo (2009) for the fraction of the average wage spent by firms on hiring and training costs in the first quarter of employment. I set h_2 to the extremely small value of 0.002 to minimize the effect of these adjustment costs on model dynamics. With $h_2 < 0.001$, I find that indeterminacy can occur in the model. Setting $h_2 = 0.002$ keeps these adjustment costs as small as possible, but safely above the threshold for indeterminacy. The value $h_2 = 0.002$ implies that adjustment costs to the level of vacancies correspond to less than 0.05% of hiring costs in every period in the calibrated model, for shock realizations of fewer than 5 standard deviations away from the mean.

I set the separation rate δ_N to roughly match the inflow rate into unemployment, which Davis et. al. (2010) find to fluctuate between 2% and 2.5% monthly in the CPS over the 1995-2005 period. The labor bargaining weight ϑ is set to match a steady state unemployment rate of 6%. I set ν so that the flow value of unemployment evaluated in consumption units $\overline{u}_t$ is equal to 40% of the wage in the steady state, as in Shimer (2005).[41] I set $\psi = 0.5$, following Shimer (2005),

adjustment costs than this. See, for instance Erickson and Whited (2000) for the case of capital adjustment costs and Eberly et al. (2012) for investment adjustment costs. The adjustment cost parameter estimated by Eberly et. al – 1.86 – appears at first glance to be modest compared to the figure of 3.5 used by Christiano and Davis (2006). However, Eberly et. al. use annual rather than quarterly data, suggesting that their estimate should be multiplied by around 4 when moving to the quarterly frequency.

[41] Many papers in the search literature, such as Christiano et al. (2015), consider a higher flow value of unemployment and justify this as including unemployment benefits. However, Chodorow-Reich and Karabarbounis (2016) find that these are negligible on average, once eligibility requirements and cost of takeup are taken into

and set $A_M = 1$, which is a normalization that has no effect on most aggregate variables.

In addition to these parameters, the behavior of wages in the model depends on the cost to the entrepreneur of proposing wage offers γ and the rate at which wage bargaining breaks down ρ. Since these parameters govern the rigidity of wages in response to aggregate shocks, I estimate them using a Bayesian approach based on the aggregate dynamics of wages in US data.

The alternating offer bargaining solution (45) implies that wages should depend upon current and expected future values of consumption, employment and productivity. Indeed, if the aggregate dynamic process governing these three variables is known, values of the wage w_t, recruiting costs h_t, vacancies v_t in each period, and the worker's continuation values $\mathscr{W}_t$ and $\mathscr{U}_t$, can all be calculated, given the model parameters, using only the equations governing labor markets and preferences. That is, the values of these five variables in each period can be inferred using only the model equations (12), (15), (32), (41), (43), (44), (45), (48), and (49). Therefore, rather than conducting a Bayesian estimation of the whole model, I assume that productivity, consumption and employment follow a reduced-form VAR process with two lags. I estimate this VAR process against the data, and use only the VAR and the eight model equations just mentioned for the purposes of Bayesian estimation of the parameters γ and ρ. The advantage of this approach is that my estimates of these two parameters will not be contaminated if other parts of the model, such as the equations governing financial markets, are misspecified.

I assume that wages are measured with iid error and jointly estimate the parameters of the VAR, the error variance of wages and the parameters ρ and γ. This entails estimating parameters in a system of eleven equations, where the eleven equations are the VAR process for consumption, employment and productivity and the eight model equations mentioned above. I assume that there are four variables whose values are observed in the data: productivity, consumption, employment and measured wages.[42]

Of course, the eight equations from the model contain other parameters in addition to ρ and γ, notably the parameters governing hiring costs and the matching function, as well as α_I, α_S, the discount factor β and the parameters ν and ϑ. For the purposes of the estimation, I fix the parameters governing hiring and matching as well as α_S, α_I and β at the values given in Table 2.[43] I also set the values of ν and ϑ to match the target moments in Table 2. However, the values of ν and ϑ that this entails are highly sensitive to the values of the wage bargaining

consideration.

[42] Assuming that these eleven equations are not misspecified, the parameters γ and ρ are identified. This is because the number of observed variables, four, is equal to the number of shocks – where the four shocks are the three error terms in the VAR, and the iid measurement error in wages.

[43] Since the calibrated values of α_S, α_I and β do depend on the values of ρ and γ, this requires an iterative process. Specifically, I set the values of α_I, α_S and β, and estimate the wage bargaining parameters. Then, using the estimated wage bargaining parameters, I recalibrate the values of α_I, α_S and β to match the target moments in Table 2. I then re-estimate the wage bargaining process and follow these steps repeatedly until convergence. In any case, I find that my calibrated values for α_S, α_I and β are not highly sensitive to the values of the wage bargaining parameters.

parameters γ and ρ. Therefore, when calculating the likelihood of each particular value of ρ and γ during the Bayesian estimation, I set ϑ and ν in each case to match the target moments in Table 2, given ρ and γ.

For the values of the observed variables, productivity, consumption, employment and wages, I use US data from 1979-2005,[44] taking aggregate consumption and output from the NIPA and employment and working hours from the CPS. I measure productivity using output per labor hour. I ignore government employment, and assume that aggregate employment corresponds to private employment in the model.[45] In order to remove trends in all variables, I use the band-pass filter of Christiano and Fitzgerald (2003) to isolate variation in the 2-120 quarter frequency band.[46]

For wages, I use the series on wages of new hires out of unemployment constructed by Haefke et al. (2013) from the CPS, adjusted for composition effects.[47] I use this series because a large empirical literature has noted that measuring the wage using average hourly earnings of all workers may generate spurious evidence that the price of labor is rigid. This is for two reasons. First, wages of long-standing employees of a firm may adjust infrequently if they are set by implicit contracts. Therefore, aggregate wages may appear rigid. However, the marginal cost of labor to employers depends on how much they would have to pay newly hired workers, which may be more volatile. As such, I consider the average wage of new hires out of unemployment rather than the the average wage of all workers. The second concern with aggregate wage series is that they may generate spurious evidence that wages are acyclical or rigid due to composition biases. For instance, since the workers who tend to lose their jobs in recessions are frequently those who earn relatively low wages, a rise in unemployment in a recession will tend to increase the average wage of all workers due to this compositional change. Failure to adjust for this generates a countercyclical bias in measured wages. Therefore, I use the series of Haefke et al. (2013), which adjusts for cyclical variation in average levels of education and demographic variables among workers.[48] Haefke et. al. argue that this adjustment appears to eliminate most sources

[44] I consider these dates because the wage series I use is not available for other dates.

[45] As an alternative, government employment can be included as an extra variable in the VAR. This makes a negligible difference to the estimates, since it is the aggregate level of employment primarily that affects the bargained wage, rather than whether the levels of government and private employment separately.

[46] 120 quarters is of course much longer than what is usually considered the length of US business cycles. However, since the focus on the model is on persistent movements in aggregate variables, it is of importance that the response of wages to other variables is consistent with the data at frequencies below the business cycle as well as at high frequencies. This argues in favor of using data across a wide frequency band, such as 2-120 quarters.

[47] I thank Thijs van Rens for making this data series publicly available at http://www.thijsvanrens.com/wage/ .

[48] For further discussion of the relevance of wages of new hires rather than all workers, see Haefke et al. (2013) and Pissarides (2009) and the references therein. For discussion of composition effects see Haefke et al. (2013) and the references therein. Arguably, the level of labor productivity should also be adjusted for compositional changes. To maintain comparability with Haefke et al. (2013), I do not make this adjustment, since they do not. However, I find that applying the same compositional adjustments to productivity as I apply to wages makes negligible difference to the estimated labor market parameters.

of compositional bias.

For the coefficients of the VAR, I use a normal prior with a standard deviation of 2 and a mean of 0, except for the coefficient of each variable on its own first lag, for which I use a normal prior with a mean of 0.9 and a standard deviation of 1. For the VAR shocks and the error term of wages, I use an inverse gamma-2 prior, with a mean of 0.01. For the parameter ρ, I use a lognormal prior. For the parameter γ, I assume that $\gamma = \gamma_0 \frac{\bar{Y}}{N}$ that is, it is equal to the proportion γ_0 of steady state productivity. For γ_0, I use a beta prior with mean of 0.45, a standard deviation of 0.225 a minimum of 0 and a maximum of 0.9 , which is close to a uniform prior on the range $[0, 0.9]$. Priors and estimates of these parameters are shown in Table 3.[49]

Table 3: Estimates of Wage Bargaining Parameters

Parameter	Prior Type	Prior Mean	Prior Standard Deviation	Posterior Mode	Posterior Stand. Dev.
$\log(\rho)$	Normal	log(0.5)	3	log(0.02)	1.09
γ_0	Beta	0.45	0.225	0.66	0.27

Although the priors are relatively loose, the posterior in ρ is relatively tight around a value of ρ close to zero. The posterior mode, $\rho = 0.02$, is very far from the standard Nash bargaining solution used in the search and matching literature, which corresponds to the special case $\rho = \infty$, as discussed in section 4.13. The much lower value of $\rho = 0.02$ implies that wages are not very sensitive at all to unemployment. Values of ρ above 0.18 can be rejected at the 5% level. This suggests that the data strongly favors wages that are substantially more rigid than the standard Nash bargaining solution. The value of γ_0 is less well identified, with the posterior standard deviation being close to the prior. For the model calibration, I use the posterior modes $\rho = 0.02$ and $\gamma_0 = 0.66$.

B.2 Calibration of Shocks

Table 4 shows the calibration of the shock processes. I calibrate the variance and autocorrelation of the shock e_ς in order to match the variance and autocorrelation of the credit spread in US data for the 1970-2012 period, where I use the measure of the credit spread constructed by Gilchrist and Zakrajšek (2012), which measures the average excess return on corporate bonds relative to treasury bonds of similar maturity.

I interpret the entrepreneurial risk shock as representing a financial shock, and a severe increase in entrepreneurial risk as representing a financial crisis. This is for several reasons.

[49] I find that if I use the average wage of all workers, instead of the average wage of new hires, as the wage series for estimation purposes, then this moderately increases the level of estimated wage rigidity. Indeed, the posterior mode for $\log(\rho)$ falls to log(0.002). Similarly, I find that failure to adjust for composition effects slightly increases estimated wage rigidity. Higher levels of wage rigidity in the model produce larger slumps following financial shocks and strengthens the conclusions of the paper.

First, the risk shock only affects the aggregate level of capital, output and employment insofar as it affects the costs of borrowing for entrepreneurs. That is, if we were to set monitoring costs equal to zero, thereby turning off financial frictions, the risk shock would have no effect on any aggregate quantities of interest. Second, the risk shock has an extremely strong effect on the credit spread as well as entrepreneurial leverage. Therefore, the risk shock induces strong comovements between credit spreads and financial conditions. Krishnamurthy and Muir (2017) have shown that movements in credit spreads are powerful predictors of the severity of financial crises. Third, Christiano et al. (2014) have found in a estimated DSGE model featuring a similar entrepreneurial risk shock, that the risk shock accounts for 95% of the volatility in the credit spread at business cycle frequencies in the US. Moreover, they find that their measured risk shock rises and falls over time simultaneously with the standard deviation of the cross-section of stock returns of non-financial firms, suggesting it is indeed related to firm-level risk.

Table 4: Calibration of Shocks

Parameter	Value Used	Target Moment	Moment Value	Source
σ_ς	0.28	St. Dev. of Spread	0.0076	Gilchrist and Zakrajsek (2012)
ρ_ς	0.84	Autocorrelation of Spread	0.84	Gilchrist and Zakrajsek (2012)
σ_G	0.023	St. Dev. of Gov. Employment	0.023	Quadrini and Trigari (2007)
ρ_G	0.94	Autocorrelation of Gov. Spending	0.94	Christiano, Motto and Rostagno (2014)
σ_Z	0.072	St. Dev. of TFP Shock	0.072	King and Rebelo (1999)
ρ_Z	0.90	Autocorrelation of TFP Shock	0.90	Michaillat (2014)

In order to compare the effects of the financial shock with those of other shocks, I also include the TFP and government employment shocks. I set the variance and autocorrelation of the public employment shock e_G so that the standard deviation of the autonomous component of public employment matches the value used by Quadrini and Trigari (2007), and the autocorrelation of government spending matches the estimate of Christiano et al. (2014). Finally, I set the variance of innovations in the TFP shock to match the value recommended by King and Rebelo (1999) and set the autocorrelation to match the autocorrelation of the cyclical component of TFP estimated by Michaillat (2014).

C Sensitivity to Modeling Assumptions

This section considers the role played by various modeling assumptions in generating the quantitative results of the model. I analyse in turn, how the impulse responses to financial shocks change when three modeling features are excluded: intangible capital with a high depreciation rate; alternating-offer wage bargaining, and adjustment costs to the level of capital and investment.

In order to provide a sense of the quantitative importance of the high depreciation rate of intangible capital in the model, Figure 5 shows the response to a financial shock when the model is re-solved and re-calibrated to have only one kind of capital, with a depreciation rate of 6.7% and a steady-state capital-output ratio of 2.3, which Khan and Thomas (2013) find to be the averages in the US fixed asset tables over 1954-2002, after adjusting for growth. For ease of comparison, Figure 5 also shows the impulse responses in the baseline model. As is evident from the figure, it is still the case in this setting that a financial shock decreases investment, leading to a deaccumulation of capital and a persistent decrease in output. However, the decrease in output is only around two-thirds as large as it is in the baseline model, in spite of a larger decrease in investment than in the baseline model. The reason is that the lower depreciation of capital in this setting means that the capital stock is more stable than the stock of intangibles is in the baseline model. Consequently, it decreases less than 2/3 as much as the stock of intangibles decreases in the baseline model, leading to a correspondingly smaller decrease in output. The aggregate stock of capital also decreases only 3/4 as much in the model with one type of capital as in the baseline model, due to the lower depreciation rate of capital, where the aggregate stock of capital in the baseline model refers to the combined stock of structures and intangibles.

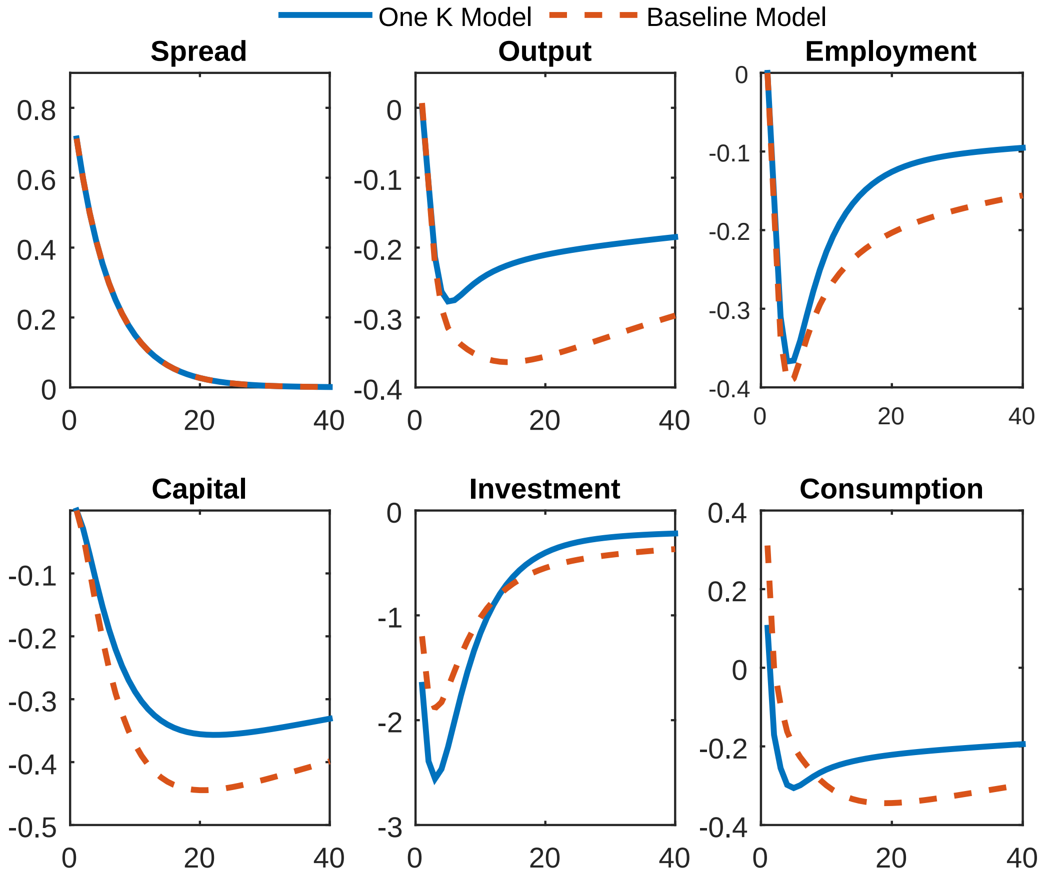
Note that while the drop in the stock of capital in the model with one type of capital is around 3/4 as large as in the baseline model, the resultant drop of output is only around 2/3 as large. The reason is that, in the baseline model, intangible capital contributes much more to production than do structures – with a share in production of 0.38, versus only 0.15 for structures.[50] Since intangible capital is disproportionately important in production in the baseline model, and falls more sharply than structures, this implies a relatively larger decrease in output in the baseline model. Therefore, the drop of output in the model with one type of capital is somewhat less than 3/4 as large as in the baseline model, even though the drop in the capital stock is fully 3/4 as large.[51]

The model with one type of capital produces a decrease in employment from the shock on impact of virtually the same size as the baseline model. However, in the model with one

[50]Section 5 above discusses why the high rate of depreciation of intangibles implies that that α_I should be high to induce entrepreneurs to invest in them.

[51]It follows from this discussion that the baseline model is not isomorphic to a model with one type of capital and a higher average depreciation rate.

Figure 5: Impulse Response to a Financial Shock: Model with One Type of Capital

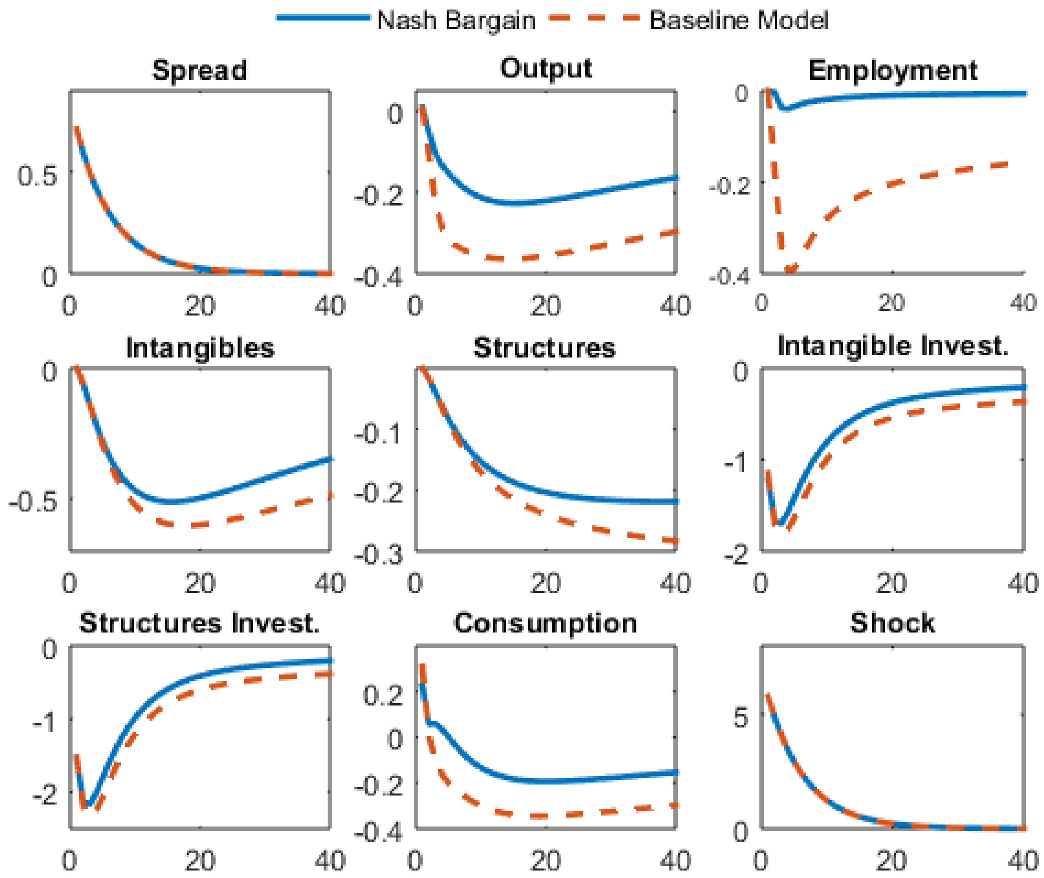


type of capital, employment recovers more quickly. Indeed, in the model with one type of capital, employment returns half way to its steady state level 14 quarters after the shock hits, as compared to 25 quarters after in the baseline model. The reason for this pattern is that, in both models, the tightening of financial constraints directly hits entrepreneurs' ability to fund hiring costs and induces a rapid decrease in employment. However, this direct effect of tighter financial constraints on hiring is relatively transitory. The persistent decrease in employment in both models comes from capital deaccumulation. Since the baseline model experiences a decrease in the stock of intangible capital of around 0.6%, compared to a decrease in aggregate capital of around 0.36% in the model with one type of capital, there is consequently a more persistent decrease in employment in the baseline model.

In order to assess the importance of wage rigidity in the model, I replace the alternating-offer bargaining protocol by the more standard Nash bargaining solution used in the search and matching literature, including Mortensen and Pissarides (1994). Moving to the standard Nash Bargaining solution is equivalent to using the alternating offer bargaining solution (45) and setting $\rho = \infty$. Therefore, I fix $\rho = \infty$ and recalibrate the rest of the model to match the same calibration targets as before.

Figure 6 shows the impulse responses to the financial shock in the model in which the Nash Bargain is used instead of the alternating offer bargain (i.e. when $\rho = \infty$). The baseline model results are also shown for ease of comparison. The most dramatic difference between the two models concerns the response of employment. In the model with standard Nash bargaining, the fall in employment is exceptionally small. This is because wages respond so strongly to changes in labor demand with Nash bargaining that the decreases in labor demand resulting from the deaccumulation of capital and tightening of financial decisions lead simply to a decrease in wages, rather than much of a decrease in employment. Since employment moves so little in the model with Nash bargaining, the response of output to the shock is also substantially smaller – output decreases after the shock around half as much as in the baseline model.

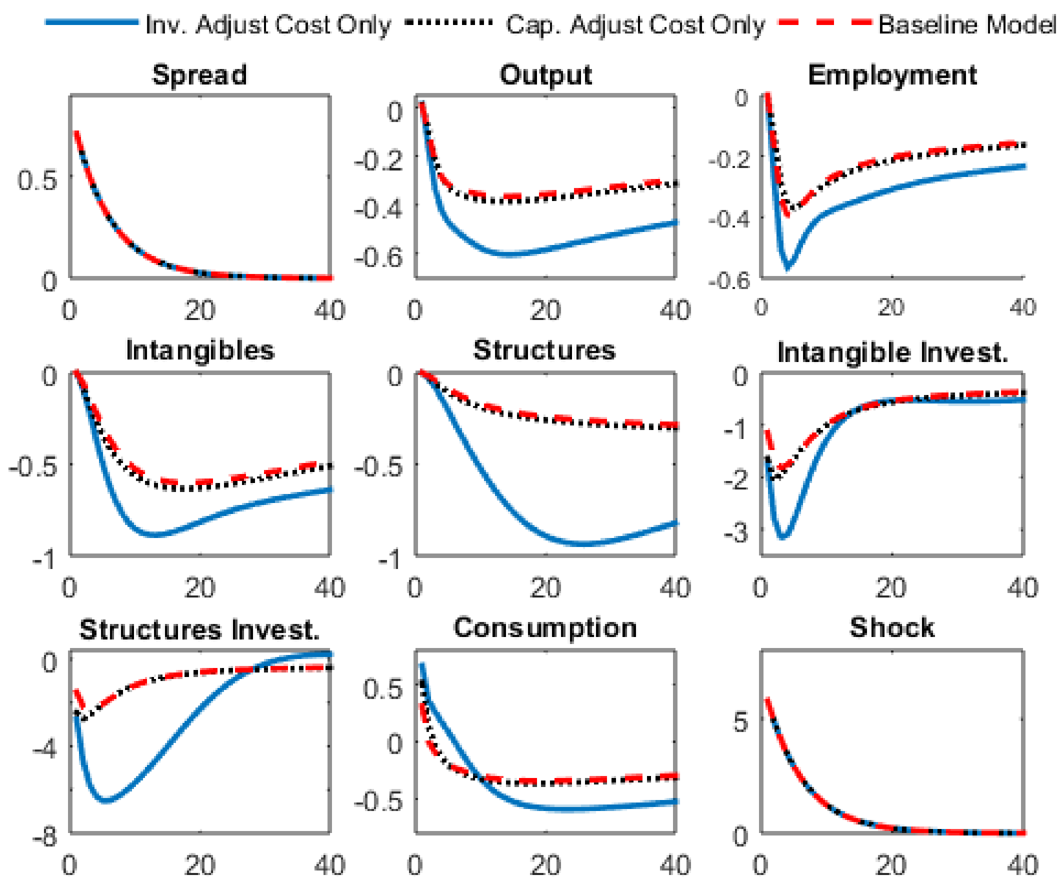
Figure 6: Impulse Response to a Financial Shock: Model with Nash Bargaining



Finally, in order to examine the role played by the adjustment costs to both the level of capital and to the level of investment in the model, Figure 7 shows how the impulse response to a financial shock changes as these are varied. The figure shows the impulse response in three cases. One case shown in the figure is the baseline model, with adjustment costs to the level of investment, $\kappa_1 = 1.2$ and adjustment costs to the level of capital $\kappa_2 = 0.7$. Another case shown is when the model is recalibrated to have only adjustment costs to the level of investment, that is $\kappa_1 = 1.2$ and $\kappa_2 = 0$. Another case shown is when the model is recalibrated to have only adjustment costs to the level of capital, $\kappa_1 = 0$, $\kappa_2 = 0.7$. In all these cases, I recalibrate the other parameters of the model to match the moments used in Tables 2 and 4.

It is evident from Figure 7 that varying adjustment costs to the level of capital affects the impulse responses much more than varying adjustment costs to the level of investment. When adjustment costs to the level of capital are removed and there are only adjustment costs to the level of investment, all variables move much more strongly in response to the shock. This is because, with fewer adjustment costs, investment can decrease much more strongly and so capital is deaccumulated much more quickly. This produces a much deeper slump. In the absence

Figure 7: Impulse Responses for Model with only Capital Adjustment Costs
or only Investment Adjustment Costs



of capital adjustment costs, it is noticeable that the stock of structures decreases particularly sharply relative to the baseline model. This is because investment in structures falls more sharply, so that the ratio of structures to intangibles does not change too much from the steady state level. This prediction of the model without capital adjustment costs is counterfactual with regard to the US Great Recession because the stock of equipment and intangibles fell much more than the stock of structures in the recession, as discussed briefly in 6.3 below.

By contrast, removing adjustment costs to the level of investment has little effect on most variables and produces similar impulse responses to the baseline model. Nevertheless, removing either kind of adjustment cost yields a sharper decline in investment and more countercyclical behavior of consumption. Essentially, reducing adjustment costs means that investment can decrease more quickly. Since output is relatively fixed in the very short run in the model, a faster decline in investment ensures that more resources are available for consumption and so consumption rises. This prediction of the models with fewer adjustment costs is at odds with the empirical fact that consumption clearly fell during the US Great Recession.

References

Abiad, A., Balakrishnan, R., Brook, P. K., Leigh, D., and Tytell, I. (2014). What's the damage? medium-term output dynamics after financial crises. *Financial Crises: Causes, Consequences, and Policy Responses*, page 277.

Acemoglu, D. and Shimer, R. (1999). Holdups and efficiency with search frictions. *International Economic Review*, 40(4):827–849.

Anzoategui, D., Comin, D., Gertler, M., and Martinez, J. (2019). Endogenous technology adoption and r&d as sources of business cycle persistence. *American Economic Journal: Macroeconomics*, 11(3):67–110.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of Macroeconomics*, 1:1341–1393.

Calvo, G. A., Izquierdo, A., and Loo-Kung, R. (2006). Relative price volatility under sudden stops: The relevance of balance sheet effects. *Journal of International Economics*, 69(1):231–254.

Cerra, V. and Saxena, S. C. (2008). Growth dynamics: The myth of economic recovery. *American Economic Review*, 98(1):439–57.

Chodorow-Reich, G. and Karabarbounis, L. (2016). The cyclicality of the opportunity cost of employment. *Journal of Political Economy*, 124(6):1563–1618.

Christiano, L. J. and Davis, J. M. (2006). Two flaws in business cycle accounting. Technical report, National Bureau of Economic Research.

Christiano, L. J., Eichenbaum, M. S., and Trabandt, M. (2015). Understanding the great recession. *American Economic Journal: Macroeconomics*, 7(1):110–67.

Christiano, L. J., Eichenbaum, M. S., and Trabandt, M. (2016). Unemployment and business cycles. *Econometrica*, 84(4):1523–1569.

Christiano, L. J. and Fitzgerald, T. J. (2003). The band pass filter. *International Economic Review*, 44(2):435–465.

Christiano, L. J., Motto, R., and Rostagno, M. (2014). Risk shocks. *American Economic Review*, 104(1):27–65.

Corrado, C. and Hulten, C. R. (2010). How do you measure a "technological revolution"? *American Economic Review: Papers and Proceedings*, 100:99–104.

Eberly, J., Rebelo, S., and Vincent, N. (2012). What explains the lagged-investment effect? *Journal of Monetary Economics*, 59(4):370–380.

Eisfeldt, A. L. and Papanikolaou, D. (2014). The value and ownership of intangible capital. *American Economic Review*, 104(5):189–194.

Erickson, T. and Whited, T. M. (2000). Measurement error and the relationship between investment and q. *Journal of Political Economy*, 108(5):1027–1057.

Furceri, D. and Mourougane, A. (2012). The effect of financial crises on potential output: New empirical evidence from oecd countries. *Journal of Macroeconomics*, 34(3):822–832.

Galí, J., Smets, F., and Wouters, R. (2012). Slow recoveries: A structural interpretation. *Journal of Money, Credit and Banking*, 44(2):9–30.

Garcia-Macia, D. (2017). The financing of ideas and the great deviation. Technical report, International Monetary Fund.

Garin, J. (2015). Borrowing constraints, collateral fluctuations, and the labor market. *Journal of Economic Dynamics and Control*, 57:112–130.

Gilchrist, S. and Zakrajšek, E. (2012). Credit spreads and business cycle fluctuations. *American Economic Review*, 102(4):1692–1720.

Haefke, C., Sonntag, M., and Van Rens, T. (2013). Wage rigidity and job creation. *Journal of monetary economics*, 60(8):887–899.

Hall, R. E. (2015). Quantifying the lasting harm to the u.s. economy from the financial crisis. In Parker, J. A. and Woodford, M., editors, *NBER Macroeconomics Annual 2014*, volume 29, chapter 2, pages 71–128. University of Chicago Press.

Hall, R. E. and Milgrom, P. R. (2008). The limited influence of unemployment on the wage bargain. *American Economic Review*, 4(98):1653–74.

Hulten, C. R. and Hao, X. (2008). What is a company really worth? intangible capital and the "market to book value" puzzle. NBER Working Paper No. 14548.

Jalil, A. J. (2015). A new history of banking panics in the united states, 1825-1929: Construction and implications. *American Economic Journal: Macroeconomics*, 7(3):295–330.

Khan, A. and Thomas, J. K. (2013). Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy*, 121(6):1055–1107.

King, R. G. and Rebelo, S. T. (1999). Resuscitating real business cycles. *Handbook of Macroeconomics*, 1:927–1007.

Krishnamurthy, A. and Muir, T. (2017). How credit cycles across a financial crisis. Technical report, National Bureau of Economic Research.

Kydland, F. E. and Zarazaga, C. E. (2016). Fiscal sentiment and the weak recovery from the great recession: A quantitative exploration. *Journal of Monetary Economics*, 79:109–125.

Lopez, J. I. and Olivella, V. (2018). The importance of intangible capital for the transmission of financial shocks. *Review of Economic Dynamics*, 30:223–238.

Masulis, R. W. (1983). The impact of capital structure change on firm value: Some estimates. *Journal of Finance*, 38(1):107–26.

McGrattan, E. R. and Prescott, E. C. (2014). A reassessment of real business cycle theory. *American Economic Review*, 104(5):177–182.

Michaillat, P. (2014). A theory of countercyclical government multiplier. *American Economic Journal: Macroeconomics*, 6(1):190–217.

Monacelli, T., Quadrini, V., and Trigari, A. (2011). Financial markets and unemployment. NBER Working Paper No. 17389.

Mortensen, D. T. and Pissarides, C. A. (1994). Job creation and job destruction in the theory of unemployment. *The Review of Economic Studies*, 61(3):397–415.

Ottonello, P. (2017). Capital unemployment. Technical report, Working paper, University of Michigan.

Perez-Orive, A. (2016). Credit constraints, firms' precautionary investment, and the business cycle. *Journal of Monetary Economics*, 78:112–131.

Petrosky-Nadeau, N. (2014). Credit, vacancies and unemployment fluctuations. *Review of Economic Dynamics*, 17(2):191–205.

Pissarides, C. A. (2009). The unemployment volatility puzzle: Is wage stickiness the answer? *Econometrica*, 77(5):1339–1369.

Quadrini, V. (2011). Financial frictions in macroeconomic fluctuations. *FRB Richmond Economic Quarterly*, 97(3):209–254.

Quadrini, V. and Trigari, A. (2007). Public employment and the business cycle*. *The Scandinavian Journal of Economics*, 109(4):723–742.

Queralto, A. (2020). A model of slow recoveries from financial crises. *Journal of Monetary Economics*, 114:1–25.

Reinhart, C. M. and Rogoff, K. (2009). *This Time is Different: Eight Centuries of Financial Folly*. Princeton University Press.

Röhe, O. (2012). *New Keynesian DSGE models: theory, empirical implementation, and specification*. PhD thesis.

Schmitt-Grohé, S. and Uribe, M. (2017). Liquidity traps and jobless recoveries. *American Economic Journal: Macroeconomics*, 9(1):165–204.

Schwartzman, F. F. (2012). When do credit frictions matter for business cycles? *FRB Economic Quarterly*, 98(3):209–230.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49.

Silva, J. I. and Toledo, M. (2009). Labor turnover costs and the cyclical behavior of vacancies and unemployment. *Macroeconomic Dynamics*, 13(1):76–96.