Entrepreneurship, Agency Frictions and Redistributive Capital Taxation

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Abstract

We study optimal capital taxation in a model with financial frictions, where the distribution of wealth across heterogeneous entrepreneurs affects how efficiently capital is used in the economy. The government sets linear taxes on wealth, consumption, capital and labor income to maximize the steady state welfare of workers, who own no wealth. In our setting, capital income taxes are particularly costly, because these taxes lead to a more inefficient allocation of capital and, ultimately, lower aggregate total factor productivity. We model financial frictions as arising endogenously as a result of an asymmetric information problem and find that the tightness of financial frictions is affected by tax rates. In our setting, optimal tax rates can be written as simple closed-form functions of pre-tax prices and parameters. We find that the optimal total tax burden on entrepreneurs should be zero, even though the government cares only about workers' welfare.

1 Introduction

This paper studies optimal redistributive capital taxation in a model where taxation affects how efficiently capital is allocated in the economy. The vast literature on optimal capital taxation in general equilibrium has typically analyzed models in which all physical capital is the same and the principal deadweight loss associated with capital taxation is its negative effect on aggregate saving. However, critics of high rates of capital taxation have long expressed concerns that it has harmful effects not only on the total level of

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investment in the economy, but also on the allocation of investment. For instance, Hayek (1960, chap. 20) argues that the taxation of profits hinders the accumulation of wealth by entrepreneurs who manage "successful new ventures", preventing these entrepreneurs from investing further. As such, he argues that "the taxation of... profits, at [high] rates, amounts to a heavy tax on that turnover of capital [between entrepreneurs] which is part of the driving force of a progressive society." Relatedly, it is often argued that taxation may affect incentives for entrepreneurs to take risks, implying that taxation may affect the allocation of capital between more and less risky uses.[1].

We analyze optimal linear capital taxation in a model which incorporates these issues. In the model, there are two types of household: workers and entrepreneurs. All wealth in the economy is held by entrepreneurs and the government sets tax rates to maximize the welfare of workers, as in Judd (1985). As such, we incorporate the possibility that taxing capital may be warranted for redistributive reasons. In the model, entrepreneurs choose how much capital to allocate to a risky technology and how much to allocate to a risk-free technology. Furthermore, entrepreneurs are heterogeneous in their total factor productivity levels and lend to one another through financial markets, but these markets are frictional due to an asymmetric information problem which we model explicitly. In particular, it is assumed that financial contracts must be written to encourage entrepreneurs to truthfully report the value of their idiosyncratic shocks, rather than to lie and divert funds to themselves (in a similar vein to e.g. Bernanke, Gertler and Gilchrist, 1999). The effect of the financial friction is that entrepreneurs are limited in their ability to borrow and are unable to fully diversify idiosyncratic risks. This discourages them from allocating capital to the risky technology, which consequently has a higher expected return than the risk-free technology in equilibrium.

Together, these modeling assumptions imply that taxes on capital affect how efficiently capital is allocated in the economy. Taxes affect how much capital entrepreneurs allocate to the risky and risk-free technologies. Furthermore, the financial friction limits entrepreneurs' abilities to borrow, implying that the amount of capital used by an entrepreneur is tied to her individual wealth. Since taxes affect the distribution of wealth, they therefore affect how far capital is allocated to high-TFP and low-TFP entrepreneurs. Finally, we allow for the possibility that entrepreneurs may choose to hide capital to avoid taxes, rather than allocate it to production, creating an additional channel through which taxation may affect the allocation of capital.

In our model, the government sets linear tax rates on capital income, wealth, labour income and consumption. Capital income taxes (i.e. taxes on the return to capital) are not equivalent to wealth taxes (i.e. taxes on the stock of capital) in our setting, unlike in traditional models. This is because entrepreneurs of different TFP levels differ in their rates of return to capital. As such, our model speaks to recent debates about whether wealth should be taxed in addition to capital income (e.g. Saez and Zucman, 2019). Our

[1]See Cullen and Gordon (2007) and Devereux (2009) and the citations therein.

model is highly tractable and we can characterize the steady state values of aggregate variables as closed form functions of prices, taxes and parameters.

For given prices, parameters and level of aggregate capital stock, we show that the efficiency with which capital is allocated is strictly decreasing in the rates of tax on capital income and consumption. As such, increases in these taxes lead to lower output for a given stock of capital and labour – they reduce the value of the Solow residual for the aggregate economy. Higher capital income taxes lead to a more inefficient allocation of capital because these taxes fall most heavily on entrepreneurs who have a high rate of return to capital. As such, capital income taxes tend to reduce the share of wealth of high TFP entrepreneurs, which also shifts the allocation of productive capital away from these entrepreneurs (due to financial frictions). Additionally, both higher capital income taxes and higher consumption taxes tend to worsen financial frictions by tightening the borrowing constraints of entrepreneurs in our setting, an effect which, to our knowledge, has not been previously studied. This effect occurs because higher values of these taxes encourage entrepreneurs to divert funds to themselves all else equal, thereby reducing the maximum amount that they can borrow in an incentive compatible contract. By tightening financial constraints, higher capital income and consumption taxes reduce the amount of capital that high TFP entrepreneurs can use, and also mean that entrepreneurs need to bear a greater share of the idiosyncratic risk of their projects, which discourages them from putting capital into the risky technology and instead shifts capital to the low return risk-free technology. The net effect of these mechanisms is that higher consumption and capital income taxes shift capital away from high TFP entrepreneurs, and away from the high return risky technology. As such, these taxes lead to a more inefficient allocation of capital in the steady state.

Furthermore, we find that the steady state capital stock is lower when taxes on capital income, consumption and wealth are higher. For capital income and wealth taxes, this is primarily a consequence of the usual mechanism that taxes on capital discourage saving and capital accumulation. However, for capital income taxes and consumption taxes, there is an additional effect that higher taxes reduce the efficiency of capital allocation, as mentioned above, thereby reducing the average return to capital and discouraging capital accumulation. That consumption taxes discourage capital accumulation in our model contrasts with the usual finding that these taxes do not distort capital accumulation when financial frictions are absent (e.g. Coleman, 2000).

In our model, the optimal steady state tax rates (those that maximize the steady state welfare of workers) can be written as simple closed form functions of prices and parameters, as in the literature on the "sufficient statistics" approach to optimal taxation (e.g. Piketty and Saez, 2013). We show that optimal steady state taxes on entrepreneurs are zero on average. This result is in a similar style to those of Chamley (1986) and Judd (1985) and occurs because the long run elasticity of the capital stock to capital tax rates is infinity in our model, as in standard neoclassical models. At the same time, while optimal steady state taxes on entrepreneurs are zero on average, we do not find that the tax rates themselves are zero. Rather, we find conditions under which optimal steady state taxes are strictly negative on capital income, weakly positive on wealth, and strictly positive on consumption. The rationale for negative capital income taxes and positive wealth taxes is that positive capital income taxes lead to a more inefficient allocation of capital by falling heavily on high return uses of capital (as discussed above) but wealth taxes do not, since a wealth tax taxes a dollar of capital at the same rate regardless of whether or not it is put to a high return use.[2] Negative capital income taxes are desirable because they shift capital towards more high return uses. We calibrate the model to data in the US national accounts and Survey of Consumer Finances. We find that the calibrated model is able to match a risk premium on risky capital equal to the equity premium in US data and produces a distribution of wealth at the top percentiles that is similar US data. In the calibrated model, we find that the optimal steady state capital income tax is -28%, the optimal steady state consumption tax is 28%, the optimal steady state wealth tax is 0% and the optimal labor income tax is -3%.

These results suggest that taxing capital income may be relatively less desirable in general than taxing either consumption or wealth due to the sizable negative effect of capital income taxes on both aggregate saving and on the allocation of capital. Since most countries today rely much more on capital income taxes than wealth taxes to raise revenue, this begs the question of whether existing tax arrangements are far from optimal, or whether abstract features of our environment fail to capture the real-world advantages of capital income taxes. Our model is quite general in a number of respects, which argues for the relevance of our results. In particular, our optimal tax formulae hold under weak assumptions about the production function, arbitrary autocorrelation of entrepreneurs' TFP levels over time, and a wide range of financial friction parameters that allow for high or low levels of external finance and equity-like or debt-like financial contracts. Nevertheless, to maintain analytical tractibility and simplicity of exposition, our model is restrictive on a number of important dimensions. We restrict attention to linear tax rates, assume logarithmic utility and constant returns to scale throughout and do not allow for endogenous changes in entry into entrepreneurship. We also do not consider bequests or aging. Finally, we do not consider the possibility of entrepreneurs declaring income as either capital or labour income in order to reduce tax liability. Relaxing these assumptions is left to future work. While these many considerations will doubtless affect the value of optimal taxes, the channels through which taxes affect the allocation of capital in this paper will presumably continue to operate.

The remainder of this paper is structured as follows. The next subsection reviews the recent related literature. Section 2 outlines the model assumptions. Section 3 derives properties of the model equilibrium and steady state and shows how the steady state is

[2] This is the 'Use it or Lose It' effect of wealth taxation studied by Guvenen et al. (2019). We discuss this closely related paper in more detail in the literature review below.

affected by tax rates. Section 4 derives formulae for the optimal tax rates and shows the values of optimal taxes in the numerical calibration. Section 5 concludes.

Related literature. This paper studies optimal taxation where aggregate output depends on the allocation of capital across heterogeneous entrepreneurs, which is affected by taxation. This is a relatively recent research avenue, with only a few papers sharing the same broad framework. Three related papers study optimal policy analytically: Shourideh (2014), Evans (2015) and Itskhoki and Moll (2019). Compared to our paper, the focus of Shourideh (2014) and Itskhoki and Moll (2019) is on different policy issues.[3] Evans (2015) studies optimal dynamic linear capital and labor income taxation in a model with persistent entrepreneurial productivity shocks. Compared to this paper, we differ in focusing on the steady state, but allow for a wider range of tax instruments and study the distinct effects of wealth, consumption and capital income taxes on the allocation of capital. Moreover, we introduce a micro-founded financial friction that changes endogenously in response to taxes. Finally, in our model, entrepreneurs have access to a risky and a risk free technology, allowing us to study how taxation affects the allocation of capital between more high risk and low risk uses.

Our paper is closely to related to work that studies the effects of taxation numerically in models with entrepreneurs with heterogeneous productivity levels. Closest among these to our paper is Guvenen et al. (2019), which numerically compares the merits of optimal wealth taxes and optimal capital income taxes in a framework with heterogeneous entrepreneurs and financial frictions. Guvenen et al. (2019) show that wealth taxes may lead to a greater efficiency of capital allocation than capital income taxes in such an environment, since wealth taxes do not fall more heavily on agents who earn a higher return to capital. This effect also exists in our model, a key mechanism behind our finding of negative optimal capital income taxes. Other related work in this literature includes Cagetti and De Nardi (2009), Kitao (2008), Rotberg and Steinberg (2019) who study the effect of changing estate, capital income and wealth taxes in related settings.

We differ from this literature in several ways. First, our model is analytically tractable and we focus on analytical rather than numerical results, with the aim of making the intuition behind the key mechanisms as transparent as possible and exploring the effects of a wider range of tax policy changes. Second, we also study to study how taxation affects the allocation of capital between high and low risk activities. Third, our financial friction arises endogenously as a consequence of asymmetric information between entrepreneurs and financial intermediaries. This turns out to make a substantial difference for our results because changes in taxes then lead to changes in the tightness of financial frictions that entrepreneurs face. As such, our results highlight that the degree to which

[3] In Shourideh (2014) there are no workers or financial intermediaries and the planner redistributes among entrepreneurs to insure against productivity risk. Itskhoki and Moll (2019) study whether governments in underdeveloped countries can accelerate economic development by market intervention, focusing their analysis on taxes on workers rather than entrepreneurs.

financial markets are frictional may itself be affected by changes in taxes and that this is of importance when considering optimal taxation.

Our paper also relates to Panousi and Reis (2014), Panousi and Reis (2019) and Phelan (2019), who study optimal taxation in the presence of idiosyncratic investment risk. In these two papers, unlike our setting, entrepreneurs do not differ in their expected productivity levels and there is only one production technology, so the allocation of capital does not itself affect aggregate output.

Lastly, our paper contributes to the wider literature on optimal capital taxation, which generally focuses on the effect of capital taxation on aggregate capital accumulation. The hallmark result of this literature is the Chamley (1986) and Judd (1985) claim that in the long run it is optimal to set the capital income tax is zero. Recently, this result has been revisited by Straub and Werning (2014), Benhabib and Szőke (2019) and Chen, Chen and Yang (2019).

A large body of work has extended this result by generalizing the results of Chamley and Judd to richer environments (see Chari and Kehoe, 1999 for a survey). Abo-Zaid (2014), Biljanovska (2019) and Biljanovska and Vardoulakis (2019) have explored how these results are affected in settings with reduced-form financial frictions while maintaining the assumptions of Chamely and Judd that capital is homogeneous and there is no idiosyncratic risk.

The rest of the paper is organized as follows. Section 2 outlines the assumptions of the model. Section 3 discusses properties of the equilibrium of this model. Section 4 presents the government's optimization problem and the optimal tax policy that results. Section 5 concludes.

2 Model

In this section we describe our model economy and derive the conditions for equilibrium.

Environment We consider a discrete time, infinite-horizon economy populated by two types of agents: entrepreneurs and workers. There is a measure one of of entrepreneurs and measure N of workers. In addition, there is a continuum of competitive final goods producers, and a continuum of competitive financial intermediaries, which we refer to as banks. Entrepreneurs own capital, while workers supply labor and live hand-to-mouth. Entrepreneurs use their capital to produce intermediate goods, which they sell to the final goods firms. In particular, each entrepreneur is the owner of two different investment projects, which produce two different types of intermediate goods: she owns a risky project, which produces 'entrepreneurial' intermediate goods denoted by y_E, and she owns a risk-free project, which produces 'standard' intermediate goods denoted by y_F. Both types of intermediate goods are sold to final goods firms.[4] Workers supply

[4]The device of having two separate types of intermediate goods is a simple way to allow entrepreneurs

labor to the final goods firms. The final goods firms use this labor and intermediate goods to produce a final good. The government levies (possibly negative) taxes on the agents and funds the fixed (exogenously given) level of government spending $\overline{G}$.

Timing Each period t is divided into three sub-periods: morning, afternoon and evening. In the morning, entrepreneurs buy and sell capital amongst themselves and each entrepreneur freely divides her capital between her risky and her risk-free investment projects. In the afternoon, each entrepreneur draws an idiosyncratic shock which affects the quantity of capital in her risky project and her two projects produce intermediate goods. Entrepreneurs sell the intermediate goods they produce to final goods firms. In the evening, the final goods firms use intermediate goods and labor to produce output of the final good, which is sold to households. Workers consume all of the final goods that they purchase, while entrepreneurs divide their final goods between consumption and investment for the next period. Existing capital depreciates at rate $\delta \in (0, 1)$. At the end of the period, some agents die and new agents are born.

Technology of Entrepreneurs At the beginning of each period t, each entrepreneur i is endowed with $k_{i,t} > 0$ units of capital. In the morning, before capital is traded, newborn entrepreneurs draw a $\theta_{i,t}$ from the ergodic discrete distribution $g(\theta)$ given by

$$g(\theta) = \begin{cases} 1 - \pi & \text{if } \theta = 0 \\ \pi & \text{if } \theta = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

We refer to θ as the entrepreneur's 'type'. At the start of each period, each continuing entrepreneur has the same θ as in the previous period with probability $1 - \lambda_\theta$ and draws a new θ from the distribution $g(\theta)$ with probability λ_θ.

After allocating capital between her risky and risk-free projects in the morning, the entrepreneur draws a idiosyncratic shock $\epsilon_{i,t}$ in the afternoon, which is independent across time and across entrepreneurs. The shock $\epsilon_{i,t}$ affects the stock of capital in the entrepreneur's risky project, so that an entrepreneur who allocates $k_{E,i,t}$ to her risky project in the morning of period t, and draws the shock $\epsilon_{i,t}$, has $\epsilon_{i,t} k_{E,i,t}$ units of capital in her risky project in the afternoon. As such, the risky project is risky, because the quantity of capital in this project changes stochastically over time. We assume that each entrepreneur's $\epsilon_{i,t}$ is drawn from a lognormal distribution H. In particular, we assume that

$$\epsilon_{i,t} = \underline{\epsilon} + (1 - \underline{\epsilon}) \exp\left(\varphi \xi_{i,t} - \frac{\varphi^2}{2}\right), \tag{2}$$

where $\xi_{i,t} \sim N(0, 1)$ and φ is a parameter determining the variance of ϵ. This assumption

to choose between allocating capital in a risky way or risk-free way. This is designed to capture the idea that some investment projects are more risky than others and that capital owners must take into account the risks associated with different projects when making investment decisions.

implies that $E[\epsilon_{i,t}] = 1$ and $\mathrm{Var}(\log(\epsilon_{i,t} - \underline{\epsilon})) = \varphi^2$. The lowest possible realization of ϵ is $\underline{\epsilon}$.

Each unit of capital in the risky project produces entrepreneurial intermediate goods equal to the entrepreneur's $\theta_{i,t}$. Therefore, if an entrepreneur with type $\theta_{i,t}$ allocates $k_{E,i,t}$ to her risky project in the morning of period t, then in the afternoon the risky project has $\epsilon_{i,t}k_{E,i,t}$ units of capital, and produces $\theta_{i,t}\epsilon_{i,t}k_{E,i,t}$ units of entrepreneurial intermediate goods. After allocating capital to her risky project, the entrepreneur is able to allocate any remaining capital to her risk-free project, or to hide it in order to evade taxes.[5] Let $k_{F,i,t}$ be the amount of capital entrepreneur i allocates to her risk-free project. In the afternoon, her risk-free project produces an output of $y_{F,i,t} = k_{F,i,t}$ standard intermediate goods. Let $k_{TE,i,t}$ denote the capital hidden by the entrepreneur at the start of period t. This capital can be stored and used in the following period. In particular, each unit of capital the entrepreneur hides at the start of period t can be transformed into $1 + \underline{r}$ units of capital the entrepreneur can use at the start of period $t + 1$, where $\underline{r} \leq 0$.

In addition to producing intermediate goods, entrepreneurs are able to hide some of the capital in their risky project after observing their shock $\epsilon_{i,t}$ and converting it directly into units of consumption.[6] In particular, instead of using units of capital in the risky project to produce intermediate goods, an entrepreneur can convert one unit of capital in the risky project into $\phi \in (0, 1)$ units of consumption. However, entrepreneurs cannot hide units of capital in the risk-free project or hide intermediate goods. It will be shown that, when taxes are set optimally, entrepreneurs will not choose to hide any units of capital in equilibrium. However, the ability of entrepreneurs to hide units of capital nevertheless affects allocations and optimal tax rates by creating frictions in financial markets in the economy and providing a vehicle for entrepreneurs to avoid taxes, which prevents the government from setting high tax rates.

The capital held by the entrepreneur i evolves from one period to the next according to the law of motion:

$$k_{i,t+1} = (1 - \delta)(\epsilon_{i,t}k_{E,i,t} + k_{F,i,t} - k_{H,i,t}) + I_{i,t} + (1 + \underline{r})\, k_{TE,i,t}, \tag{3}$$

where $I_{i,t}$ denotes the quantity of investment by the entrepreneur at the end of the period. We let $k_{H,i,t}$ denote the quantity of capital the entrepreneur hides. The term $\epsilon_{i,t}k_{E,i,t}$ appears in this equation because this is the quantity of capital in the risky project that the entrepreneur has at the end of the period, if the entrepreneur does not hide any capital.

Technology of Final Goods Firms We assume that the final goods producer buys entrepreneurial intermediate goods from the entrepreneur at price $r_{E,t}$ per unit, and

[5] As will be discussed below, entrepreneurs do not pay taxes on the units of capital they hide.

[6] As we subsequently discuss, the realization of $\epsilon_{i,t}$ is private information to the entrepreneurs, so they are able to hide capital.

buys standard intermediate goods from the entrepreneur at price $r_{F,t}$ per unit. Since an entrepreneur's output of standard intermediate goods is $y_{F,i,t} = k_{F,i,t}$, a consequence of this is that $r_{F,t}$ is the market rate of return to capital in risk-free projects per period. The final goods producer also hires workers at wage rate w_t. The representative final goods producer produces final output according to the production function

$$Y_t = F\left(Y^S_{E,t}, Y^S_{F,t}, N\right),$$

where Y_t is the aggregate final output per period, N is aggregate labor, $Y^S_{E,t}$ is aggregate input of the entrepreneurial intermediate good and $Y^S_{F,t}$ is aggregate input of the standard intermediate good. We assume that F is concave and strictly increasing in all arguments, and satisfies the Inada conditions.

Demographics At the end of each period fraction $\gamma \in (0,1)$ of entrepreneurs and workers die. They are replaced by an equal measure of newborn entrepreneurs and newborn workers. An agent's occupation as entrepreneur or worker is fixed exogenously from birth and never changes. The capital of dead entrepreneurs is redistributed equally between newborn entrepreneurs, so that each newborn entrepreneur i starts period t with capital $k_{i,t} = K_t$, where K_t is the aggregate capital stock at the start of period t, which is also equal to the average capital per entrepreneur. Workers are born with no wealth and live hand-to-mouth every period.[7]

Preferences Each worker has a constant labor endowment $n = 1$ which he supplies inelastically. Entrepreneurs do not work. The consumption of an entrepreneur i is denoted $c_{i,t}$ and the consumption of a worker is denoted $c_{N,t}$. Workers are identical and so each will have the same consumption. A worker born in period t values his future consumption stream according to: $\sum_{j=0}^{\infty} (1-\rho)^j (1-\gamma)^j u(c_{N,t+j})$, where u is a strictly increasing function. An entrepreneur born in period t values her future consumption stream according to $\sum_{j=0}^{\infty} (1-\rho)^j (1-\gamma)^j U(c_{i,t+j})$ and we assume that the entrepreneur's utility function satisfies $U(c) \equiv \log(c)$.

Government The government sets four different types of tax: a consumption tax $\tau_{C,t}$, a labor income tax $\tau_{N,t}$, a capital income tax $\tau_{K,t}$ and a wealth tax $\tau_{W,t}$ per period. Any of these tax rates can be negative. The government has to finance exogenous expenditure $\overline{G}$, and must balance its budget every period. Taxes are paid in the evening and government spending also takes place in the evening. The government is not allowed to trade in

[7] Provided that workers are born with no wealth, the assumption that they live hand-to-mouth is not critical for the formal results. If we instead allowed workers to hold risk-free bonds earning the risk-free interest rate, they would always choose to hold zero wealth in the steady state, because the steady state risk-free rate is lower than the discount rate $\frac{1}{(1-\rho)(1-\gamma)} - 1$. Therefore, all the steady state results derived below under the assumption that workers live hand-to-mouth would carry through to this alternative case, and steady state optimal taxes would be unchanged.

financial assets at any time. The government's budget constraint each period is

$$\begin{aligned}\overline{G} &= \tau_{N,t} w_t N + \tau_{K,t}(r_{E,t} Y^S_{E,t} + r_{F,t} Y^S_{F,t} - \delta\left(K_t - K_{TE,t}\right)) \\ &+ \tau_{W,t}(K_t - K_{TE,t}) + \tau_{C,t}(C_t - C_{H,t}). \end{aligned} \tag{4}$$

It is instructive to examine the right hand side term by term. The labor income tax revenue is simply $\tau_{N,t} w_t N$. The total gross capital income earned by entrepreneurs each period is equal to the total revenue they earn by selling intermediate goods, $r_{E,t} Y^S_{E,t} + r_{F,t} Y^S_{F,t}$. The government taxes their capital income at rate $\tau_{K,t}$, after deducting for depreciation. The total private wealth of the economy at the start of each period is the total capital held by entrepreneurs, K_t. The government taxes this at rate $\tau_{W,t}$. At the same time, it is assumed that capital that is hidden by entrepreneurs in the morning cannot be taxed by the government. The consequence is that government revenue from the wealth tax is proportional to aggregate capital K_t, net of aggregate capital hidden by the entrepreneurs in the morning $K_{TE,t}$. Finally, all agents pay the tax $\tau_{C,t}$ in proportion to their consumption. As such, government revenue from the consumption tax is equal to aggregate consumption C_t, net of the consumption $C_{H,t}$ which is generated by entrepreneurs from hiding capital, since this consumption cannot be taxed. We show below that $C_{H,t}$ is always zero in the steady state if the tax policy is set optimally.

Financial Markets Entrepreneurs may fund capital purchases each morning by writing one-period state-contingent financial contracts with banks. We assume that banks are risk neutral and perfectly competitive and live for only one period each, so they have no interest in multi-period financial contracts. New banks are created at the start of each period. The financial market opens immediately after each entrepreneur's ability $\theta_{i,t}$ is revealed. If she writes a financial contract with the bank, the entrepreneur receives from the bank some quantity $b_{i,t}$ (possibly negative) in the morning and in exchange she agrees to return to the bank the quantity $\hat{b}_{i,t}$ (possibly negative) at the end of the period, where $\hat{b}_{i,t}$ may depend on the realization of the entrepreneur's shock $\epsilon_{i,t}$. In this way, financial contracts function as a within-period loan for entrepreneurs, and entrepreneurs can also use them to insure themselves against the idiosyncratic risk associated with the shock $\epsilon_{i,t}$. We refer to an entrepreneur as a borrower if she chooses $b_{i,t} > 0$ and a saver if she chooses $b_{i,t} < 0$.

It is convenient to write the entrepreneur's choices of $b_{i,t}$ and $\hat{b}_{i,t}$ as policy functions of the relevant state variables. In general, an entrepreneur's choice of $b_{i,t}$ will depend on her ability $\theta_{i,t}$, her start of period capital $k_{i,t}$ and the aggregate state of the economy, which we label X_t. Therefore, abusing notation slightly, we write $b_{i,t} \equiv b(\theta_{i,t}, k_{i,t}, X_t)$. Likewise, we write $\hat{b}_{i,t} \equiv \hat{b}(\theta_{i,t}, k_{i,t}, \epsilon_{i,t}, X_t)$, since $\hat{b}_{i,t}$ will depend on the variables $\theta_{i,t}, k_{i,t}, X_t$, as well as the shock $\epsilon_{i,t}$.

Since banks are risk-neutral, perfectly competitive and profit maximizing, a bank will agree to a financial contract written by an entrepreneur if and only if the financial contract delivers it non-negative profits in expectation at the end of the period. As such, banks will only lend to entrepreneurs in the morning if the expected return on the loan in the evening is equal to the market risk-free rate. This implies the following constraint

$$\int_{\epsilon} \hat{b}\left(k_{i,t}, \theta_{i,t}, \epsilon, X_t\right) \mathrm{d}H\left(\epsilon\right) \geq R_{F,t} b\left(k_{i,t}, \theta_{i,t}, X_t\right), \tag{5}$$

where $R_{F,t}$ denotes the gross market risk-free rate of interest per period. Since entrepreneurs have no desire to pay the banks more than is necessary, the inequality (5) will be satisfied with equality. The consequence is that banks make exactly zero profits in equilibrium.[8]

Budget Constraints Workers live hand-to-mouth. Thus, the consumption of each worker satisfies

$$c_{N,t}(1+\tau_{C,t}) = w_t(1-\tau_{N,t}). \tag{6}$$

In the morning of each period the entrepreneur may buy and sell capital, divide her capital between a risky and risk-free project, hide some capital and write a financial contract with a bank. Her choices in the morning must satisfy the budget constraint

$$k_{E,i,t} + k_{F,i,t} + k_{TE,i,t} = k_{i,t} + b_{i,t}. \tag{7}$$

After receiving the $\epsilon_{i,t}$ shock, the entrepreneur chooses how many units (if any) of capital in the risky project to hide. We let $k_{H,i,t}$ denotes the quantity of capital the entrepreneur hides in the afternoon. In the evening, the entrepreneur chooses how much to consume, which we denote by $c_{i,t}$, and how much to invest, $I_{i,t}$. Capital hidden in the afternoon is transformed into $c_{H,i,t}$ units of consumption. Finally, in the evening the entrepreneur repays the bank $\hat{b}_{i,t}$ (or is paid by the bank if $\hat{b}_{i,t} < 0$) and pays her taxes to the government. Consequently, in the evening the entrepreneur's budget constraint is

$$\begin{aligned}(1+\tau_{C,t})(c_{i,t} - c_{H,i,t}) + I_{i,t} + \hat{b}_{i,t} &= (1-\tau_K)\left(r_{E,t} y_{E,i,t} + r_{F,i,t} y_{F,i,t}\right) \\ &+ \tau_K \delta(\epsilon_{i,t} k_{E,i,t} + k_{F,i,t}) - \tau_W(\epsilon_{i,t} k_{E,i,t} + k_{F,i,t}),\end{aligned} \tag{8}$$

where $c_{H,i,t}, y_{E,t}, y_{F,i,t}$ satisfy

$$c_{H,i,t} = \phi k_{H,i,t} \tag{9}$$

$$y_{E,t} = \theta_{i,t} \epsilon_{i,t} k_{E,i,t} \tag{10}$$

$$y_{F,i,t} = k_{F,i,t}. \tag{11}$$

[8] Since banks make zero profits, it makes no difference to the equilibrium behavior of the economy who owns the banks. We may assume that they are owned either by workers or by entrepreneurs.

It is convenient to define the entrepreneur's end-of-period resources $\omega_{i,t}$ as

$$\begin{aligned}(1+\tau_C)\,\omega_{i,t} &= (1+\tau_C)\,c_{H,i,t}-\hat{b}_{i,t}+(1-\tau_K)\,(r_{E,t}y_{E,i,t}+r_{F,i,t}y_{F,i,t}) \\ &+ (1-\delta\,(1-\tau_K)-\tau_W)\,(\epsilon_{i,t}k_{E,i,t}+k_{F,i,t}) \\ &- (1-\delta)\,k_{H,i,t}+(1+\underline{r})\,k_{TE,i,t} \end{aligned} \tag{12}$$

Then, using the capital accumulation equation (3), the entrepreneur's evening budget constraint can be re-written as

$$c_{i,t}+\frac{k_{i,t+1}}{1+\tau_{C,t}}=\omega_{i,t} \tag{13}$$

We can write the entrepreneur's decisions as policy functions of the relevant variables, as was done in the previous section for $b_{i,t}$ and $\hat{b}_{i,t}$. That is, we define the capital functions $k_E(k_{i,t},\theta_{i,t},X_t)\equiv k_{E,i,t},\ k_F(k_{i,t},\theta_{i,t},X_t)\equiv k_{F,i,t},\ k_{TE}(k_{i,t},\theta_{i,t},X_t)\equiv k_{TE,i,t},\ k_H(k_{i,t},\theta_{i,t},X_t)\equiv k_{H,i,t}$, consumption and investment functions $c(k_{i,t},\theta_{i,t},\epsilon_{i,t},X_t)\equiv c_{i,t},\ c_H(k_{i,t},\theta_{i,t},\epsilon_{i,t},X_t)\equiv c_{H,i,t},\ I(k_{i,t},\theta_{i,t},\epsilon_{i,t},X_t)\equiv I_{i,t}$, output functions $y_E(k_{i,t},\theta_{i,t},\epsilon_{i,t},X_t)\equiv y_{E,i,t},\ y_F(k_{i,t},\theta_{i,t},\epsilon_{i,t},X_t)\equiv y_{F,i,t}$, and the cash-on-hand function $\omega(k_{i,t},\theta_{i,t},\epsilon_{i,t},X_t)\equiv\omega_{i,t}$. Furthermore, we let $k'(k_{i,t},\theta_{i,t},\epsilon_{i,t},X_t)$ denote the choice of $k_{i,t+1}$.

Agency Friction During the period, an entrepreneur's realization of ϵ, the quantity of capital in the risky sector she hides, her end-of-period investment and the consumption she obtains from converting hidden capital are all private information. In particular, after observing the shock ϵ, an entrepreneur can choose to honestly report the amount of capital she has in the risky sector, but she can also lie by under-reporting the amount of capital she has and hiding more capital than she admits to. However, the quantity of capital allocated to the entrepreneur's projects initially, and the quantity of intermediate goods she produces are assumed to be public information.[9]

When an entrepreneur writes a financial contract in the morning, the market will expect the entrepreneur to repay $\hat{b}(k_{i,t},\theta_{i,t},\epsilon_{i,t},X_t)$ in the evening, given her realization of ϵ. In equilibrium, the market must be correct in expecting this, and so the entrepreneur must have an incentive to repay this amount, rather than lying about ϵ and repaying too little. Therefore, it is without loss of generality to restrict attention to contracts where the entrepreneur honestly reports her ϵ, and pays the promised amount $\hat{b}(k_{i,t},\theta_{i,t},\epsilon_{i,t},X_t)$. Such a contract is only incentive compatible if it is optimal for the entrepreneur to report ϵ honestly, rather than lying by reporting some $\hat{\epsilon}\neq\epsilon$ and hiding more (or fewer) units of capital than she admits to. The entrepreneur will be tempted to lie about ϵ only if doing so increases her available resources for consumption and/or her next period capital, which will be true if and only if by lying she is able to increase her end of period

[9] In the extreme case $\phi=0$ there would be no informational friction, since the entrepreneur has no incentive to hide capital.

$\omega_{i,t}$. This gives rise to the following incentive compatibility constraint

$$\omega(k,\theta,\epsilon,X) \geq \omega(k,\theta,\hat{\epsilon},X) + \phi k_E(k,\theta,X)(\epsilon - \hat{\epsilon}), \tag{14}$$

for any ϵ and $\hat{\epsilon} > 0$ satisfying

$$y_E(k,\theta,\hat{\epsilon},X) \leq y_E(k,\theta,\epsilon,X). \tag{15}$$

This constraint arises from the fact that an entrepreneur who reports $\hat{\epsilon} \neq \epsilon$ will find herself with $k_E(k,\theta,X)(\epsilon - \hat{\epsilon})$ more units of capital than she claimed to have, which she can hide and then transform into $\phi k_E(k,\theta,X)(\epsilon - \hat{\epsilon})$ units of consumption. In that case, her end of period cash-on-hand will be the amount $\omega(k,\theta,\hat{\epsilon},X)$ prescribed by the contract if she drew $\hat{\epsilon}$, plus the value of the extra consumption she produces from the hidden entrepreneurial intermediate goods. For truth telling to be optimal, this must be less than the end of period cash-on-hand she would obtain from reporting truthfully, $\omega(k,\theta,\epsilon,X)$. This incentive compatibility constraint need only be satisfied for $\hat{\epsilon}$ consistent with equation (15) because the entrepreneur cannot convincingly claim to have an $\hat{\epsilon}$ so high that the contract would mandate her delivering more intermediate goods to the final goods producer than she has actually produced.[10]

Entrepreneur's Optimization Problem We write the entrepreneur's optimization problem recursively. Let $V(k,\theta,X)$ denote the continuation value of the entrepreneur who starts the period with capital k and draws ability θ, when the aggregate state is X. Then, the entrepreneur's optimization problem is to choose functions $k_E(\cdot)$, $k_F(\cdot)$, $k_{TE}(\cdot)$, $k_H(\cdot)$, $b(\cdot)$, $\hat{b}(\cdot)$, $c(\cdot)$, $c_H(\cdot)$, $I(\cdot)$, $y_E(\cdot)$, $y_F(\cdot)$, $\omega(\cdot)$ and $k'(\cdot)$ to solve

$$\begin{aligned} V(k,\theta,X) &= \sup \int_{\epsilon>0} \Bigg(\log(c(k,\theta,\epsilon,X)) \\ &+ (1-\rho)(1-\gamma)E\left[V(k'(k,\theta,\epsilon,X),\theta',X')\middle|\theta\right]\Bigg)\mathrm{d}H(\epsilon), \end{aligned} \tag{16}$$

subject to the budget constraints (7) and (8), the law of motion for capital (3), the production functions for c_H in (9), y_E in (10) and y_F in (11), the definition of ω in (12), the incentive compatibility constraint (14), the break-even condition for the banks (5), and the non-negativity conditions $k_E(\cdot) \geq 0$, $k_F(\cdot) \geq 0$, $k_H(\cdot) \geq 0$, $c(\cdot) \geq 0$, $c_H(\cdot) \geq 0$, $I(\cdot) + k_{TE}(\cdot) \geq 0$, $y_E(\cdot) \geq 0$, $y_F(\cdot) \geq 0$, $\omega(\cdot) \geq 0$ and $k'(\cdot) \geq 0$.[11,12]

[10]She cannot do this because sales to the final goods producer are publicly observed.

[11]We do not impose the constraint $c_H(\cdot) \leq c(\cdot)$ for the entrepreneur, although we do impose $C_{H,t} \leq C_t$ in the aggregate in equation (18). To not impose $c_H(\cdot) \leq c(\cdot)$ is justified if the entrepreneur is able to covertly sell the consumption goods generated from hidden capital to other entrepreneurs and/or workers, which would allow the entrepreneur to choose $c_H(\cdot) > c(\cdot)$. Regardless, we conjecture that this assumption is of no consequence in practice, because the constraint $c_H(\cdot) \leq c(\cdot)$ would not bind at realistic parameter values even if it were imposed.

[12]The restriction $I(\cdot) + k_{TE}(\cdot) \geq 0$ amounts to the assumption that the entrepreneur could sell hidden

Here, by having the entrepreneur choose the functions $b(\cdot)$ and $\hat{b}(\cdot)$ subject to the incentive compatibility constraint and break-even condition for the bank, we are assuming that the entrepreneur designs a financial contract and proposes it to a bank. The bank accepts provided that the contract is incentive compatible and the bank breaks even in expectation.

Aggregation and Market Clearing The aggregate level of consumption C_t and of $C_{H,t}$ satisfy

$$C_t = Nc_{N,t} + \int_i c_{i,t}\mathrm{d}i \tag{17}$$

$$C_{H,t} = \int_i c_{H,i,t}\mathrm{d}i \leq C_t. \tag{18}$$

The aggregate levels of capital devoted to each use satisfy

$$K_t = \int_i k_{i,t}\mathrm{d}i \tag{19}$$

$$K_{E,t} = \int_i k_{E,i,t}\mathrm{d}i \tag{20}$$

$$K_{F,t} = \int_i k_{F,i,t}\mathrm{d}i \tag{21}$$

$$K_{TE,t} = \int_i k_{TE,i,t}\mathrm{d}i \tag{22}$$

$$K_{H,t} = \int_i k_{H,i,t}\mathrm{d}i. \tag{23}$$

In each period, the asset market must clear. This requires that the net amount banks lend to entrepreneurs must equal zero

$$\int_i b_{i,t}\mathrm{d}i = 0. \tag{24}$$

The market for intermediate goods of each type must clear each period

$$Y^S_{E,t} = \int_i y_{E,i,t}\mathrm{d}i \tag{25}$$

$$Y^S_{F,t} = \int_i y_{F,i,t}\mathrm{d}i. \tag{26}$$

The first order conditions of the representative final goods producer imply that the (be-

capital at the end of the period in order to finance investment.

fore tax) returns on capital and wage rate are given by:

$$r_{E,t} = F_{Y^S_{E,t}}\left(Y^S_{E,t}, Y^S_{F,t}, N\right) \tag{27}$$

$$r_{F,t} = F_{Y^S_{F,t}}\left(Y^S_{E,t}, Y^S_{F,t}, N\right) \tag{28}$$

$$w_t = F_N\left(Y^S_{E,t}, Y^S_{F,t}, N\right). \tag{29}$$

The final goods market clearing condition then follows by Walras' law[13]

$$C_t + K_{t+1} + \overline{G} = F\left(Y_{E,t}, Y_{F,t}, N\right) + (1-\delta)\left(K_t - K_{TE,t} - K_{H,t}\right) + (1+\underline{r})\, K_{TE,t} + C_{H,t} \tag{30}$$

Equilibrium We are now in the position to define an equilibrium for our model economy.

Definition 1. *For a given sequence of tax rates* $\{\tau_{W,t}, \tau_{K,t}, \tau_{C,t}, \tau_{N,t}\}_{t=0}^{\infty}$, *an equilibrium* $\mathcal{E}$ *of this economy is a sequence of prices* $\{r_{E,t}, r_{F,t}, w_t\}_{t=0}^{\infty}$, *policy functions giving entrepreneurs' decisions, a sequence of worker consumption* $\{c_{N,t}\}_{t=0}^{\infty}$, *and a sequence of aggregate variables* $\{C_t, C_{H,t}, K_t, K_{E,t}, K_{F,t}, K_{TE,t}, K_{H,t}, Y_t, Y^S_{E,t}, Y^S_{F,t}\}_{t=0}^{\infty}$ *such that:*

1. *The government's budget constraint* (4) *is balanced every period.*[14]
2. *Worker consumption satisfies* (6).
3. *Entrepreneurs' decision rules are given by the solution to the entrepreneur's problem* (16).
4. $\{C_t, C_{H,t}, K_t, K_{E,t}, K_{F,t}, K_{TE,t}, K_{H,t}\}_{t=0}^{\infty}$ *represent the aggregate of agents' decisions given by equations* (17)-(23).
5. *The asset market clears, according to equation* (24).
6. *The markets for intermediate goods clear, according to equations* (25) *and* (26).
7. *Prices of intermediate goods* $r_{E,t}$ *and* $r_{F,t}$, *and wages* w_t *are determined by the first order conditions of the final goods firms* (27)-(29).

[13] In particular, the goods market clearing condition can be obtained by summing the budget constraints of workers, entrepreneurs and government, substituting the other market clearing conditions, aggregation conditions and first order conditions of the final goods firm and using that F displays constant returns to scale.

[14] Naturally only some sequences of tax rates $\{\tau_{W,t}, \tau_{K,t}, \tau_{C,t}, \tau_{N,t}\}_{t=0}^{\infty}$ will be consistent with this condition and, therefore, with existence of an equilibrium.

3 Properties of the Model Equilibrium

In this section, we characterize the optimal decisions of entrepreneurs in the model and the aggregate steady state. We derive comparative static results for how aggregate steady state variables change in response to changes in taxes.

3.1 Entrepreneur's Optimal Decisions

We now solve the entrepreneur's optimization problem in (16). To simplify the problem, note first that all the entrepreneur ultimately cares about this period is her level of consumption c and her level of capital for the next period, k'. These are the only variables over which she has influence that enter directly into the entrepreneur's Bellman equation in (16). Now, the level of c and k' that the entrepreneur can afford at the end of the period depend solely on her total resources ω at the end of the period, according to the equation (13). That is, her within-period choices of how much capital to put into each project, how much to borrow and how much capital to hide only impact on the level of c and k' she can afford insofar as they affect the ω she will have at the end of the period. Therefore, we can split the entrepreneur's problem into a within-period choice of trying to achieve a high value of ω, and a between period choice of how to divide her resources ω between consumption and next-period capital. To this end, let $\tilde{V}(\omega, X)$ denote the value in the evening of a period of an entrepreneur with resources ω, who is yet to divide her resources between consumption and next period capital. Then, we can write the entrepreneur's between period problem recursively as:

$$\tilde{V}(\omega, X) = \sup_{c,k' \geq 0} \left(\log(c) + (1-\rho)(1-\gamma) E\left[V(k', \theta', X') \right] \right), \tag{31}$$

$$\text{s.t.} \quad c + \frac{k'}{1+\tau_C} = \omega. \tag{32}$$

The entrepreneur's recursive within period problem is to choose non-negative functions $k_E(k,\theta,X)$, $k_F(k,\theta,X)$, $k_T E(k,\theta,X)$, $k_H(k,\theta,X)$, $\omega(k,\theta,\epsilon,X)$ and functions $b(k,\theta,X)$ and $\hat{b}\,(k,\theta,\epsilon,X)$ to solve:

$$V(k,\theta,X) = \sup \int_{\epsilon>0} \tilde{V}(\omega, X) \mathrm{d}H(\epsilon), \tag{33}$$

subject to the constraints:

$$
\begin{aligned}
k_E + k_F + k_{TE} &= k + b && (34)\\
\int_\epsilon \hat{b} \mathrm{d}H(\epsilon) &= R_F b && (35)\\
(1+\tau_C)\omega &= (1+\tau_C)\phi k_H - \hat{b}(k,\theta,\epsilon,X) + (1-\tau_K)(r_E \epsilon\theta k_E + r_F k_F) \\
&+ (1-\delta(1-\tau_K) - \tau_W)(\epsilon k_E + k_F) \\
&- (1-\delta) k_H + (1+\underline{r}) k_{TE} && (36)\\
\omega(k,\theta,\epsilon,X) &\geq \omega(k,\theta,\hat{\epsilon},X) + \phi k_E(\epsilon - \hat{\epsilon}), && (37)
\end{aligned}
$$

for any ϵ and $\hat{\epsilon} > 0$ satisfying:

$$\theta\hat{\epsilon}k_E \leq \theta\epsilon k_E. \quad (38)$$

The constraints (34), (35), (36) and (37) are simply the equations (7), (5), (12) and (14), respectively, where we substituted in the equations (9), (10) and(11) to eliminate C_H, y_E and y_F.

The constant returns to scale assumptions on the entrepreneur's technology for producing intermediate goods means that the value function must take a particular form, as shown in the following lemma. This considerably simplifies the solution to the entrepreneur's problem.

Lemma 1. *There exists a function $\overline{V}(\theta, X)$ such that, for any k, θ and X,*

$$E\left[V(k,\theta,X)\middle|k,\theta,X\right] = \overline{V}(\theta,X) + \frac{\log(k)}{1-(1-\rho)(1-\gamma)}.$$

Proof. See Appendix A.1. □

Using Lemma 1, the solution to the between period problem can be found immediately by taking the first order condition

$$\frac{1}{(1+\tau_C)\omega - k'} = \frac{(1-\rho)(1-\gamma)}{1-(1-\rho)(1-\gamma)}\frac{1}{k'},$$

and combining it with equation (32) to conclude that the entrepreneur chooses

$$
\begin{aligned}
c &= (1-(1-\rho)(1-\gamma))\omega && (39)\\
k' &= (1+\tau_C)(1-\rho)(1-\gamma)\omega. && (40)
\end{aligned}
$$

Substituting these choices into the Bellman equation (31), we have that

$$\begin{aligned}\tilde{V}(\omega, X) = \frac{\log(\omega)}{1-(1-\rho)(1-\gamma)} &+ \log(1-(1-\rho)(1-\gamma)) \\ &+ \frac{(1-\rho)(1-\gamma)\log((1+\tau_C)(1-\rho)(1-\gamma))}{1-(1-\rho)(1-\gamma)} \\ &+ (1-\rho)(1-\gamma)E[\overline{V}(\theta', X')]. \end{aligned} \tag{41}$$

This completes the solution of the between period problem.

To solve the within period problem, we first take the first order approach to the incentive compatibility constraint.[15] In particular, we can replace the equation (37) with the associated first order condition for the choice of $\hat{\epsilon}$, evaluated at the honest report $\hat{\epsilon} = \epsilon$

$$\frac{\partial \omega(k, \theta, \hat{\epsilon}, X)}{\partial \hat{\epsilon}} = \phi k_E(k, \theta, X).$$

Integrating with respect to ϵ, it follows that there must exist some function $\underline{\omega}(k, \theta, X)$ such that:

$$\omega(k, \theta, \epsilon, X) \equiv \underline{\omega}(k, \theta, X) + \phi k_E(k, \theta, X)\epsilon. \tag{42}$$

Thus, the entrepreneur's end-of-period resources ω must linearly increase with ϵ, so that it is optimal for her to report her ϵ honestly, rather than under-reporting ϵ, hiding some capital, and converting it into units of consumption. Note that the more an entrepreneur's end-of-period resources are sensitive to ϵ, the more risk the entrepreneur faces. Since the entrepreneur has log utility, she is risk averse. On the other hand, the bank is risk neutral. Therefore, in the absence of agency frictions, the entrepreneur and bank would prefer a contract in which the bank took all the risk and the entrepreneur's ω was independent of ϵ. The agency friction prevents this, leading the entrepreneur to face the level of risk implied by the equation (42).

To simplify the within period problem further, note that during the period t the entrepreneur can do four different activities each of which yields a riskless return to her capital. First, she can hide capital at the beginning of the period. Second, she can sell capital and lend to a bank, setting $b < 0$. The bank's break even condition (35) implies that the bank would be willing to borrow from her and pay back the risk-free gross interest rate of R_F at the end of the period, for each unit lent to the bank. Third, the entrepreneur can allocate capital to her risk-free project and sell the output of this to the final goods firms. Fourth, the entrepreneur can hide her capital after observing ε and transform it into units of consumption. Lemma 2 below establishes that, for the asset market and market for standard intermediate goods to be in equilibrium, the entrepreneur's return from lending to the bank must be identical to the return she

[15] A proof that the first order approach provides the correct solution to this problem is available upon request.

obtains from hiding capital in the beginning of the period and allocating capital to her risk-free project and must be the same or greater than the return from hiding capital after observing the realization of ε. The consequence is that the entrepreneur will weakly prefer to lend to the bank over all other risk-free activities.

Lemma 2. *In an equilibrium of the economy, it must hold every period that*

$$R_F \;=\; 1+\underline{r} = 1 + [(1-\tau_K)\,(r_F-\delta) - \tau_W] \geq \phi(1+\tau_C). \tag{43}$$

In equilibrium, all entrepreneurs are indifferent over their choices of the level of k_F. If the inequality in (43) is strict, all entrepreneurs strictly prefer to set $k_H = 0$. If the inequality in (43) holds with equality, entrepreneurs will be indifferent over the level they choose of k_H.

Proof. See Appendix A.2. □

Equation (43) establishes that in a equilibrium of the economy each entrepreneur is indifferent between hiding capital in the beginning of the period, allocating capital to her risk-free project (and selling the intermediate goods that result) and lending to a bank. If ϕ is sufficiently high, an entrepreneur may also be indifferent between these and hiding capital after observing the realization of ϵ. On the other hand, if the inequality in (43) is strict, the entrepreneur will choose to hide no capital in the middle of the period.

Given that the entrepreneur weakly prefers to lend to the bank than the other risk-free activities, we can solve her optimization problem under the assumption that she chooses $k_{TE} = k_F = k_H = 0$, with the understanding that in equilibrium some entrepreneurs may borrow from banks to engage in these other risk-free activities, to the extent needed to clear markets. Combining the rewritten incentive compatibility constraint (42) with the definition of ω in (36), and integrating with respect to ϵ using that $k_{TE} = k_F = k_H = c_H = 0$ and $E[\epsilon] = 1$, reveals that $\underline{\omega}(\cdot)$ must satisfy

$$(1+\tau_C)(\underline{\omega}+\phi k_E) \;=\; (1-\delta(1-\tau_K)-\tau_W)k_E + (1-\tau_K)r_E\theta k_E - \int_\epsilon \hat{b}\,(k,\theta,\epsilon,X)\,\mathrm{d}H\,(\epsilon)\,.$$

Combining this with the bank zero profit condition (35) and the budget constraint (34) and rearranging gives

$$(1+\tau_C)\underline{\omega} = (1-\delta(1-\tau_K)-\tau_W-\phi(1+\tau_C))k_E + (1-\tau_K)r_E\theta k_E + R_F(k-k_E). \tag{44}$$

Using equations (41), (42) and (44), we can rewrite the entrepreneur's within period problem more compactly. The entrepreneur seeks to choose functions $k_E(\cdot)$ and $\underline{\omega}(\cdot)$ to solve:

$$\sup \int_\epsilon \log(\underline{\omega}+\phi\epsilon k_E)\mathrm{d}H(\epsilon), \tag{45}$$

subject to the constraints:

$$
\begin{aligned}
(1+\tau_C)\underline{\omega} &= \left[1-\delta(1-\tau_K)-\tau_W-\phi(1+\tau_C)+(1-\tau_K)r_E\theta-R_F\right]k_E+R_Fk && (46)\\
k_E &\geq 0 && (47)\\
\underline{\omega}+\rho\underline{\epsilon}k_E &\geq 0 && (48)
\end{aligned}
$$

Here, we used that the only part of $\tilde{V}$ in (41) which depends on the entrepreneur's decisions is the term $\frac{\log(\omega)}{1-(1-\rho)(1-\gamma)}$. Therefore, maximizing the expected value of $\tilde{V}$ amounts to maximizing the expected value of this term. The constraint (48) arises because the minimum value of ϵ is $\underline{\epsilon}>0$ so, the entrepreneur can be sure of non-negative consumption if and only if $\underline{\omega}+\rho\underline{\epsilon}k_E$.

Now, we note that, in an equilibrium of this economy, it must be that the price of entrepreneurial intermediate goods is sufficiently low that entrepreneurs do not wish to produce and sell infinite quantities of entrepreneurial intermediate goods. This condition is contained in the following lemma.

Lemma 3. *In an equilibrium of the economy, it must be the case every period that:*

$$
1-\delta(1-\tau_K)-\tau_W+(1-\tau_K)\bar{\theta}r_E-\phi(1+\tau_C)-R_F \;\leq\; 0 \tag{49}
$$

Proof. See Appendix A.3. □

The entrepreneur's optimization problem (45) amounts to determining the optimal choice of k_E. This is simply a problem of a trade-off between risk and return. Choosing higher k_E increases the variance of ω, since $\omega=\underline{\omega}+\phi\epsilon k_E$, but higher k_E may carry a higher expected return. The optimal decision rule of the entrepreneur can be written more compactly in terms of after-tax prices which we define as follows

$$
\begin{aligned}
\tilde{r}_F &\equiv \frac{r_F\left(1-\tau_K\right)}{1+\tau_C} && (50)\\
\tilde{r}_E &\equiv \frac{r_E\left(1-\tau_K\right)}{1+\tau_C} && (51)\\
\tilde{p} &\equiv \frac{\delta(1-\tau_K)+\tau_W}{1+\tau_C}=\frac{1-R_F}{(1+\tau_C)}+\tilde{r}_F. && (52)
\end{aligned}
$$

Then, the entrepreneur's within-period problem is to to choose functions $k_E(k,\theta,X)$ and $\underline{\omega}(k,\theta,X)$ to solve:

$$
\sup\int_\epsilon \log(\underline{\omega}+\phi\epsilon k_E)\mathrm{d}H(\epsilon), \tag{53}
$$

subject to the constraints:

$$
\begin{aligned}
\underline{\omega} &= \left(-\phi + \tilde{r}_E\theta - \tilde{r}_F\right) k_E + \left(\frac{1}{1+\tau_C} - \tilde{p} + \tilde{r}_F\right) k & (54)\\
k_E &\geq 0 & (55)\\
\underline{\omega} + \phi\underline{\epsilon}k_E &\geq 0. & (56)
\end{aligned}
$$

3.2 Continuous Time Limit

To describe the environment, the equilibrium conditions and to simplify the contracting problem between the entrepreneurs and banks it was natural to make the assumption of discrete time. In the remainder of the paper, which is devoted to characterizing the entrepreneur's optimal decision rule, the steady state of the economy, and to solving for the optimal taxes, it is more convenient to work in continuous time. We formally derive a continuous-time version of our discrete-time model in Appendix B. There, we assume that each period is of length Δ and obtain solutions to the entrepreneur's problem and characterize the steady state of the economy. We then take the limit as Δ goes to zero. This leads to the following optimal decision rule for the entrepreneur in the continuous-time version of the model.

Proposition 1. *In equilibrium, the entrepreneur's problem has a unique solution which depends continuously on the parameters. The entrepreneur's optimal choices are:*

$$
\begin{aligned}
k_E &= \frac{k}{(1+\tau_C)\phi(1-\underline{\epsilon})} \times \max\left\{0; \min\left\{\frac{\theta\tilde{r}_E - \tilde{r}_F}{\phi(1-\underline{\epsilon})\varphi^2}; 1\right\}\right\} & (57)\\
c &= (\rho+\gamma)\frac{k}{1+\tau_C} & (58)\\
dk &= \left[(1+\tau_C)\left(\tilde{r}_F - \tilde{p}\right)k + (1+\tau_C)\left(k_E\left(\theta\tilde{r}_E - \tilde{r}_F\right) - c\right)\right]dt \\
&\quad + k_E\phi\left(1+\tau_C\right)\left(1-\underline{\epsilon}\right)\varphi dW & (59)
\end{aligned}
$$

where dW is the difference of a standard Brownian motion.

Proof. See Appendix B.4. □

As such, the solution to the entrepreneur's optimization problem implies that if the after-tax expected return on the risky project $\theta\tilde{r}_E$ is lower than the after-tax risk-free net return on the risk-free project $\tilde{r}_F$, then it is optimal for the entrepreneur to set $k_E = 0$ and either hide all capital in the beginning of the period, or allocate all her capital to her risk-free project, or sell it and use the revenue from the sale to lend to a bank. If the after-tax excess return from investing the risky project is positive, i.e. $\theta\tilde{r}_E - \tilde{r}_F > 0$, then the entrepreneur allocates an amount of capital to her risky project which is proportional to her initial capital k. As in many other models with financial market frictions, it therefore follows that the allocation of capital in the economy depends on the wealth distribution across entrepreneurs – capital is not necessarily allocated to its most productive uses. If the after-tax excess return from investing the risky project is sufficiently high, the entrepreneur will choose the largest k_E that guarantees non-negative consumption under the lowest possible realization of ϵ.

Unsurprisingly, all else equal, a richer entrepreneur invests more in her risky project. Furthermore, entrepreneurs invest more in risky projects and less in risk-free projects when: *(i)* the after-tax return to risky projects is relatively higher (i.e. higher $\tilde{r}_E$), *(ii)* the after-tax return to risk-free projects is relatively lower (lower $\tilde{r}_F$) or *(iii)* the agency friction is less severe (i.e. lower ϕ).

3.3 Aggregate Steady State

In this section, we formally characterize a steady state of the model. We derive comparative static results for how aggregate steady state variables change in response to changes in taxes. In Section 4 below, we use these results to derive the government's optimal steady state tax policy. We define a steady state as follows:

Definition 2. *A* ***steady state*** $\mathscr{S}$ *of the economy is a set of values of tax rates* $\{\tau_W^*, \tau_K^*, \tau_C^*, \tau_N^*\}$*, prices* $\{r_E^*, r_F^*, w^*\}$*, aggregate variables* $\{K^*, K_E^*, C^*, Y_E^{S*}, Y_F^{S*}\}$ *and an equilibrium* $\mathcal{E}$ *in which all tax rates, prices and aggregate variables are equal to these steady state values in every period.*

We focus on steady states in which no capital or intermediate goods are hidden because, as argued below, the government will never design a tax policy to select a steady state in which capital and intermediate goods are hidden. The full set of conditions that must be satisfied in a steady state are summarized in Proposition 2 below. To write the conditions compactly, we express them in post-tax prices. The steady state post tax prices $\tilde{r}_E^*$, $\tilde{r}_F^*$ and $\tilde{p}^*$ are defined according to equations (50)-(52) above. We define the steady state post-tax wage as:

$$\tilde{w}^* \equiv \frac{w^*(1-\tau_N^*)}{1+\tau_C^*} \tag{60}$$

Using these definitions, Proposition 2 summarizes the necessary and sufficient conditions for a steady state:

Proposition 2. *There exists a steady state* $\mathscr{S}$ *which is consistent with the particular values of aggregate variables* $\{K^*, K_E^*, C^*, Y_E^{S*}, Y_F^{S*}\}$*, post-tax prices* $\{\tilde{r}_E^*, \tilde{r}_F^*, \tilde{w}^*, \tilde{p}^*\}$ *and consumption tax rate* τ_C^* *and in which no entrepreneurs hide capital or intermediate goods, if and only if the* *following conditions hold:*

$$\mu_K^*(\theta) = \frac{\lambda_\theta g(\theta)}{\lambda_\theta+\rho+\gamma-(1+\tau_C)\left(\tilde{r}_F-\tilde{p}+\hat{k}_E(\theta)(\theta\tilde{r}_E-\tilde{r}_F)\right)} \tag{61}$$

$$\frac{\tilde{r}_F^*}{\tilde{r}_E^*} = \frac{F'_{Y_F^{S*}}}{F'_{Y_E^{S*}}} \tag{62}$$

$$Y_F^{S*} = K^* - K_E^* \tag{63}$$

$$C^* = \tilde{w}^* N + \tilde{r}_E^* Y_E^{S*} + \tilde{r}_F^* Y_F^{S*} - \tilde{p}^* K^* \tag{64}$$

$$C^* = \tilde{w}^* N + (\rho+\gamma)\frac{K^*}{1+\tau_C} \tag{65}$$

$$C^* + \delta K^* + \overline{G} = F(Y_E^{S*}, Y_F^{S*}, N) \tag{66}$$

$$Y_E^{S*} = K^* \textstyle\sum_\theta \theta \hat{k}_E(\theta)\mu_K^*(\theta) \tag{67}$$

$$K_E^* = K^* \textstyle\sum_\theta \hat{k}_E(\theta)\mu_K^*(\theta) \tag{68}$$

$$\hat{k}_E(\theta) = \frac{1}{(1+\tau_C)\phi(1-\epsilon)} \times \max\left\{0; \min\left\{\frac{\theta\tilde{r}_E-\tilde{r}_F}{\phi(1-\epsilon)\varphi^2}; 1\right\}\right\} \tag{69}$$

where $\tilde{r}_E^*\overline{\theta} > \tilde{r}_F^*$, $K_E^* < K^*$, $(1+\tau_c^*)(\tilde{r}_F^* - \tilde{p}^*) \geq \underline{r}$ *and* $\frac{1}{1+\tau_C} \geq \phi$, *with strict inequality if* $\tilde{r}_F > \tilde{p}$.

Proof. See Appendix B.5. □

Equation (61) describes the steady state fraction of wealth held by entrepreneurs of ability θ, which we denote by $\mu_K^*(\theta)$. This can be found by deriving the law of motion for the fraction of wealth held by entrepreneurs of each θ, and solving for the steady state of this law of motion. Unsurprisingly, $\mu_K^*(\theta)$ is increasing in $g(\theta)$, which is the fraction of entrepreneurs of ability θ, and is also increasing in $(1+\tau_C)\left(\tilde{r}_F - \tilde{p} + \hat{k}_E(\theta)(\theta\tilde{r}_E - \tilde{r}_F)\right)$, which is the average rate of return earned by entrepreneurs of ability θ, measured in units of capital goods. Equation (62) states that the ratio of prices of entrepreneurial and standard intermediate goods post-tax is the same as the ratio of marginal products of these intermediate goods. This follows from the first order conditions of the final goods firms. Equation (63) is simply the consequence of the fact that each unit of capital not placed in risky projects is placed in risk-free projects (since no capital is hidden) and each unit of capital in risk-free projects yields one unit of standard intermediate goods. Equations (64), (65), (67) and (68) come from integrating the entrepreneur's optimal decision rules in Proposition 1 across entrepreneurs, while noting that workers live hand-to-mouth. Equation (66) is the goods market clearing condition. Finally the inequalities $\tilde{r}_E^* \overline{\theta} > \tilde{r}_F^*$ and $K_E^* < K^*$ ensure that the Inada conditions on the final goods production function are satisfied, and the inequalities $(1+\tau_C)(\tilde{r}_F^* - \tilde{p}^*) \geq \underline{r}$ and $\frac{1}{1+\tau_C} \geq \phi$ are the continuous time analog of the inequalities that were shown to hold in Lemma 2.

Comparative Statics of Capital Allocation Using the results in Proposition 2, this section analyses how the allocation of capital in the economy changes in response to changes in post-tax prices, treating post-tax prices as exogenous for the moment. In particular, we derive expressions for the elasticity of key aggregate variables with respect to changes in tax rates. These elasticities are critical to deriving the optimal tax rates studied in Section 4.

First, combining equations (61)-(69), we can derive the following expression for K_E^* in a steady state in terms of parameters, taxes and pre-tax prices:[16]

$$K_E^* = \frac{\frac{K^*}{(1+\tau_C^*)\phi(1-\underline{\epsilon})}}{\max\left\{1 + \frac{\lambda_\theta(1-\pi)}{\lambda_\theta\pi + (1-\tau_K^*)(r_E^* - r_F^*)\frac{K_E^*}{K^*}}; \left[\left(1 + \frac{\lambda_\theta(1-\pi)}{\lambda_\theta\pi + (1-\tau_K^*)(r_E^* - r_F^*)\frac{K_E^*}{K^*}}\right)\frac{\varphi^2}{(1-\tau_K)\left(r_E^* - r_F^*\right)\frac{K_E^*}{K^*}}\right]^{1/2}\right\}}. \tag{70}$$

It may seem surprising that the wealth tax τ_W^* does not appear in this expression. The reason is that the wealth tax τ_W^* affects the total steady state stock of capital K^*, but does not affect the ratio $\frac{K_E^*}{K^*}$, given the values of other taxes and pre-tax prices.

Using equation (70), we define $e_{\tau_K}^{K_E}$ and $e_{\tau_C}^{K_E}$ as the elasticity of K_E^* with respect to capital income and consumption taxes, holding constant pre-tax prices and aggregate steady state capital K^*.[17] That is, we define:

$$e_{\tau_K}^{K_E} = \frac{1-\tau_K^*}{K_E^*} \cdot \frac{\partial K_E^*}{\partial \tau_K^*}$$
$$e_{\tau_C}^{K_E} = \frac{1+\tau_C^*}{K_E^*} \cdot \frac{\partial K_E^*}{\partial \tau_C^*}.$$

Implicitly differentiating equation (70) and rearranging, it follows that these satisfy:

$$e_{\tau_K}^{K_E} < 0 \tag{71}$$
$$e_{\tau_C}^{K_E} = -1 + e_{\tau_K}^{K_E} < -1. \tag{72}$$

These $e_{\tau_K}^{K_E}$ and $e_{\tau_C}^{K_E}$ are the crucial elasticities that determine the effect of taxes on how

[16]See Appendix B.5 for a derivation of this result.

[17]We treat K^* as fixed because it turns out that this is relevant for the government's optimization problem in Section 4, since the government can effectively set the tax rate τ_W^* to achieve its desired level of K^*.

efficiently capital is allocated in the economy. The effect of tax changes on aggregate output, if τ_W^* is adjusted in order to keep K^* fixed, can be directly expressed in terms of these elasticities. That is, for instance:

$$\frac{\partial Y^*}{\partial \tau_K^*} = \frac{\partial Y^*}{\partial K_E^*} \cdot \frac{\partial K_E^*}{\partial \tau_K^*} - \frac{\partial Y^*}{\partial K_F^*} \cdot \frac{\partial K_E^*}{\partial \tau_K^*} = \left(\frac{r_E^* - r_F^*}{1 - \tau_K^*}\right) K_E^* e_{\tau_K}^{K_E},$$

and similarly

$$\frac{\partial Y^*}{\partial \tau_C^*} = \left(\frac{r_E^* - r_F^*}{1 + \tau_C^*}\right) K_E^* e_{\tau_C}^{K_E}.$$

Since $e_{\tau_K}^{K_E}$ and $e_{\tau_C}^{K_E}$ are both negative, we may conclude that an increase in capital or consumption taxes serves to decrease the aggregate output of the economy, when τ_W^* is adjusted to keep K^* constant. The reason is that an increase in capital and consumption taxes distorts the allocation of capital in the economy by affecting both the fraction of aggregate capital allocated to entrepreneurs' risky projects and risk-free projects, and also by affecting the fraction of wealth held by high θ entrepreneurs. The general effect of tax increases is to reduce aggregate output, by reducing the proportion of capital allocated to risky projects, which on average carry a higher expected return than risk-free projects. Since tax increases can decrease aggregate output in this economy even conditional on the value of aggregate capital K^* and aggregate labor N, an economist conducting a growth accounting exercise would attribute the resulting decrease in output following a tax rise to a decrease in measured aggregate total factor productivity. The elasticity of measured steady state total factor productivity with respect to a change in tax rates would therefore be exactly equal to the elasticity of aggregate steady state output Y^* with respect to a change in tax rates, holding fixed K^*. This effect of tax increases of reducing the total factor productivity of the economy by distorting the allocation of capital is a distinct effect of taxation on capital owners which until recently has not been emphasized in earlier literature on optimal taxation.[18] The literature has instead focused on the effect of capital taxation on aggregate investment. In our model, taxation of capital owners affects aggregate output via both channels.

We now consider the elasticity of entrepreneurs' consumption with respect to taxes. Since workers live hand-to-mouth, equation (65) implies that the aggregate consumption of entrepreneurs is equal to $(\rho + \gamma) \frac{K^*}{1+\tau_C^*}$. Then, if we define $e_{\tau_K}^{C_E}$, $e_{\tau_C}^{C_E}$ as the elasticity of aggregate entrepreneurial consumption with respect to capital income and consumption

[18]This channel is also active in Guvenen et al. (2019), Rotberg and Steinberg (2019).

taxes, holding constant K^*, we have that:

$$e^{C_E}_{\tau_K} = \frac{(1-\tau_K^*)}{C_E^*} \cdot \frac{\partial C_E^*}{\partial \tau_K^*} = 0 \tag{73}$$

$$e^{C_E}_{\tau_C} = \frac{(1+\tau_C)}{C_E^*} \cdot \frac{\partial C_E^*}{\partial \tau_C} = -1. \tag{74}$$

Therefore, the consumption of entrepreneurs, fixing K^*, decreases in consumption taxes but doesn't depend on capital income taxes.

An additional elasticity that is highly relevant for the optimal tax rates is the elasticity of the post-tax risk-free return to capital with respect to taxes, where the post-tax risk-free return to capital is given by $\tilde{R}_F = (r_F - \delta)(1-\tau_K) - \tau_W = (1+\tau_C)\left(\tilde{r}_F - \tilde{p}\right)$. This is important for the government's choice of optimal taxes because, for entrepreneurs to not evade taxes in the steady state, it must hold that $\tilde{R}_F \geq \underline{r}$, as shown in Proposition 2. Using equations (64) and (65), it follows that, in a steady state, $\tilde{R}_F^*$ must satisfy:

$$\tilde{R}_F^* = \rho + \gamma - (1-\tau_K^*)(r_E^* - r_F^*)\frac{K_E^*}{K^*}. \tag{75}$$

Intuitively, in order to encourage entrepreneurs to invest enough, the steady state post-tax risk-free rate must be higher if entrepreneurs discount the future more (higher ρ and γ) and lower if the return to risky capital is higher (τ_K^* is lower). Again, τ_W^* does not appear here directly, but matters indirectly since it influences K^*.

Using equation (75) and defining $e^{R_F}_{\tau_K}$ and $e^{R_F}_{\tau_C}$ as the semi-elasticities of $\tilde{R}_F^*$ with respect to τ_K^* and τ_C^*, holding constant K^* (but not K_E^*), we have:

$$e^{R_F}_{\tau_K} \equiv (1-\tau_K^*) \cdot \frac{\partial \tilde{R}_F^*}{\partial \tau_K^*} = (1-\tau_K^*)(r_E^* - r_F^*)\frac{K_E^*}{K^*}(1 - e^{K_E}_{\tau_K}) \tag{76}$$

$$e^{R_F}_{\tau_C} \equiv (1+\tau_C^*) \cdot \frac{\partial \tilde{R}_F^*}{\partial \tau_C^*} = -(1-\tau_K^*)(r_E^* - r_F^*)\frac{K_E^*}{K^*}e^{K_E}_{\tau_C}. \tag{77}$$

4 Government Optimization

In this section, we formulate and solve the government's problem of choosing optimal taxes. We focus on an optimal steady state tax policy. In particular, we assume that the government chooses steady state tax rates $\left\{\tau_K^*, \tau_C^*, \tau_W^*, \tau_N^*\right\}$, and an aggregate steady state $\mathscr{S}$ of the economy consistent with these tax rates, in order to maximize the present discounted utility of a newborn worker in the steady state.[19]

[19]Provided that newborn entrepreneurs are better off than newborn workers (which will be the case provided the number of entrepreneurs is small relative to K^*), this amounts to maximizing the steady state value of a Rawlsian social welfare function. Equally, provided the number of entrepreneurs is small

To solve the government's optimization problem, we first remark that the assumptions made on the final goods production function in Section 2 ensure that it is inefficient for the aggregate economy if entrepreneurs hide capital or intermediate goods. Furthermore, Lemma 2 and Lemma 3 ensure that, in any steady state, entrepreneurs weakly prefer not to hide capital or intermediate goods. Then, the government will always prefer to select a steady state in which entrepreneurs do not hide capital or intermediate goods.

The first order conditions for the government's optimal choice of taxes are given by the following Lemma.

Lemma 4. *If there exist tax rates $\tau_W^*, \tau_K^*, \tau_N^* \in (-\infty, 1), \tau_C^* \in (-1, \infty)$ which solve the government's optimization problem, then the taxes τ_K^*, τ_W^*, τ_N^*, τ_C^* must satisfy the following first order conditions:*

$$\left(r_E^* - r_F^*\right) K_E^* e_{\tau_K}^{K_E} - C_E^* e_{\tau_K}^{C_E} = -\mu_1 e_{\tau_K}^{R_F} \tag{78}$$

$$\left(r_E^* - r_F^*\right) K_E^* e_{\tau_C}^{K_E} - C_E^* e_{\tau_C}^{C_E} = -\mu_1 e_{\tau_C}^{R_F} - \mu_2 \tag{79}$$

$$r_E^* K_E^* + r_F^* (K^* - K_E^*) - \delta K^* - C_E^* = 0, \tag{80}$$

where μ_1 and μ_2 are Lagrange multipliers on the constraints $\frac{1}{1+\tau_C^*} \geq \phi$ and $\tilde{R}^F \geq \underline{r}$, respectively.

Proof. See Appendix B.6 □

Intuitively, these three first order conditions should be viewed as first order conditions for τ_K^*, τ_C^*, and τ_W^*, respectively. The left hand sides of the first two first order conditions reflect the fact that the change in worker lifetime utility from a tax increase is equal to the increase in total output after the tax increase minus the increase in entrepreneurs' consumption after the increase in taxes. The first term on the left hand sides of the first two first order conditions represents the change in aggregate output from an increase in capital income and consumption taxes, respectively, as discussed in the previous section. The second left hand side terms for each of these represents the change in entrepreneurs' consumption after an increase in these taxes. The right hand sides of the first two first order conditions reflect the fact that the constraints $\frac{1}{1+\tau_C^*} \geq \phi$ and $\tilde{R}^F \geq \underline{r}$ may bind.

It may seem surprising that these first two first order conditions seem to ignore the effect of a tax change on K^*. This is because, the third first order condition (80) is that the wealth tax τ_W^* is set to ensure that K^* is at the level consistent with maximizing

relative to K^* and relative to the number of workers, this is also a close approximation to maximizing a utilitarian social welfare function, since entrepreneurs' consumption will then be relatively high compared to the consumption of workers, and so the marginal utility of consumption of entrepreneurs multiplied by the mass of entrepreneurs will be relatively low compared to the marginal utility of workers multiplied by their mass.

workers' steady state consumption.[20] Then, a small change in K^* has no marginal effect on workers' steady state consumption. The first three left hand side terms of (80) signify the marginal increase in aggregate national income, net of depreciation, associated with an increase in K^*. The last left hand side term denotes the marginal increase in entrepreneur's steady state consumption associated with an increase in K^*. Then, the left hand side is the marginal increase in workers' consumption associated with an increase in K^*. It is natural that this should be zero at an optimum.

Combining the first order conditions with the characterizations of elasticities in Section 3.3 and the characterization of the steady state from Proposition 2, it can be shown that the optimal taxes must satisfy four properties, which are laid out in Proposition 3.

Proposition 3. *If there exist tax rates $\tau_W^*, \tau_K^*, \tau_N^* \in (-\infty, 1), \tau_C^* \in (-1, \infty)$ which solve the government's optimization problem, then the taxes τ_K^*, τ_W^*, τ_N^*, τ_C^* must satisfy the following four conditions:*

$$\begin{aligned}
\left(\tau_N + \frac{\tau_C (1-\tau_N)}{1+\tau_C}\right) w^* N + \tau_K (r_E K_E + r_F K_F - \delta K) + \tau_W K + \tau_C C_E^* &= G && (81)\\
(r_F^* - \delta)(1-\tau_K) - \tau_W &= \underline{r} && (82)\\
C_E^* \tau_C + (\bar{r} - \delta)\tau_K K^* + \tau_W K^* &= 0 && (83)\\
\max\{(\rho+\gamma)\phi; \bar{r} - r_F\} K^* &= C_E^*, && (84)
\end{aligned}$$

where

$$\begin{aligned}
C_E^* &= \frac{(\bar{r}-\delta)(1-\tau_K^*) K^* - \tau_W^* K^*}{1+\tau_C^*} = \frac{(\rho+\gamma) K^*}{1+\tau_C^*} && (85)\\
\bar{r} &= r_E^* \frac{K_E^*}{K^*} + r_F^* \left(1 - \frac{K_E^*}{K^*}\right). && (86)
\end{aligned}$$

Proof. The results follow from substituting the characterizations of elasticities in Section 3.3 into the first order conditions (78)-(80) and rearranging, using equations (64)-(66) from Proposition 2 and the definitions of post-tax prices in (50) -(52) and (60). □

Here C_E^* is the steady state consumption of entrepreneurs, and $\bar{r}$ is the average pre-tax return to all capital.[21] Equation (81) is simply the government's budget balance condition as in (4), where we take into account that entrepreneurs do not hide capital, and so do not evade taxes. Equation (82) is a important result. This states that entrepreneurs should be indifferent between hiding capital and not hiding capital. This implies that the government should tax wealth at the highest rate consistent with agents not evading taxes by concealing their wealth. The reason for this result is that, as shown in Section

[20]The reason that the wealth tax is set so that K^* is at the optimal steady state level is because K^* is infinitely elastic with respect to the wealth tax.

[21]Equation (85) comes directly from equations (64)-(66), rather than relying per se on taxes being set at an optimal level.

3.3, an increase in capital income taxes reduces aggregate output when the wealth tax is adjusted to keep the aggregate capital stock K^* constant. Then, it is optimal for the government to cut capital income taxes and increase wealth taxes (while holding K^* constant) until it can no longer do this without entrepreneurs choosing to hide their capital. That is, taxes on wealth are set at the highest possible level consistent with agents not evading these high wealth taxes.

Equation (83) states that the total sum of all taxes paid by entrepreneurs is zero at the optimal steady state. This comes directly from equation (80), in combination with Proposition 2 and is essentially a result in the spirit of Chamley and Judd. Intuitively, aggregate K^* is perfectly elastic with respect to average tax on all capital (whether this is aggregate capital income or wealth taxes), so taxes on entrepreneurs are very costly in terms of their effect on aggregate K^*. The government therefore sets taxes to achieve its ideal level of capital stock, which occurs when total taxes on entrepreneurs are set to zero.[22]

Finally, equation (84) states that the level of entrepreneurs' steady state consumption chosen by the government is equal to the term on the left hand side, which is roughly a measure of the severity of financial frictions. Financial frictions are more severe if ϕ is higher– since this means entrepreneurs can hide capital more easily. When the constraint $\frac{1}{1+\tau_C^*} \geq \phi$ binds (recall that higher consumption taxes than this would persuade entrepreneurs to hide capital), then the optimal set of taxes entrails $C_E^* = \phi(\rho + \gamma)$, with entrepreneur's consumption depending on ϕ because $\frac{1}{1+\tau_C^*} = \phi$. If this constraint does not bind, entrepreneurs' consumption is determined by the capital market wedge $\bar{r} - r_F^*$, which is the gap between the average rate of return and the risk-free return. If capital was efficiently allocated, we would have that $\bar{r} = r_F^*$. However, due to the non-diversifiable risk associated with risky capital investment, we have instead in general in a steady state that $r_E^* > \bar{r} > r_F^*$.[23] The larger is $\bar{r} - r_F^*$, the more inefficient is the allocation of capital, and so the more the government would prefer to cut taxes on entrepreneurs' consumption to induce a more efficient capital allocation (since $e_{\tau_C}^{K_E} < 0$), thereby increasing entrepreneurs' consumption.

Rearranging the equations in Proposition 3, we can solve for the four optimal taxes $\tau_W^*, \tau_K^*, \tau_N^*, \tau_C^*$ in terms of pre-tax prices and parameters. The results are shown in the following corollary.

Corollary 1. *If there exists tax rates $\tau_W^*, \tau_K^*, \tau_N^* \in (-\infty, 1), \tau_C^* \in (-1, \infty)$ which solve the*

[22]The reason it is optimal to set total taxes on entrepreneurs equal to zero is that government chooses to cut taxes on capital income and increase K until the marginal tax revenue per additional unit of K is equal to zero, which will only be true if total taxes on entrepreneurs equal zero.

[23]This follows directly from Proposition 2.

government's optimization problem, then the taxes τ_K^*, τ_W^*, τ_N^*, τ_C^* must satisfy:

$$\tau_C^* = \frac{\rho+\gamma}{\max\{(\rho+\gamma)\,\phi;\bar{r}-r_F^*\}} - 1 \tag{87}$$

$$\tau_K^* = 1 - \frac{\rho+\gamma-\underline{r}}{\bar{r}-r_F^*} \tag{88}$$

$$\tau_W^* = (\rho+\gamma)\left(\frac{\max\{(\rho+\gamma)\,\phi;\bar{r}-r_F^*\}}{\bar{r}-r_F^*} - 1\right) - \underline{r}\left(\frac{\max\{(\rho+\gamma)\,\phi;\bar{r}-r_F^*\}}{\bar{r}-r_F^*}\right) \tag{89}$$

$$\tau_N^* = \frac{\frac{G}{wN} - \tau_C^*}{1-\tau_C^*} \tag{90}$$

where $\bar{r}$ is as defined in Proposition 3.

Using that $\underline{r} \leq 0$, these formulae immediately imply that $\tau_W^* \geq 0$, which is unsurprising since wealth taxes are set to the highest level consistent with no tax evasion. Furthermore, provided that $\bar{r} - r_F^* < \rho + \gamma$ (that is, the allocation of capital is not too inefficient), the formulae imply that $\tau_C^* > 0$ and $\tau_K^* < 0$. Intuitively, this is because consumption taxes reduce entrepreneurs' consumption for a given value of K^* (by equation (85)) but capital income taxes do not, so consumption taxes are generally preferable to capital income taxes. However, if the allocation of capital is very inefficient (i.e. if $\bar{r} - r_F^* > \rho + \gamma$) then $\tau_C^* < 0 < \tau_K^*$ is possible. This is because the elasticity of K_E^* with respect to consumption taxes is large ($|e_{\tau_C}^{K_E}| > |e_{\tau_K}^{K_E}|$ according to the results of Section 3.3) and so if the capital allocation is highly inefficient, the government does not desire to make it even more inefficient through consumption taxes.

To provide a sense of what level of taxes these formulae might imply in practice, we consider a numerical calibration in the next section.

4.1 A Numerical Calibration

To interpret the magnitudes of these optimal taxes in practice, we undertake a numerical calibration. To that end, we construct a benchmark economy that has the same primitives as the economy outlined in Section 2 and in which we set taxes at their current levels in the United States. We calibrate the benchmark economy at annual frequency and summarize parameter values in Table 1.

Demographics We set the mortality rate γ to 1%, corresponding to a life expectancy of 100 years. We choose the discount rate ρ so that in the steady state the average return to capital, weighted by capital shares and net of depreciation, is 4% (McGrattan and Prescott, 2001).

Technology We set the depreciation rate δ to 7%, approximately the average depreciation rate in the US fixed asset tables, and the autocorrelation of the productivity shock $1-\lambda_\theta$ to 0.885, as in Cooper and Haltiwanger (2006). Following Panousi (2012), we calibrate ϕ to 0.15, to match the variance of entrepreneurial capital. We choose the lowest possible realization of the idiosyncratic productivity shock, $\underline{\epsilon}$, to match a debt-to-asset ratio for entrepreneurs of 0.35 (Mehrotra and Crouzet, 2017, Boar and Midrigan, 2019). Lastly, we assume that the final output production technology is Cobb-Douglas $Y = AY_E^{\alpha_E} Y_F^{\alpha_F} N^{1-\alpha_E-\alpha_F}$ and jointly calibrate α_E, α_F and π, the probability of a high type (i.e. $\theta = 1$), to match a labor share of 2/3, the return to risky capital and the risk-free interest rate. We calibrate to an average return to capital in risky projects gross of depreciation of 15% so that the return net of depreciation is 8%, approximately the annual rate of return to equities in the US over the twentieth century (Mehra and Prescott, 2003). We calibrate to a risk-free return gross of depreciation of 8%, consistent with a risk-free interest rate of 1% which is close to the average return of relatively riskless securities in the US over the twentieth century (Mehra and Prescott, 2003).

Financial friction We calibrate the parameter ϕ, indicating the severity of the agency friction, to 0.76 so as to match the equity share of business owners in the US data. We use the Survey of Consumer Finances (National) Survey of Small Business Finances to document that entrepreneurs own, on average, 84% of their firm's equity.[24] We set $\underline{r} = 0$.

Tax system We set the consumption tax τ_C to 11%, following Altig et al. (2001) and Cagetti and Nardi (2009). We set τ_E to 20%, in line with the US corporate tax rate for small businesses reported in the OECD Tax Database, and τ_W to zero, in line with the current practice in the US .[25] We choose $\bar{G}$, so that the share of government spending is 20% of GDP, the historical average in the US over the past four decades.

Table 2 shows that the calibrated model produces a relatively similar wealth distribution to the data at the top end, despite this not being targeted in the calibration. Specifically, households in the top 1% of the wealth distribution hold 43% of the wealth in the model and between 31 and 42% in the data. At the same time, households in the top 0.01% of the wealth distribution hold 20% of the wealth in the model and between 15 and 22% in the data. It is reassuring that the model matches the extreme wealth concentration at the top, given the focus on the taxation of wealth and capital income.

Figures 1-3 depict the effect of the wealth tax τ_W, the capital income tax τ_K and the consumption tax τ_C on steady-state TFP, capital stock K, relative investment in the risky sector $\frac{K_E}{K}$ and post-tax wage $\tilde{w}$, our measure of welfare. For each of these figures we vary one of these three tax rates, set the other two tax rates at current US levels, and vary the labour income tax τ_N to balance the government's budget in the steady state.

Figure 1 shows that increases in the wealth tax lead to a decrease in aggregate capital but an increase in aggregate TFP. The decrease in aggregate capital is a natural consequence of the fact that the wealth tax reduces the post-tax average rate of return to capital and therefore reduces aggregate saving. This decrease in the aggregate capital

[24] See Appendix C for a detailed discussion of our treatment of the data.

[25] The calibrated value of τ_K is an average of the tax rate between 2000 and 2016 and includes both federal and state taxes.

Table 1: Parameter Values

Parameter	Value used	Target moment
γ	0.010	Lifespan 100 Years
ρ	0.022	Average net return to capital 4%
δ	0.070	Depreciation
λ_θ	0.115	Profitability autocor. (Cooper and Haltiwanger, 2006)
φ	0.150	Small Bus. Risk (Panousi, 2015)
$\underline{\epsilon}$	0.350	Debt-to-asset ratio (Boar and Midrigan, 2019)
α_E	0.193	Labor share 2/3
α_F	0.137	Risk-free rate
π	0.074	Return to Equity
τ_C	0.110	Consumption tax rate (Cagetti and Nardi, 2009)
τ_K	0.200	Corp. tax rate small businesses (OECD Tax Database)
τ_W	0	Current US level
$\bar{G}$	0.200	Govt. spending/GDP
ϕ	0.757	Small Bus. Owner Equity Share (SSBF)
$\underline{r}$	0	$<$ Risk Free Rate

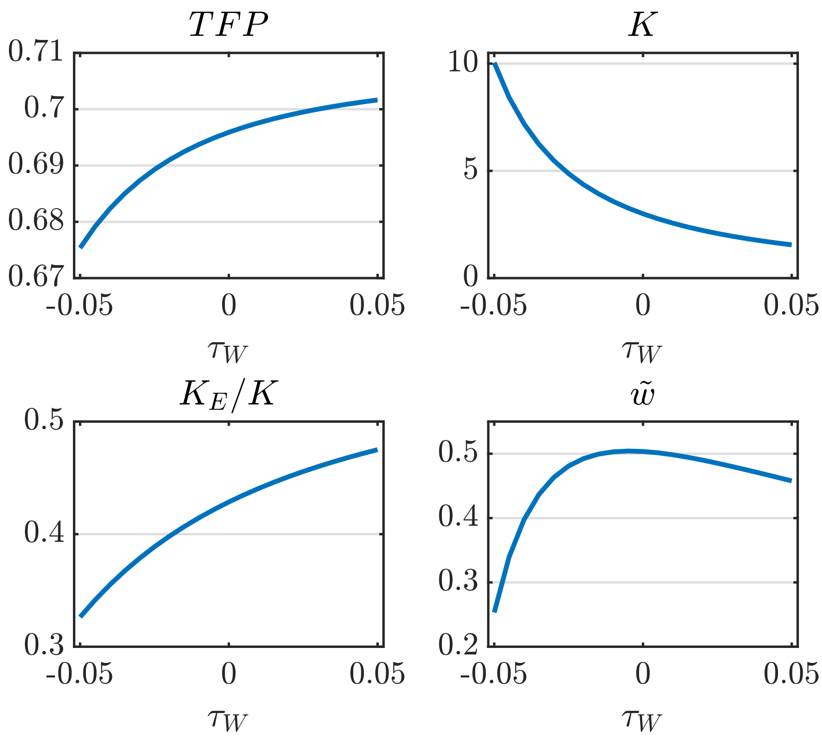
Table 2: Wealth Inequality

Wealth Share	Model	Data (Saez and Zucman, 2016)	Data (Smith, Zidar and Zwick, 2019)
Top 10%	98%	77%	71.4%
Top 1%	43%	42%	30.9%
Top 0.1%	20%	22%	15.1%

stock leads to a general equilibrium effect that raises the pre-tax rate of return to capital in the risky technology more than rate of return to capital in the risk-free technology all else equal. This is due to calibrating the production function as Cobb-Douglas. As a consequence, capital is diverted from the low-return risk-free technology to the high-return risky technology and so the increase in wealth taxes raises aggregate TFP. Nevertheless, the growth in steady state welfare due to higher aggregate TFP is slightly smaller than the decrease in welfare due to a lower aggregate capital stock. As such, we find that the steady state welfare of workers $\tilde{w}$ would be higher in this case if the wealth tax were set to a slightly negative level (given the levels of τ_K and τ_C).

Figure 2 shows the effect of varying τ_K. As expected, an increase in τ_K reduces the

Figure 1: The effect of the wealth tax on investment, efficiency and welfare



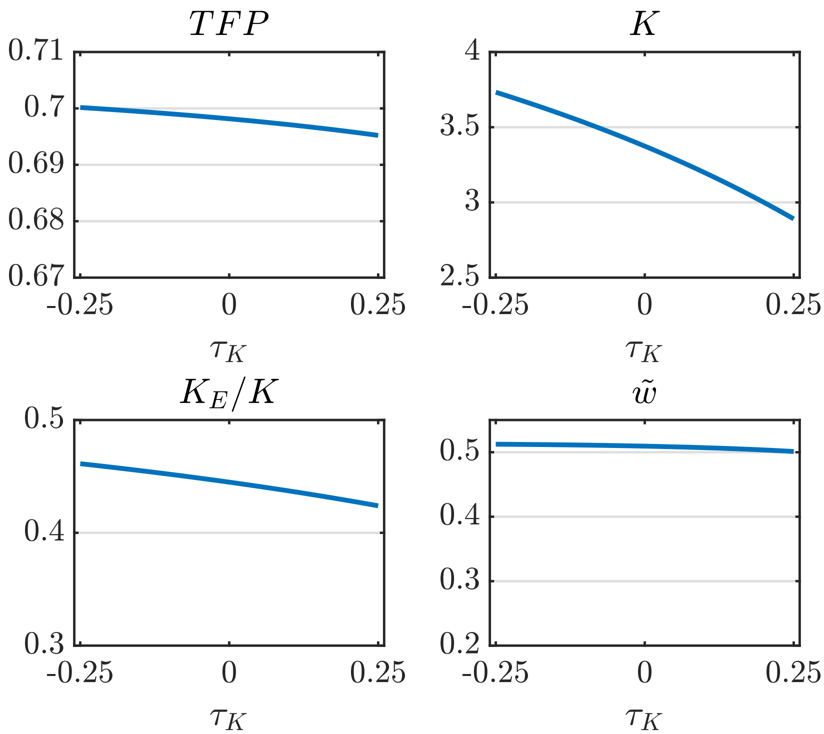
Notes: τ_K and τ_C are fixed at their current values.

steady state capital stock and also reduces aggregate TFP, by diverting capital from the high-return risky technology to the low-return risk-free technology. As a consequence, we find that the steady state welfare of workers $\tilde{w}$ is decreasing in τ_K.

Figure 3 shows the effect of varying τ_C. An increase in τ_C reduces aggregate TFP by tightening financial frictions, thereby diverting capital from the high-return risky technology to the low return risk-free technology. Since this decrease in aggregate TFP reduces the average return to capital, we find that the steady state capital stock slightly decreases. However, the resulting decrease in aggregate output is small relative to the benefits of the consumption tax: it raises government revenue which finances decreases in the labor income tax τ_N, thereby increasing worker welfare $\tilde{w}$. As such, we find that worker welfare $\tilde{w}$ is increasing in τ_C in this case.

Using the optimal tax formulae found in Proposition 1, the calibration implies an optimal value of τ_C^* of 28.4%, and optimal value of τ_K^* of -28.4% and an optimal value of τ_W^* of 0. Government budget balance then implies that the steady state optimal labor income tax is -3.4%.

Figure 2: The effect of the capital income tax on investment, efficiency and welfare



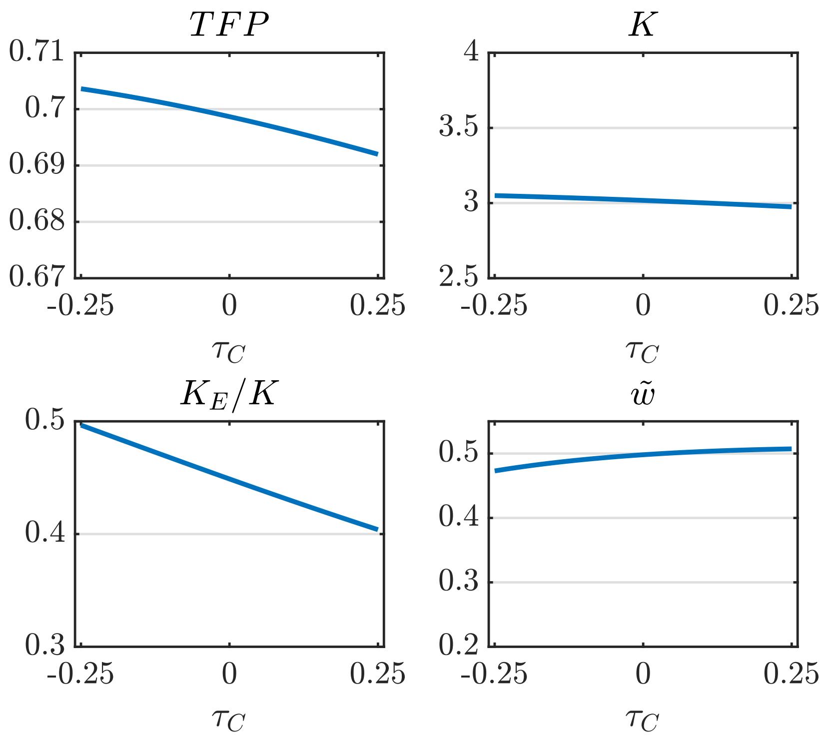
Notes: τ_W and τ_C are fixed at their current values.

5 Conclusion

We examine the implications of entrepreneurial financial frictions for optimal linear capital taxation, in a setting where all the wealth in the economy is owned by entrepreneurs and the government wants to maximize the welfare of workers, who live hand-to mouth, as in Judd (1985). Allowing for financial frictions implies that capital taxation can affect the efficiency of the allocation of capital – a force missing from models without financial frictions. In our framework, entrepreneurs can invest capital in projects which yield risky return or in risk-free projects. The government seeks to maximize the steady state welfare of workers using a mixture of (possibly negative) taxes on wealth, consumption, capital income and on labor, and has to finance exogenous government expenditure while balancing its budget.

Our model is relatively tractable analytically and we characterize optimal steady state taxes as functions of prices and parameters. We find that the government optimally sets zero total taxes on entrepreneurs in the steady state, because the steady state stock of capital is infinitely elastic with respect to the post-tax rate of return, as it is in the models of Chamley (1986) and Judd (1985). At the same time, we find that the government chooses to set wealth taxes at the highest possible level consistent with no tax evasion.

Figure 3: The effect of the consumption tax on investment, efficiency and welfare



Notes: τ_W and τ_K are fixed at their current values.

The reason for this is that capital income taxes entail greater welfare costs than wealth taxes, because capital income taxes reduce the fraction of capital used in risky projects by the most efficient entrepreneurs, by particularly taxing these individuals more heavily. As such, the government prefers to raise wealth taxes and cut capital income taxes as long as there is no tax evasion. We show that, provided financial frictions are not too severe, the optimal tax on capital income is strictly negative, the optimal tax on consumption is strictly positive and the optimal tax on wealth is weakly positive. When we calibrate our model to match data on entrepreneurial finance from the Survey of Consumer Finances, we find that the optimal tax on capital income is -28%, the optimal tax on wealth is 0%, the optimal tax on consumption is 28% and the optimal labor income tax is -3%.

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Appendices

A Discrete Time Model

A.1 Proof of Lemma 1

Note that the technology associated with an entrepreneur's risky and risk-free projects displays constant returns to scale. Consider an entrepreneur Alice, who at some period t has $a > 0$ times as much capital as an entrepreneur Bob. The constant returns to scale properties imply that Alice can make the same decisions as Bob, for each ϵ and θ, except do everything a times as much: she can consume a times as much each period, devote a times as much capital to each project, hide a times as many units of capital etc. If Alice does this she can consume a times as much as Bob each period, for the same draws of θ and ϵ. Since, for any c, $\log(ac) \equiv \log(a) + \log(c)$, Alice's present discounted utility from these choices would then be the same as Bob's plus an additional $\sum_{j=0}^{\infty}(1-\rho)^j(1-\gamma)^j \log(a) = \frac{\log(a)}{1-(1-\rho)(1-\gamma)}$. Since these choices are possible for Alice, it must be that $V(ak, \theta, X) \geq \frac{\log(a)}{1-\beta(1-\gamma)} + V(k, \theta, X)$. However, on the other hand, Bob can choose to do everything that Alice does only $\frac{1}{a}$ times as much. By the same logic as before, doing so would yield Bob a present discounted utility equal to Alice's minus $\frac{\log(a)}{1-(1-\rho)(1-\gamma)}$. Therefore, it must be the case that $V(k, \theta, X) \geq V(ak, \theta, X) - \frac{\log(a)}{1-(1-\rho)(1-\gamma)}$. Comparing these two inequalities that V must fulfil, it is immediate that it cannot satisfy both unless $V(ak, \theta, X) = \frac{\log(a)}{1-(1-\rho)(1-\gamma)} + V(k, \theta, X)$. In that case, it must be that $V(k, \theta, X) \equiv \frac{\log(k)}{1-(1-\rho)(1-\gamma)} + V(1, \theta, X)$. Let $\overline{V}(\theta, X)$ denote $E[V(1, \theta, X)|\theta, X]$. Then it follows that $E\left[V(k, \theta, X)\middle| k, \theta, X\right] = \overline{V}(\theta, X) + \frac{\log(k)}{1-(1-\rho)(1-\gamma)}$. □

A.2 Proof of Lemma 2

During the period t the entrepreneur can do four different activities each of which yields a riskless return to her capital. First, she can hide capital in the beginning of the period. Equation (36) implies that hiding one unit of capital in the beginning of the period increases the entrepreneur's ω at the end of the period by $\frac{1+r}{1+\tau_C}$. Second, she can sell capital and lend to a bank, setting $b < 0$. The bank's break even condition (35) implies that the bank would be willing to borrow from her and pay back the risk-free gross interest rate of R_F at the end of the period, for each unit lent to the bank. Then, equation (36) implies that to do this would increase the entrepreneur's ω at the end of the period by $\frac{R_F}{1+\tau_C}$. Third, the entrepreneur can allocate capital to her risk-free project and sell the output of this to the final goods firms. Examining the equation (36) reveals that for each additional unit the entrepreneur can allocate to k_F she is able to increase her ω by $\frac{(1-\tau_K)r_F+1-\delta(1-\tau_K)-\tau_W}{1+\tau_C}$. Fourth, the entrepreneur can hide her capital. Each additional unit of hidden capital allows her to increase her ω by ϕ.

Since these four activities are risk-free, that is they increase ω by the same amount regardless of the realization of ϵ, they do not affect $\frac{\partial \omega(k,\theta,\epsilon,X)}{\partial \epsilon}$. Consequently, which of these activites the entrepreneur participates in will have no bearing on the incentive compatibility constraint (42), for a given choice of k_E by the entrpreneur. Therefore, agency frictions will not prevent the entrepreneur from allocating her capital to whichever of these three activities offers the highest return.

Since the entrepreneur optimally allocates her funds between these three activities, it must be the case in equilibrium that lending to the bank offers at least as high a return as the other three activities:

$$R_F \geq \max\left\{1+\underline{r}; (1-\tau_K)r_F + 1 - \delta(1-\tau_K) - \tau_W; \phi(1+\tau_C)\right\}. \tag{A.1}$$

To verify this, suppose otherwise. Then, the entrepreneur would be able to borrow without limit at the start of the period, buy capital which she allocates to one of the other three activities that offers higher return, repay her debts at the end of the period, and obtain an arbitrarily high ω at the end of the period. This cannot be a equilibrium of the economy.

Equally, in equilibrium, it must be the case that the entrepreneur's risk-free project offers at least as high a return as lending to a bank and as hiding capital in the beginning of the period. Suppose otherwise, then no entrepreneurs would allocate capital to their risk-free projects, instead they would sell and lend to banks or hide capital. The consequence of this would be that the economy's output of standard intermediate goods would fall to zero, driving the price of standard intermediate goods to infinity, because of the Inada conditions on the final goods production function. This also cannot be a competitive equilibrium. As such, we may conclude that, in equilibrium:

$$R_F \quad = \quad 1+\underline{r} = (1-\tau_K)r_F + 1 - \delta(1-\tau_K) - \tau_W \geq \phi(1+\tau_C).$$

Equation (A.2) establishes that in equilibrium each entrepreneur is indifferent between hiding capital in the beginning of the period, allocating capital to her risk-free project (and selling the intermediate goods that result) and lending to a bank. If the inequality in (A.2) holds with equality then the entrepreneur obtains an equally high return by hiding capital after observing the realization of ϵ, in which case she is indifferent about the level of k_H that she chooses. On the other hand, if the inequality in is strict, the entrepreneur obtains a strictly lower return by hiding capital in the middle of the period so will not do it. □

A.3 Proof of Lemma 3

In an equilibrium of the economy, it must be the case every period that:

$$1 - \delta(1-\tau_K) - \tau_W + (1-\tau_K)\bar{\theta} r_E - \phi(1+\tau_C) - R_F < 0 \tag{A.2}$$

If this were not the case, then equations (42) and (46) imply that ω is strictly increasing in k_E for all $\epsilon > 0$ for entrepreneurs with θ close enough to $\bar{\theta}$. Then, by choosing an arbitrarily high k_E, such entrepreneurs will be able to achieve arbitrarily high consumption and therefore utility. Therefore, if this condition held, some entrepreneurs would desire to allocate an infinite amount of capital to their risky projects, which cannot be an equilibrium, since the capital stock is finite each period. □

B Continuous Time Model

B.1 Environment and Equilibrium with Period Length Δ

Let $\Delta \in (0,1]$ denote the length of a period. Over a period of length Δ, agents value future consumption at rate $(1-\rho\Delta)$, die with probability $\gamma\Delta$, capital depreciates at rate $\delta\Delta$, entrepreneurs draw a new productivity θ with probability $\lambda_\theta\Delta$ and shocks ϵ satisfy

$$\epsilon = \underline{\epsilon} + (1-\underline{\epsilon}) \exp\left(\varphi\sqrt{\Delta}\cdot\xi - \frac{\varphi^2\Delta}{2}\right), \tag{B.1}$$

where $\xi \sim N(0,1)$ and φ is a parameter determining the variance of ϵ. This assumption implies that $E[\epsilon] = 1$ and $\mathrm{Var}(\log(\epsilon - \underline{\epsilon})) = \varphi^2\Delta$. Risky projects produce $\theta\epsilon k_E\Delta$ and risk-free projects produce $k_F\Delta$. A worker's preferences are described by the lifetime utility function $\sum_{j=0}^{\infty}(1-\rho\Delta)^j(1-\gamma\Delta)^j u\left(C_{N,t+j\Delta}\right)$. An entrepreneur values her future consumption stream according to $\sum_{j=0}^{\infty}(1-\rho\Delta)^j(1-\gamma\Delta)^j U\left(C_{i,t+j\Delta}\right)$. The government sets taxes τ_C, τ_N, τ_K and $\tau_W\Delta$ per period, and has to finance exogenous expenditure $\overline{G}\Delta$.

As in the main text, where $\Delta = 1$, the entrepreneur's problem is to solve:

$$\begin{aligned} V(k,\theta,X) &= \sup \int_{\epsilon>0} \Bigg(\log(c(k,\theta,\epsilon,X)) + (1-\rho\Delta)(1-\gamma\Delta) \\ &\quad \times\ E\left[V(k'(k,\theta,\epsilon,X),\theta',X')\middle|\theta\right]\Bigg) \mathrm{d}H(\epsilon), \end{aligned} \tag{B.2}$$

subject to the budget constraints

$$
\begin{aligned}
c\Delta + \frac{k_{+\Delta}}{1+\tau_C} &= \omega \\
(1+\tau_C)\,\omega &= (1+\tau_C)\,c_H\Delta - \hat{b} + (1-\tau_K)\,(r_E Y_E \Delta + r_F y_F \Delta) \\
&+ (1-\delta\Delta\,(1-\tau_K) - \tau_W\Delta)\,(\epsilon k_E + k_F) - (1-\delta\Delta)\,k_H + (1+\underline{r}\Delta)\,k_{TE},
\end{aligned}
$$

the production functions $c_H\Delta = \phi k_H$, $y_E = \theta\epsilon k_E\Delta$, $y_F = k_F\Delta$, the incentive compatibility constraint

$$\omega(k,\theta,\epsilon,X) \geq \omega(k,\theta,\hat{\epsilon},X) + \phi k_E(k,\theta,X)(\epsilon - \hat{\epsilon}),$$

the break-even condition for the banks

$$\int_{\epsilon} \hat{b}\,(k,\theta,\epsilon,X_t)\,\mathrm{d}H\,(\epsilon) \geq R_F\Delta b\,(k,\theta,X_t)\,,$$

and non-negativity constraints on k_E, k_F, k_{TE}, k_H, c, c_H, y_E, y_F, ω and $k_{+\Delta}$.

The equilibrium conditions of the model with period length Δ are summarized below.

Definition 3. *For a given sequence of tax rates $\{\tau_{W,t}, \tau_{K,t}, \tau_{C,t}, \tau_{N,t}\}_{t=0}^{\infty}$, an **equilibrium** $\mathcal{E}_\Delta$ of the economy with period length Δ is a sequence of prices $\{r_{E,t}, r_{F,t}, w_t\}_{t=0}^{\infty}$, policy functions giving entrepreneurs' decisions, a sequence of worker consumption $\{c_{N,t}\}_{t=0}^{\infty}$, and a sequence of aggregate variables $\{C_t, C_{H,t}, K_t, K_{E,t}, K_{F,t}, K_{TE,t}, K_{H,t}, Y_t, Y_{E,t}^S, Y_{F,t}^S\}_{t=0}^{\infty}$ such that:*

1. *The government's budget constraint below is balanced every period:*

$$\overline{G} = \tau_{N,t} w_t N + \tau_{K,t}(r_{E,t} Y_{E,t}^S + r_{F,t} Y_{F,t}^S - \delta K_t) + \tau_{W,t}(K_t - K_{H,t}) + \tau_{C,t}(C_t - C_{H,t}) \tag{B.3}$$

2. *Worker consumption satisfies $c_{N,t}\,(1+\tau_{C,t}) = w_t\,(1-\tau_{N,t})$.*

3. *Entrepreneurs' decision rules are given by the solution to the entrepreneur's problem (B.2).*

4. *$\{C_t, C_{H,t}, K_t, K_{E,t}, K_{F,t}, K_{TE,t}, K_{H,t}\}_{t=0}^{\infty}$ represent the aggregate of agents' decisions given by equations (17)-(23).*

5. *The asset market clears, according to equation (24).*

6. *The markets for intermediate goods clear, according to equations (25) and (26).*

7. *Prices of intermediate goods $r_{E,t}$ and $r_{F,t}$, and wages w_t are determined by the first order conditions of the final goods firms (27)-(29).*

The final goods market clearing condition then follows by Walras' law:

$$C_t\Delta + K_{t+\Delta} + \overline{G}\Delta = F\left(Y^S_{E,t}, Y^F_{E,t}, N\right)\Delta + (1-\delta\Delta)\left(K_{E,t} + K_{F,t} - K_{H,t}\right) + C_{H,t}\Delta \tag{B.4}$$

B.2 Solution to Entrepreneur's Problem with Period Length Δ

Following the same derivation as in the main text, where $\Delta = 1$, it can readily be shown that the solution to the entrepreneur's between period problem is given by

$$c\Delta = (1-(1-\rho\Delta)(1-\gamma\Delta)) = (\gamma+\rho+\gamma\rho\Delta)\,\Delta\omega \tag{B.5}$$

$$k' = (1+\tau_C)(1-\rho\Delta)(1-\gamma\Delta)\omega = (1+\tau_C)\,(\omega - c\Delta)\,. \tag{B.6}$$

and that entrepreneur's within period problem is to choose functions $k_E\,(k,\theta,X)$ and $\omega\,(k,\theta,X)$ to solve:

$$\sup \int_\epsilon \log(\underline{\omega} + \phi\epsilon k_E)\mathrm{d}H(\epsilon), \tag{B.7}$$

subject to the constraints:

$$\underline{\omega} = \left(-\phi + \tilde{r}_E\theta\Delta - \tilde{r}_F\Delta\right)k_E + \left(\frac{1}{1+\tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta\right)k \tag{B.8}$$

$$k_E \geq 0 \tag{B.9}$$

$$\underline{\omega} + \phi\underline{\epsilon}k_E \geq 0. \tag{B.10}$$

The following proposition summarizes the entrepreneur's optimal decisions.

Proposition 4. *In equilibrium, the entrepreneur's problem has a unique solution for $c(k,\theta,\epsilon,X)$, $k'(k,\theta,\epsilon,X)$, $\omega(k,\theta,\epsilon,X)$ and $k_E(k,\theta,X)$ which depends continuously on the parameters. The entrepreneur's optimal choice of k_E is:*

$$k_E = \frac{S^{-1}\left(\max\left\{0;\min\left\{\frac{\theta\tilde{r}_E\Delta - \tilde{r}_F\Delta}{\phi};S^\star\right\}\right\}\right)k\left(\tilde{r}_F\Delta + \frac{1}{1+\tau_C} - \tilde{p}\Delta\right)}{\phi - \left(\theta\tilde{r}_E\Delta - \tilde{r}_F\Delta\right)S^{-1}\left(\max\left\{0;\min\left\{\frac{\theta\tilde{r}_E\Delta - \tilde{r}_F\Delta}{\phi};S^\star\right\}\right\}\right)}, \tag{B.11}$$

where $S^\star = S\left(\frac{1}{1-\underline{\epsilon}}\right)$. For any equilibrium values of $\tilde{r}_E, \tilde{r}_F$, the entrepreneur's choices entail

$$\omega = \left(\phi\,(\epsilon-1) + \tilde{r}_E\Delta\theta - \tilde{r}_F\Delta\right)k_E + \left(\frac{1}{1+\tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta\right)k \tag{B.12}$$

$$c = (\gamma+\rho+\gamma\rho\Delta)\omega \tag{B.13}$$

$$k' = (1+\tau_C)(\omega - c\Delta), \tag{B.14}$$

where

$$S(x) = 1 - \frac{\int_{\epsilon} \left(1 + x(\epsilon - 1)\right)^{-1} \epsilon h(\epsilon)\, d\epsilon}{\int_{\epsilon} \left(1 + x(\epsilon - 1)\right)^{-1} h(\epsilon)\, d\epsilon}, \tag{B.15}$$

and

$$S^{\star} = S\left(\frac{1}{1 - \underline{\epsilon}}\right). \tag{B.16}$$

Here $S : \left[0, \frac{1}{1-\epsilon}\right] \to [0, S^{\star}]$ *is a differentiable and strictly increasing (and therefore invertible) function.*

Proof. Note that

$$\int_{\epsilon} (\underline{\omega} + \phi\epsilon k_E)\, \mathrm{d}H(\epsilon) = \underline{\omega} + \phi k_E = (\tilde{r}_E\theta\Delta - \tilde{r}_F\Delta)\, k_E + \left(\frac{1}{1 + \tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta\right) k.$$

If $\tilde{r}_E\theta\Delta \leq \tilde{r}_F\Delta$, then it follows immediately that $\int_{\epsilon} (\underline{\omega} + \phi\epsilon k_E)\, \mathrm{d}H(\epsilon)$ is decreasing in k_E. By Jensen's inequality, it follows that $\int_{\epsilon} \log(\underline{\omega} + \phi\epsilon k_E)\mathrm{d}H(\epsilon)$ is strictly decreasing in k_E and so the entrepreneur will optimally choose $k_E = 0$.

In the case $\tilde{r}_E\theta\Delta > \tilde{r}_F\Delta$ we take the first order condition. Given the strict concavity of the objective function, the first order condition is a sufficient condition for a unique optimum. We guess that the constraints $k_E \geq 0$ and $\underline{\omega} + \phi\underline{\epsilon}k_E$ will not bind, and will verify that this guess is correct by showing that the first order condition has a solution where they do not bind. In this case, the first order condition is

$$\int_{\epsilon} \frac{\phi\,(\epsilon - 1) + \tilde{r}_E\theta\Delta - \tilde{r}_F\Delta}{(\phi\,(\epsilon - 1) + \tilde{r}_E\theta\Delta - \tilde{r}_F\Delta)\, k_E + \left(\frac{1}{1+\tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta\right) k} \mathrm{d}H(\epsilon) = 0. \tag{B.17}$$

Now, note that, in any equilibrium in which the entrepreneur optimizes, it must be the case that

$$\left(\frac{1}{1 + \tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta\right) k + (\tilde{r}_E\theta\Delta - \tilde{r}_F\Delta)\, k_E > 0. \tag{B.18}$$

This follows because the entrepreneur can convert units of capital into consumption at rate ϕ and the risk free rate of return is $\frac{1}{1+\tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta$, so there can only be an equilibrium in which some entrepreneurs put a positive amount of capital in the risk-free sector if

$$\frac{1}{1 + \tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta \geq \phi > 0,$$

guaranteeing that the first term in (B.18) is strictly positive. Furthermore, if $\tilde{r}_E\theta\Delta \leq \tilde{r}_F\Delta$ then the entrepreneur's objective function is strictly decreasing in k_E, so the entrepreneur will choose $k_E = 0$ in that case. Therefore, it must always be the case that, at the optimal solution,

$$(\tilde{r}_E\theta\Delta - \tilde{r}_F\Delta)k_E \geq 0,$$

guaranteeing that the first term in (B.18) is positive.

Since we have established (B.18), we may multiply the entrepreneur's first order condition by $\left(\frac{1}{1+\tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta\right)k + (\tilde{r}_E\theta\Delta - \tilde{r}_F\Delta)\,k_E$ and write it as:

$$\int_\epsilon \left(\frac{\left(\frac{1}{1+\tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta\right)k + (\tilde{r}_E\theta\Delta - \tilde{r}_F\Delta)\,k_E + \phi(\epsilon-1)k_E}{\left(\frac{1}{1+\tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta\right)k + (\tilde{r}_E\theta\Delta - \tilde{r}_F\Delta)\,k_E}\right)^{-1} (\phi\,(\epsilon-1) + \tilde{r}_E\theta\Delta - \tilde{r}_F\Delta)\,\mathrm{d}H(\epsilon) = 0.$$

This is the same as:

$$\int_\epsilon \Big(1 + x(\epsilon-1)\Big)^{-1} (\phi\,(\epsilon-1) + \tilde{r}_E\theta - \tilde{r}_F)\,\mathrm{d}H(\epsilon) = 0, \tag{B.19}$$

where

$$x = \frac{\phi k_E}{\left(\frac{1}{1+\tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta\right)k + (\tilde{r}_E\theta\Delta - \tilde{r}_F\Delta)\,k_E}. \tag{B.20}$$

We can rearrange (B.19) further, to get

$$\int_\epsilon \Big(1 + x(\epsilon-1)\Big)^{-1} \phi\epsilon \mathrm{d}H(\epsilon) = (\phi - \tilde{r}_E\theta\Delta + \tilde{r}_F\Delta)\int_\epsilon \Big(1 + x(\epsilon-1)\Big)^{-1} \mathrm{d}H(\epsilon). \tag{B.21}$$

Now, inspecting the formula for the entrepreneur's end of period resources ω, and comparing with (B.20), we see that the entrepreneur's end of period resources are proportional to $1 + x(\epsilon-1)$. Since the entrepreneur will not make decisions which give her zero (or negative) consumption with positive probability, it must be that $1 + x(\epsilon-1) > 0$ with probability 1. Therefore $\int_\epsilon \Big(1 + x(\epsilon-1)\Big)^{-1} \mathrm{d}H(\epsilon) > 0$ and so we can divide through by it. Likewise, $\phi > 0$, so we can rearrange the entrepreneur's first order condition as:

$$\frac{\int_\epsilon \Big(1 + x(\epsilon-1)\Big)^{-1} \epsilon\,\mathrm{d}H(\epsilon)}{\int_\epsilon \Big(1 + x(\epsilon-1)\Big)^{-1} \mathrm{d}H(\epsilon)} = \frac{\phi - \tilde{r}_E\theta\Delta + \tilde{r}_F\Delta}{\phi}. \tag{B.22}$$

We can write this as

$$S(x) = \frac{\tilde{r}_E\theta\Delta - \tilde{r}_F\Delta}{\phi}, \tag{B.23}$$

where

$$S(x)=1-\frac{\int_{\epsilon}\Big(1+x(\epsilon-1)\Big)^{-1}\epsilon\mathrm{d}H(\epsilon)}{\int_{\epsilon}\Big(1+x(\epsilon-1)\Big)^{-1}\mathrm{d}H(\epsilon)},\tag{B.24}$$

and is a differentiable and strictly increasing function (as discussed below) that satisfies $S(0)=1-\frac{E[\epsilon]}{1}=0$ and $S(1)=1-\frac{1}{E[\epsilon^{-1}]}<1$, since $E[\epsilon^{-1}]>1$ by Jensen's inequality.

We now show that $S(x)$ is differentiable and strictly increasing. To show that $S(x)$ is differentiable, recall that the entrepreneur's objective function is continuously differentiable over some compact set containing the optimum. Furthermore, the entrepreneur's choice set can easily be verified to be compact and depends on the rates of return and parameters in an upper hemi-continous way. Thus, the conditions required for the application of Berge's theorem of the maximum are satisfied. By the theorem of the maximum, $k_E(k,\theta)$ is differentiable with respect to θ,ϕ and the rates of return. Since we showed above that $k_E(k,\theta)$ is the solution to

$$S(x)=\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi},$$

where x is given by (B.20), it is immediately evident that if $S(x)$ is not differentiable at x, then k_E cannot be a differentiable function of the parameters, contradicting Berge's maximum theorem. Therefore $S(x)$ is differentiable. To show that $S(x)$ is strictly increasing, note that there exists a unique k_E satisfying B.20 and that the k_E solving this equation is a monotonically increasing function of x. Then it must follow that there is only one solution to the equation $S(x)=y$ and thus $S(x)$ is invertible. Since $S(x)$ is invertible as well as differentiable, it must be strictly monotone. It was shown above that $S(0)=0$ and $S(1)\in(0,1)$. Therefore, $S(x)$ must be strictly increasing.

Let $x^\star$ denote the x that makes the entrepreneur's consumption equal to zero for the lowest realization of ϵ. That is, $1+x^\star(\underline{\epsilon}-1)=0$, which rearranges to $x^\star=\frac{1}{1-\underline{\epsilon}}$. Additionally, let $S^\star$ denote $S(x^\star)$.

Above we showed above, the entrepreneur's first order condition is

$$S(x)=\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi},$$

where x is given by (B.20). Then, since S is invertible, we can write:

$$x=\frac{\phi k_E}{\left(\frac{1}{1+\tau_C}-\tilde{p}\Delta+\tilde{r}_F\Delta\right)k+\left(\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta\right)k_E}=S^{-1}\left(\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi}\right),$$

which rearranges to

$$k_E\left[\phi-(\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta)\,S^{-1}\left(\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi}\right)\right]=k\left(\frac{1}{1+\tau_C}-\tilde{p}\Delta+\tilde{r}_F\Delta\right)S^{-1}\left(\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi}\right)$$

We know that the FOC is only relevant when $\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi}>0$, since otherwise the entrepreneur chooses $k_E=0$. Furthermore, since $S(0)=0$ and S is strictly increasing, it follows that $S^{-1}(0)=0$ and S^{-1} is strictly increasing. Therefore, if the FOC is relevant then $S^{-1}\left(\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi}\right)>0$. Now, if

$$\phi\leq(\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta)\,S^{-1}\left(\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi}\right),$$

then the left hand side of the FOC is negative and the right hand side is positive, so the FOC has no solution, and the entrepreneur's optimization problem must be the corner solution where k_E is at the level that makes consumption exactly zero when $\epsilon=\underline{\epsilon}$ (this is the largest k_E the entrepreneur could possibly choose). On the other hand, if

$$\phi>(\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta)\,S^{-1}\left(\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi}\right),$$

then we can divide through to solve for k_E. That is,

$$k_E=\frac{k\left(\frac{1}{1+\tau_C}-\tilde{p}\Delta+\tilde{r}_F\Delta\right)S^{-1}\left(\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi}\right)}{\phi-(\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta)\,S^{-1}\left(\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi}\right)}$$

This gives k_E in the case where the first order condition holds. Note that, if θ is sufficiently high, the FOC will not hold and the entrepreneur will choose k_E such that the entrepreneur's consumption is zero when $\epsilon=\underline{\epsilon}$. After some rearrangement of the entrepreneur's budget constraints, this is

$$\begin{aligned}k_E &= \frac{k\left(\frac{1}{1+\tau_C}-\tilde{p}\Delta+\tilde{r}_F\Delta\right)\frac{1}{1-\underline{\epsilon}}}{\phi-(\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta)\frac{1}{1-\underline{\epsilon}}}=\frac{k\left(\frac{1}{1+\tau_C}-\tilde{p}\Delta+\tilde{r}_F\Delta\right)x^\star}{\phi-(\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta)\,x^\star}\\ &= \frac{k\left(\frac{1}{1+\tau_C}-\tilde{p}\Delta+\tilde{r}_F\Delta\right)S^{-1}(S^\star)}{\phi-(\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta)\,S^{-1}(S^\star)}.\end{aligned}$$

This is the highest possible value k_E can take, while respecting $c\geq 0$. We know also that if $\left(\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi}\right)\leq 0$, the entrepreneur chooses $k_E=0$, and in all other cases, the entrepreneur chooses k_E satisfying the rearranged first order condition above.

Then, since $S^{-1}(0)=0$ and $S^{-1}(\cdot)$ is strictly increasing, we can collect all three cases into one equation, and say that the entrepreneur's k_E *always* satisfies:

$$k_E=\frac{S^{-1}\left(\max\left\{0;\min\left\{\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi};S^{\star}\right\}\right\}\right)k\left(\frac{1}{1+\tau_C}-\tilde{p}\Delta+\tilde{r}_F\Delta\right)}{\phi-\left(\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta\right)S^{-1}\left(\max\left\{0;\min\left\{\frac{\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta}{\phi};S^{\star}\right\}\right\}\right)} \tag{B.25}$$

From equations (42) and (B.8), we have that

$$\omega=\left(\phi\left(\epsilon-1\right)+\tilde{r}_E\theta\Delta-\tilde{r}_F\Delta\right)k_E+\left(\frac{1}{1+\tau_C}-\tilde{p}\Delta+\tilde{r}_F\Delta\right)k.$$

Combining these results with equations (B.5) and (B.6) yields all the results of the proposition. □

B.3 Steady-state with Period Length Δ

In this section, we formally characterize a steady state of the model in which the length of the period is Δ.

Proposition 5. *There exists a steady state $\mathscr{S}$ which is consistent with the particular values of aggregate variables $\{K^*,K_E^*,C^*,Y_E^{S*},Y_F^{S*}\}$, post-tax prices $\{\tilde{r}_E^*,\tilde{r}_F^*,\tilde{w}^*,\tilde{p}^*\}$ and consumption tax rate τ_C^* and in which no entrepreneurs hide capital or intermediate goods, if and only if the following conditions hold:*

$$\frac{\tilde{r}_F^*}{\tilde{r}_E^*}=\frac{F'_{Y_F^{S*}}}{F'_{Y_E^{S*}}} \tag{B.26}$$

$$Y_F^{S*}=K^*-K_E^* \tag{B.27}$$

$$C^*=\tilde{w}^*N+K^*\left(\tilde{r}_F^*-\tilde{p}^*\right)+\tilde{r}_E^*Y_E^*-\tilde{r}_F^*K_E^* \tag{B.28}$$

$$C^*=\tilde{w}^*N+\left(\rho+\gamma-\rho\gamma\Delta\right)\left(\tilde{r}_E^*Y_E^{S*}\Delta+\tilde{r}_F^*Y_F^{S*}\Delta+\frac{K^*}{1+\tau_C}-\tilde{p}^*K^*\Delta\right) \tag{B.29}$$

$$C^*+\delta K^*+\overline{G}=F(Y_E^{S*},Y_F^{S*},N) \tag{B.30}$$

$$Y_E^{S*}=K\textstyle\sum_\theta\theta\hat{k}_E(\theta)\mu_K^*(\theta) \tag{B.31}$$

$$K_E^*=K\textstyle\sum_\theta\hat{k}_E(\theta)\mu_K^*(\theta) \tag{B.32}$$

where

$$\mu_K^*(\theta) = \frac{\lambda_\theta \Delta g(\theta)}{1-(1-\lambda_\theta\Delta)(1-\Delta(\rho+\gamma-\rho\gamma\Delta))\left[1+(1+\tau_C)\Delta\left(\tilde{r}_F-\tilde{p}+\hat{k}_E(\theta)(\theta\tilde{r}_E-\tilde{r}_F)\right)\right]} \tag{B.33}$$

$$\hat{k}_E(\theta) = \frac{S^{-1}\left(\max\left\{0;\min\left\{\frac{\theta\tilde{r}_E\Delta-\tilde{r}_F\Delta}{\phi};S^\star\right\}\right\}\right)\left(\tilde{r}_F\Delta+\frac{1}{1+\tau_C}-\tilde{p}\Delta\right)}{\rho-(\theta\tilde{r}_E\Delta-\tilde{r}_F\Delta)S^{-1}\left(\max\left\{0;\min\left\{\frac{\theta\tilde{r}_E\Delta-\tilde{r}_F\Delta}{\phi};S^\star\right\}\right\}\right)} \tag{B.34}$$

and where $\phi - \tilde{r}_E^*\overline{\theta}\Delta + \tilde{r}_F^*\Delta > 0$, $\tilde{r}_E^*\overline{\theta} > \tilde{r}_F^*$, $\frac{1}{1+\tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta \geq \phi$, $(1+\tau_C)\left(\tilde{r}_F^* - \tilde{p}^*\right) \geq \underline{r}$ *and* $K_E^* < K^*$.

Proof. First we prove that these conditions are necessary for a steady state. We do this equation by equation.

For equation (B.26) note that $r_F^* = F'_{Y_F^{S*}}$ and $r_E^* = F'_{Y_E^{S*}}$, because of the first order conditions of the final goods firms. Now, by definition:

$$\begin{aligned} \tilde{r}_F^* &= \frac{(1-\tau_K^*)r_F^*}{1+\tau_C^*} \\ \tilde{r}_E^* &= \frac{(1-\tau_K^*)r_E^*}{1+\tau_C^*} \end{aligned}$$

Dividing the first of these equations by the second yields (B.26).

Equation (B.27) follows from the fact that each unit of capital in risk-free projects produces 1 unit of standard intermediate goods. Furthermore, since, by assumption, no units of capital are hidden, all capital must either find its way into entrepreneurs' risky projects or risk-free projects. Therefore $K = K_F^* + K_E^*$ and (B.27) follows immediately.

To derive equation (B.28), first integrate equation (B.14) across entrepreneurs. We obtain:

$$K' = (1+\tau_C)\int_i \omega_i di - (1+\tau_C)\Delta\int_i c_i di$$

Using equation (B.12) and noting that, in a steady state, $K' = K = K^\star$, we obtain

$$K = (1+\tau_C)K\left(\tilde{r}_F\Delta + \frac{1}{1+\tau_C} - \tilde{p}\Delta\right) + (1+\tau_C)\left(\tilde{r}_E Y_E\Delta - \tilde{r}_F K_E\Delta\right) - (1+\tau_C)\Delta\int_i c_i di$$

where we assume a large of large numbers holds so that idiosyncratic shocks to each ϵ_i do not affect $\int_i \omega_i di$ and we use that $\int_i \theta_i k_{E,i} di = Y_E$. Subtracting K from both sides and dividing through by $\Delta(1+\tau_C)$, we get:

$$0 = K\left(\tilde{r}_F - \tilde{p}\right) + \left(\tilde{r}_E Y_E - \tilde{r}_F K_E\right) - \int_i c_i di$$

Finally, note that aggregate consumption is equal to entrepreneur consumption plus worker consumption. Therefore:

$$C = \int_i c_i di + \tilde{w}N$$

Combining these expressions, and using asterisks to denote steady state values, we obtain (B.28).

To derive equation (B.29), first integrate equation (B.13) across entrepreneurs. We obtain:

$$\int_i c_i di = (\rho + \gamma - \rho\gamma\Delta) \int_i \omega_i di$$

Using equation (B.12) to eliminate ω_i, and noting that aggregate consumption satisfies $C = \int_i c_i di + \tilde{w}N$, and $Y_F^S = K - K_E$, we obtain (B.29).

Equation (B.30) is just the steady state form of the goods market clearing condition (B.4), where we impose that $K_E^* + K_F^* = K_{t+1} = K^*$, since by assumption no capital is hidden, and impose $C_H^* = 0$, since by assumption no capital is hidden. This immediately yields (B.30).

To show (B.31) and (B.32) , note that Proposition 4 implies that $k_E(k, \theta, X)$ is proportional to k. For the proof, it is convenient to define $\hat{k}(\theta) = \frac{k_E(k,\theta,X)}{k}$. Thus, two entrepreneurs with the same θ will always choose the same $\hat{k}(\theta)$.

Now, in equilibrium aggregate $K_{E,t}$ and $Y_{E,t}^S$ are given by equations (20) and (25). Since $\theta_{i,t}$ and $\epsilon_{i,t}$ are drawn independently of $k_{i,t}$ each period, and by assumption no intermediate goods are hidden, we use the standard abuse of the law of large numbers to infer that:

$$K_E^* = K^* \int_i \hat{k}_E(\theta) \mathrm{d}i = K^* \sum_\theta \hat{k}_E(\theta) \mu_K^*(\theta) \tag{B.35}$$

$$Y_E^{S*} = K^* \sum_\theta \theta \hat{k}_E(\theta) \mu_K^*(\theta) \tag{B.36}$$

where $\mu_K^*(\theta)$ is the proportion of aggregate capital held by entrepreneurs of ability level θ in the steady state. Thus, we have shown (B.31) and (B.32). From Proposition 4, it follows immediately that $\hat{k}_E(\theta)$ satisfies (B.34).

Now, to derive equation (B.33), note that Proposition 4 implies that two entrepreneurs with the same θ and ϵ will choose the same k'. Let $\hat{k}'(\theta, \epsilon) = \frac{k'(k,\theta,\epsilon,X)}{k}$. Let $\mu_{K,t}(\theta)$ denote the proportion of aggregate capital held by entrepreneurs of ability level θ at time t. Our assumptions on the evolution of θ for each entrepreneur imply that, for each θ,

$$K_{t+\Delta}\mu_{K,t+\Delta}(\theta) = \lambda_\theta \Delta g(\theta) K_{t+\Delta} + (1 - \lambda_\theta \Delta) K_t \mu_{K,t}(\theta) \int_\epsilon \hat{k}'(\theta, \epsilon) \mathrm{d}H(\epsilon) \tag{B.37}$$

The justification for equation (B.37) is as follows. Recall that, at the start of each period, entrepreneurs retain the same θ with probability $(1-\lambda_\theta\Delta)$ and draw a new θ with probability $\lambda_\theta\Delta$. The left hand side corresponds to the total capital held by entrepreneurs of ability level θ at time $t+\Delta$. The first right hand term indicates the capital held at time $t+\Delta$ by entrepeneurs who draw a new ability level that period, who happen to draw θ. Since the total capital stock at the start of $t+\Delta$ is $K_{t+\Delta}$, the total capital of entrepreneurs who draw a new ability level is $\lambda_\theta\Delta K_{t+\Delta}$, and the proportion of these that draw ability level θ is given by $g(\theta)$, so the first right hand side term equation (B.37) follows. The second right hand side term corresponds to the capital held by entrepreneurs who had ability level θ in period t, and who do not draw a new ability level in period $t+\Delta$. In period $t+\Delta$, an entrepreneur who has capital k at the start of period t, who has ability θ and draws shock ϵ, will have capital $\hat{k}'(\theta,\epsilon)$ at the start of period $t+\Delta$. Note that the total capital at the start of period t of entrepreneurs who had ability θ in period t corresponds to $K_t\mu_{K,t}(\theta)$. Then, the total capital of such entrepreneurs at the start of period $t+\Delta$ corresponds to $K_t\mu_{K,t}(\theta)\int_\epsilon \hat{k}'(\theta,\epsilon)\mathrm{d}H(\epsilon)$. Since fraction $(1-\lambda_\theta\Delta)$ of such entrepreneurs retains the ability θ into period $t+\Delta$, the second right hand side term of equation (B.37) follows.

In a steady state, $\mu_{K,t+\Delta}=\mu_{K,t}=\mu_K^*$ and $K_{t+\Delta}=K_t=K^*$. In that case, equation (B.37) simplifies to:

$$K^*\mu_K^*(\theta)=\lambda_\theta\Delta g(\theta)K^*+(1-\lambda_\theta\Delta)K^*\mu_K^*(\theta)\int_\epsilon \hat{k}'(\theta,\epsilon)\mathrm{d}H(\epsilon)$$

Rearranging to solve for μ_K^*, we get:

$$\mu_K^*=\frac{\lambda_\theta\Delta g(\theta)}{1-(1-\lambda_\theta\Delta)\int_\epsilon \hat{k}'(\theta,\epsilon)\mathrm{d}H(\epsilon)} \tag{B.38}$$

Now, Proposition 4 implies that an entrepreneur's choice of k' is given by:

$$k'=(1+\tau_C)\left[k\left(\tilde{r}_F\Delta+\frac{1}{1+\tau_C}-\tilde{p}\Delta\right)+k_E\left(\phi\left(\epsilon-1\right)+\theta\tilde{r}_E\Delta-\tilde{r}_F\Delta\right)\right](1-\Delta(\rho+\gamma-\rho\gamma\Delta))$$

Diving through by k, we have that $\hat{k}'(\epsilon,\theta)$ is given by:

$$\hat{k}'(\theta,\epsilon)=(1+\tau_C)\left[\tilde{r}_F\Delta+\frac{1}{1+\tau_C}-\tilde{p}\Delta+\hat{k}_E(\theta)\left(\phi\left(\epsilon-1\right)+\theta\tilde{r}_E\Delta-\tilde{r}_F\Delta\right)\right](1-\Delta(\rho+\gamma-\rho\gamma\Delta))$$

Integrating across ϵ:

$$\int_\epsilon \hat{k}'(\theta,\epsilon)\mathrm{d}H(\epsilon)=\left[1+(1+\tau_C)\Delta\left(\tilde{r}_F-\tilde{p}+\hat{k}_E(\theta)\left(\theta\tilde{r}_E-\tilde{r}_F\right)\right)\right](1-\Delta(\rho+\gamma-\rho\gamma\Delta))$$

Substituting this into (B.38), we obtain (B.33).

To show that the conditions of Proposition 5 are necessary for a steady state, it remains only to show that the conditions $\phi - \tilde{r}_E^* \overline{\theta} \Delta + \tilde{r}_F^* \Delta > 0$, $\tilde{r}_E^* \overline{\theta} > \tilde{r}_F^*$, $\frac{1}{1+\tau_C} - \tilde{p}\Delta + \tilde{r}_F \Delta \geq \phi$, $(1+\tau_C)(\tilde{r}_F^* - \tilde{p}^*) \geq \underline{r}$ and $K_E^* < K^*$ are necessary. The condition $\phi - \tilde{r}_E^* \overline{\theta} \Delta + \tilde{r}_F^* \Delta > 0$ is analogous to the inequality (49), which was shown in Lemma 3 to be necessary for any equilibrium, for a generic period length Δ. The only change is that we have rewritten this inequality in terms of post-tax prices using the definitions (50)-(52). Hence, this inequality must be necessary for a steady state following the same argument in the Proof of Lemma 3. The inequality $\tilde{r}_E^* \overline{\theta} > \tilde{r}_F^*$ must hold since otherwise Proposition 4 implies that all entrepreneurs will set $k_E = 0$. In that case, the Inada conditions on the final goods production function imply that r_E^* goes to infinity, which cannot be an equilibrium. Similarly, the inequality $K_E^* < K^*$ must hold in equilibrium since otherwise $K_F^* \leq 0$, in which case the Inada conditions imply that r_F^* goes to infinity (or is undefined), which also cannot be an equilibrium. The condition $(1+\tau_C)(\tilde{r}_F^* - \tilde{p}^*) \geq \underline{r}$ was shown in Lemma 2 to be necessary in order for entrepreneurs not to strictly prefer to hide capital to evade taxes in the beginning of the period. As above, the only change is that we have rewritten this inequality in terms of post-tax prices using the definitions (50)-(52).

It remains to show that $\frac{1}{1+\tau_C} - \tilde{p}\Delta + \tilde{r}_F \Delta \geq \phi$ is necessary for a steady state. This inequality is simply the same inequality as (43), which was shown in Lemma 2 to be necessary for any equilibrium, but for a generic period length Δ. The only change is that we have rewritten this inequality in post-tax prices using the definitions (50)-(52). Hence, it must be necessary for a steady state following the same argument as in the Proof of Lemma 2. This completes the proof that the conditions in Proposition 5 are necessary for a steady state.

To show that these conditions are sufficient for a steady state, we assume that they hold and use them to construct an equilibrium in which aggregate variables are stable over time, thereby proving sufficiency by construction. In particular, we set $K_F^* = K^* - K_E^*$ and $Y^* = F(Y_E^{S*}, Y_F^{S*}, N)$ and we set pre-tax prices r_E^*, r_F^* and w^* to be consistent with the final goods firms' first order conditions (27)-(29). We set the tax rates τ_N^*, τ_K^* and τ_W^* to be consistent with these values of pre-tax prices, and the already assumed values of post-tax prices, according to equations (50)-(52) and (60). Since these aggregate variables are, by assumption, all stable over time, it follows that they form a steady state provided that these values are all consistent with equilibrium. Therefore, it remains to show only that these values of aggregate variables and prices form an equilibrium in the sense of Definition 3.

Government budget balance can be shown by combining the condition (B.30) with the conditions (B.28) and (B.29) using the final goods firms' first order conditions (27)-(29)

and definitions of post-tax prices (50)-(52) and (60) to eliminate post-tax prices from the resulting equation, and using that constant returns to scale of final goods firms imply that $Y = r_E Y_E^S + r_F Y_F^S + wN$ to eliminate $F(Y_E^{S*}, Y_F^{S*}, N)$. After a little rearrangement of the resulting equation, we obtain the government budget balance condition (B.3).

That workers and entrepreneurs are optimizing in the proposed steady state allocation follows because for the necessity section of this proof, we showed that equations (B.28) and (B.29) are consistent with workers' and entrepreneurs' optimization. Likewise for the necessity section of this proof we showed that the equations (B.28)-(B.32) come directly from integrating entrepreneurs' and workers' decision rules, and therefore must be consistent with the aggregation and market clearing conditions (17)-(23), (25) and (26), which were also direct integration and summation of the decision rules of individual agents.

The pre-tax prices are consistent with the final goods frims' first order conditions, since we explicitly defined the pre-tax prices so that these conditions would hold.

Finally, the definitions (50)-(52) imply that entrepreneurs are indifferent between lending to banks and putting resources into their risk-free projects. Therefore, we may assume that they lend to banks to exactly the level needed to ensure that asset markets clear. For instance, we can consider an equilibrium in which only one entrepreneur, whose wealth is negligible compared to aggregate wealth, puts any resources into a risk-free project, and all the remaining entrepreneurs either put funds into risky projects or lend to the bank. Then, the total net quantity that the other entrepreneurs will wish to lend to the bank is equal to their total wealth, minus the amount they allocate to their risky projects, that is $K^* - K_E^*$. This will be precisely the amount that the one entrepreneur who manages a risk-free project will want to borrow, since $K^* - K_E^* = K_F^*$. Therefore, this is consistent with asset market clearing. □

B.4 Proof of Proposition 1

This proof makes use of the following two lemmas.

Lemma 5. *The following holds, for any* $x \in \left[0; \frac{1}{1-\underline{\epsilon}}\right]$:

$$\lim_{\Delta \to 0} \frac{S(x)}{\Delta} = (1-\underline{\epsilon})^2 \varphi^2 x. \tag{B.39}$$

Proof. To prove this, note first that

$$\frac{S(x)}{\Delta} = \frac{1}{\Delta}\left(1 - \frac{\int_{\epsilon}(1+x(\epsilon-1))^{-1}\epsilon \mathrm{d}H(\epsilon)}{\int_{\epsilon}(1+x(\epsilon-1))^{-1}\mathrm{d}H(\epsilon)}\right) = -\left(\frac{\int_{\epsilon>0}\left(\frac{\epsilon-1}{\Delta}\right)(1+x(\epsilon-1))^{-1}\mathrm{d}H(\epsilon)}{\int_{\epsilon>0}(1+x(\epsilon-1))^{-1}\mathrm{d}H(\epsilon)}\right).$$

Then, it remains to show that, for any x in the domain of S,

$$\lim_{\Delta\to 0}\left[\frac{\int_{\epsilon>0}\left(\frac{\epsilon-1}{\Delta}\right)(1+x(\epsilon-1))^{-1}\mathrm{d}H(\epsilon)}{\int_{\epsilon>0}(1+x(\epsilon-1))^{-1}\mathrm{d}H(\epsilon)}\right] = -x(1-\underline{\epsilon})^2\varphi^2. \tag{B.40}$$

To prove this, we prove the following results, from which equation (B.40) follows trivially:

$$\begin{aligned} \lim_{\Delta\to 0}\int_{\epsilon>0}\left(\frac{\epsilon-1}{\Delta}\right)(1+x(\epsilon-1))^{-1}\mathrm{d}H(\epsilon) &= -x(1-\underline{\epsilon})^2\varphi^2 && \text{(B.41)} \\ \lim_{\Delta\to 0}\int_{\epsilon>0}(1+x(\epsilon-1))^{-1}\mathrm{d}H(\epsilon) &= 1 && \text{(B.42)} \end{aligned}$$

To prove these, recall that the $\epsilon_{i,t}$ is given by:

$$\epsilon_{i,t} = \underline{\epsilon} + (1-\underline{\epsilon})\exp\left(\varphi\sqrt{\Delta}\xi_{i,t} - \frac{\varphi^2\Delta}{2}\right).$$

where $\xi_{i,t} \sim N(0,1)$. This implies that

$$\epsilon_{i,t} - 1 = (1-\underline{\epsilon})\left(\exp\left(\varphi\sqrt{\Delta}\xi_{i,t} - \frac{\varphi^2\Delta}{2}\right) - 1\right).$$

Therefore, the left hand side of (B.41) is:

$$\lim_{\Delta\to 0}\int_{-\infty}^{\infty}(1-\underline{\epsilon})\left(\frac{\exp\left(\varphi\sqrt{\Delta}\xi - \frac{\varphi^2\Delta}{2}\right) - 1}{\Delta}\right)\left(1 + x(1-\underline{\epsilon})\left(\exp\left(\varphi\sqrt{\Delta}\xi - \frac{\varphi^2\Delta}{2}\right) - 1\right)\right)^{-1}\mathrm{d}\Phi(\xi),$$

where $\Phi(\cdot)$ is the standard normal cdf. Likewise, the left hand side of (B.42) is:

$$\lim_{\Delta\to 0}\int_{-\infty}^{\infty}\left(1 + x(1-\underline{\epsilon})\left(\exp\left(\varphi\sqrt{\Delta}\xi - \frac{\varphi^2\Delta}{2}\right) - 1\right)\right)^{-1}\mathrm{d}\Phi(\xi).$$

Now, we consider a first order approximation of $\exp\left(\varphi\sqrt{\Delta}\xi - \frac{\varphi^2\Delta}{2}\right)$ in units of $\sqrt{\Delta}$, around the point $\sqrt{\Delta}=0$:

$$\exp\left(\varphi\sqrt{\Delta}\xi-\frac{\varphi^2\Delta}{2}\right)\simeq 1+\varphi\sqrt{\Delta}\xi. \tag{B.43}$$

Similarly, in the neighborhood of $\sqrt{\Delta}=0$:

$$\left(1+x(1-\underline{\epsilon})\left(\exp\left(\varphi\sqrt{\Delta}\xi-\frac{\varphi^2\Delta}{2}\right)-1\right)\right)^{-1}\simeq 1-x(1-\underline{\epsilon})\varphi\sqrt{\Delta}\xi. \tag{B.44}$$

Multiplying the term in the limit on the left hand side of (B.41) by Δ, and ignoring terms of order greater than Δ, we therefore can write it as:

$$\begin{aligned}
&\int_{-\infty}^{\infty}(1-\underline{\epsilon})\varphi\sqrt{\Delta}\cdot\xi\left(1-x(1-\underline{\epsilon})\varphi\sqrt{\Delta}\xi\right)\mathrm{d}\Phi(\xi)\\
&=\int_{-\infty}^{\infty}(1-\underline{\epsilon})\varphi\sqrt{\Delta}\cdot\xi-x(1-\underline{\epsilon})^2\varphi^2\Delta\xi^2\mathrm{d}\Phi(\xi)\\
&=(1-\underline{\epsilon})\varphi\sqrt{\Delta}E[\xi]-x(1-\underline{\epsilon})^2\varphi^2\Delta E[\xi^2]\\
&=-x(1-\underline{\epsilon})^2\varphi^2\Delta.
\end{aligned}$$

since ξ is normally distributed. Hence (B.41) follows immediately.

Likewise, considering the term on the left hand side of (B.42), we have:

$$\begin{aligned}
&\int_{-\infty}^{\infty}1-x(1-\underline{\epsilon})\varphi\sqrt{\Delta}\xi\mathrm{d}\Phi(\xi)\\
=&1-x(1-\underline{\epsilon})\varphi\sqrt{\Delta}E[\xi]\\
=&1.
\end{aligned}$$

This proves (B.42). □

Lemma 6. *For any $z\in(-\infty,\infty)$, it holds that*

$$\lim_{\Delta\to 0}S^{-1}(\max\{0;\min\{\Delta z;S^\star\}\})=\max\left\{0;\min\left\{\frac{z}{(1-\underline{\epsilon})^2\varphi^2};\frac{1}{1-\underline{\epsilon}}\right\}\right\}. \tag{B.45}$$

Proof. To prove (B.45), we first note that

$$\lim_{\Delta\to 0}\frac{S^\star}{\Delta}=(1-\underline{\epsilon})\varphi^2. \tag{B.46}$$

This follows immediately from the definition of $S^\star$ in (B.16) and from Lemma 5.

Now, we show that

$$\forall z \in [0, (1-\underline{\epsilon})\varphi^2), \ \lim_{\Delta \to 0} S^{-1}(\Delta z) = \frac{z}{\varphi^2 (1-\underline{\epsilon})^2}. \tag{B.47}$$

To prove (B.47) , define the function $\mathscr{F}(x)$ according to:

$$\mathscr{F}(x) = \frac{S(x)}{\Delta}. \tag{B.48}$$

where $x \in \left(0, \frac{1}{1-\underline{\epsilon}}\right)$. Given that S is continuous and strictly increasing, it follows that function $\mathscr{F}(\cdot)$ is continuous and strictly increasing, and therefore invertible. We now show that, for any z in the range of $\mathscr{F}$,

$$\mathscr{F}^{-1}(z) \equiv S^{-1}(\Delta z). \tag{B.49}$$

To show this let $x = \mathscr{F}^{-1}(z)$. Then $z = \mathscr{F}(x) = \frac{S(x)}{\Delta}$, so $S(x) = \Delta z$ and $x = S^{-1}(\Delta z)$, giving us (B.49).

Define the function $\overline{\mathscr{F}}(x) = (1-\underline{\epsilon})^2 \varphi^2 x$. It follows from simple rearrangement that its inverse is:

$$\overline{\mathscr{F}}^{-1}(x) = \frac{x}{(1-\underline{\epsilon})^2 \varphi^2}. \tag{B.50}$$

We know from Lemma 5 that, as $\Delta \to 0$, $\mathscr{F}(x)$ converges to $\overline{\mathscr{F}}(x)$. Since $\mathscr{F}$ is continuous, this convergence is uniform, and the inverse $\mathscr{F}^{-1}(x)$ converges to $\overline{\mathscr{F}}^{-1}(x)$. Then, using equations (B.49) and (B.50), we obtain (B.47) for values of z in the relevant domain.

Note that, for $\Delta > 0$, the domain of $S^{-1}(\cdot)$ is $[0, S^\star]$. Therefore, the result (B.47) must hold for all $z \in \left(0, \lim_{\Delta \to 0} \frac{S^\star}{\Delta}\right) \equiv (0, (1-\underline{\epsilon})\varphi^2)$ since, for any such z, Δz will be in will be in the domain of S^{-1} for sufficiently small $\Delta > 0$. Equally, it must be true that $\lim_{\Delta \to 0} S^{-1}(0) = 0$, since $S^{-1}(0) = 0$ for any value of $\Delta > 0$. Therefore, (B.47) follows for $z \in [0, (1-\underline{\epsilon})\varphi^2)$.

Now, using (B.46) and (B.47), we prove (B.45) for $z \in (-\infty, \infty)$. We proceed in cases. First, consider the case $z \leq 0$. Then, for sufficiently small $\Delta > 0$, equation (B.46) implies that $\Delta z \leq 0 < S^\star$. Then,

$$\lim_{\Delta \to 0} S^{-1}(\max\{0; \min\{\Delta z; S^\star\}\}) = \lim_{\Delta \to 0} S^{-1}(0) = 0, \tag{B.51}$$

where the second equality used (B.47).

Now, suppose that $z \in \left(0, \frac{1}{1-\underline{\epsilon}}\right)$. Then, for sufficiently small $\Delta > 0$, equation (B.46) implies that $0 < \Delta z < S^\star$. Then,

$$\lim_{\Delta \to 0} S^{-1}(\max\{0; \min\{\Delta z; S^\star\}\}) = \lim_{\Delta \to 0} S^{-1}(\Delta z) = \frac{z}{(1-\underline{\epsilon})^2 \varphi^2}, \tag{B.52}$$

where the second equality used (B.47).

Now, suppose that $z > \frac{1}{1-\underline{\epsilon}}$. Then, for sufficiently small $\Delta > 0$, equation (B.46) implies that $\Delta z > S^\star > 0$. Then,

$$\lim_{\Delta \to 0} S^{-1}(\max\{0; \min\{\Delta z; S^\star\}\}) = \lim_{\Delta \to 0} S^{-1}(S^\star) = \lim_{\Delta \to 0} \frac{1}{1-\underline{\epsilon}} = \frac{1}{1-\underline{\epsilon}}, \tag{B.53}$$

where the second equality used that, for any $\Delta > 0$, $S^{-1}(S^\star) = \frac{1}{1-\underline{\epsilon}}$, since $S^\star = S\left(\frac{1}{1-\underline{\epsilon}}\right)$, by definition.

Comparing equation (B.45) with equations (B.51),(B.52) and (B.53), we see that we have proven (B.45) for any $z \in (-\infty, \infty)$ except for $z = \frac{1}{1-\underline{\epsilon}}$. In particular, (B.45) must hold for all $z \neq \frac{1}{1-\underline{\epsilon}}$ in the neighborhood of $\frac{1}{1-\underline{\epsilon}}$. Now, note that the left hand side of (B.45) is continuous and weakly increasing in z for any $\Delta > 0$. Equally, the right hand side of (B.45) is continuous and weakly increasing in z. Continuity arguments then imply that (B.45) also holds at $z = \frac{1}{1-\underline{\epsilon}}$. $\square$

Proof of the Propositon Using Lemma 6, the proof of Proposition 1 becomes straightforward. Taking the limit of equation (B.11) as $\Delta \to 0$, and using Lemma 6, we obtain (57). Combining (B.12) and (B.13), we get:

$$c = (\rho + \gamma - \rho\gamma\Delta)\left[k\left(\tilde{r}_F\Delta + \frac{1}{1+\tau_C} - \tilde{p}\Delta\right) + k_E\left(\phi\left(\epsilon - 1\right) + \theta\tilde{r}_E\Delta - \tilde{r}_F\Delta\right)\right].$$

Taking the limit of this as $\Delta \to 0$ and noting that, as $\Delta \to 0$, $\epsilon \to 1$ in probability, we obtain (58). Finally, we note that Proposition 4 implies that:

$$k' = (1+\tau_C)\left[k\left(\tilde{r}_F\Delta + \frac{1}{1+\tau_C} - \tilde{p}\Delta\right) + k_E\left(\phi\left(\epsilon - 1\right) + \theta\tilde{r}_E\Delta - \tilde{r}_F\Delta\right)\right] - (1+\tau_C)c\Delta,$$

so that

$$\frac{k'-k}{\Delta} = (1+\tau_C)k\left(\tilde{r}_F - \tilde{p}\right) + (1+\tau_C)k_E\left[\phi\left(\frac{\epsilon-1}{\Delta}\right) + \theta\tilde{r}_E - \tilde{r}_F\right] - c(1+\tau_C). \tag{B.54}$$

Now, for sufficiently small Δ, $\frac{\epsilon i,t-1}{\Delta} \simeq (1-\underline{\epsilon})\varphi\sqrt{\Delta}\xi_{i,t}$. Furthermore, $\frac{\xi_{i,t}}{\sqrt{\Delta}}$ corresponds to the difference of a standard Brownian motion, since $\frac{\xi_{i,t}}{\sqrt{\Delta}} \sim N(0, \Delta)$. Therefore, as $\Delta \to 0$, equation (B.54) simplifies to (59). $\square$

B.5 Proof of Proposition 2

Of the equations describing the steady state in Proposition 5, most do not contain Δ, so these equations are unchanged in the limit as $\Delta \to 0$. The only equations in Proposition 5, containing Δ are equations (B.29), (B.33) and (B.34). Taking limits, it is immediate that (B.29) and (B.33) become equations (65) and (61). Furthermore, using that $\hat{k}_E(\theta) = \frac{k_E(\theta,k,X)}{k}$ it follows from Proposition 1 that $\hat{k}_E(\theta)$ is given by (69) in the limit as $\Delta \to 0$.

The inequalities $(1+\tau_C)(\tilde{r}_F^* - \tilde{p}^*) \geq \underline{r}$, $\tilde{r}_E^*\overline{\theta} > \tilde{r}_F^*$ and $K_E^* < K^*$ do not contain Δ and so are unchanged as $\Delta \to 0$. Since $\phi > 0$, the inequality $\phi - \tilde{r}_E^*\overline{\theta}\Delta + \tilde{r}_F^*\Delta > 0$ is automatically satisfied in the limit as $\Delta \to 0$.

The only remaining condition for a steady state in Proposition 5 is

$$\frac{1}{1+\tau_C} - \tilde{p}\Delta + \tilde{r}_F\Delta \geq \phi.$$

Evidently, this is satisfied in the limit as $\Delta \to 0$ if $\frac{1}{1+\tau_C} > \phi$. Furthermore, it is not satisfied in the limit as $\Delta \to 0$ if $\frac{1}{1+\tau_C} < \phi$. Finally, if $\frac{1}{1+\tau_C} = \phi$, then the inequality is satisfied in the limit as $\Delta \to 0$ if and only if $\tilde{p} < \tilde{r}_F$. $\square$

B.5.1 Derivation of K_E^*

Recall that the entrepreneurial type θ is drawn from the discrete distribution $g(\theta)$ given by

$$g(\theta) = \begin{cases} 1-\pi & \text{if } \theta = 0 \\ \pi & \text{if } \theta = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{B.55}$$

The, equation (69) immediately implies that

$$\hat{k}_E(\theta) = \begin{cases} 0 & \text{if } \theta = 0 \\ \frac{1}{(1+\tau_C)\phi(1-\underline{\epsilon})} \times \max\left\{0; \min\left\{\frac{\tilde{r}_E - \tilde{r}_F}{\phi(1-\underline{\epsilon})\varphi^2}; 1\right\}\right\} & \text{if } \theta = 1. \end{cases}$$

Since, by assumption, $\tilde{r}_E > \tilde{r}_F$, we can simplify further to obtain

$$\hat{k}_E(\theta) = \begin{cases} 0 & \text{if } \theta = 0 \\ \frac{1}{(1+\tau_C)\phi(1-\underline{\epsilon})} \times \min\left\{\frac{\tilde{r}_E - \tilde{r}_F}{\phi(1-\underline{\epsilon})\varphi^2}; 1\right\} & \text{if } \theta = 1. \end{cases} \tag{B.56}$$

Using this equation, we can rewrite equations (67) and (68) as

$$Y_E^{S*} = K_E^* = K\hat{k}_E(1)\mu_K^*(1) = K\hat{k}_E(1)(1-\mu_K^*(0)) \tag{B.57}$$

where the last equality follows because, since $\mu_K^*(\theta)$ is the proportion of capital held by entrepreneurs of type θ, it must, by definition, be the case that $\sum_\theta \mu_K^*(\theta) = 1$. [26]

Using equation (61) and that $\hat{k}_E(0) = 0$, we can re-write (B.57) as:

$$K_E^* = K\hat{k}_E(1)\left(1 - \frac{\lambda_\theta(1-\pi)}{\lambda_\theta + \rho + \gamma - (1+\tau_C)(\tilde{r}_F - \tilde{p})}\right) \tag{B.58}$$

$$= K\hat{k}_E(1)\left(\frac{\lambda_\theta \pi + \rho + \gamma - (1+\tau_C)(\tilde{r}_F - \tilde{p})}{\lambda_\theta + \rho + \gamma - (1+\tau_C)(\tilde{r}_F - \tilde{p})}\right) \tag{B.59}$$

Now, combine equations (64) and (65) to get:

$$\tilde{r}_E^* Y_E^{S*} + \tilde{r}_F^* Y_F^{S*} - \tilde{p}^* K^* = (\rho + \gamma)\frac{K^*}{1+\tau_C}$$

Since, in this case, $Y_E^{S*} = K_E^*$, and $Y_F^{S*} = K_F^* = K^* - K_E^*$, this equation can be rewritten as:

$$\tilde{r}_E^* - \tilde{r}_F^* = \left(\frac{\rho+\gamma}{1+\tau_C} - (\tilde{r}_F^* - \tilde{p}^*)\right)\frac{K^*}{K_E^*}.$$

Note that rearranging this gives an alternative expression for m:

$$m = (1+\tau_C)(\tilde{r}_E^* - \tilde{r}_F^*)\frac{K_E^*}{K^*} = (1-\tau_K)(r_E^* - r_F^*)\frac{K_E^*}{K^*}.$$

Substituting this into equation (B.56), we have:

$$\hat{k}_E(1) = \frac{1}{(1+\tau_C)\phi(1-\underline{\epsilon})} \times \min\left\{\frac{1}{\phi(1-\underline{\epsilon})\varphi^2}\left(\frac{\rho+\gamma}{1+\tau_C} - (\tilde{r}_F^* - \tilde{p}^*)\right)\frac{K^*}{K_E^*};1\right\},$$

and combining with equation (B.59), we get:

$$\frac{K_E^*}{K^*}\left(\frac{\lambda_\theta + \rho + \gamma - (1+\tau_C)(\tilde{r}_F - \tilde{p})}{\lambda_\theta \pi + \rho + \gamma - (1+\tau_C)(\tilde{r}_F - \tilde{p})}\right)$$
$$= \frac{1}{(1+\tau_C)\phi(1-\underline{\epsilon})} \times \min\left\{\frac{1}{\phi(1-\underline{\epsilon})\varphi^2}\left(\frac{\rho+\gamma}{1+\tau_C} - (\tilde{r}_F^* - \tilde{p}^*)\right)\frac{K^*}{K_E^*};1\right\},$$

or, equivalently

$$1 = v\left(\frac{\lambda_\theta \pi + m}{\lambda_\theta + m}\right)\min\left\{\frac{mv}{\varphi^2};1\right\}, \tag{B.60}$$

[26] That $\sum_\theta \mu_K^*(\theta) = 1$ also can also be derived algebraically from equations (61), (64) and (65).

where $v = \frac{1}{(1+\tau_C)\phi(1-\epsilon)}\left(\frac{K^*}{K_E^*}\right)$ and $m = \rho + \gamma - (1+\tau_C)(\tilde{r}_F^* - \tilde{p}^*)$. Now, note that equation (B.60) implies that

$$1 \in \left\{ v^2 \frac{\lambda_\theta \pi + m}{\lambda_\theta + m} \frac{m}{\varphi^2}; v \frac{\lambda_\theta \pi + m}{\lambda_\theta + m} \right\},$$

which in turn implies that

$$v \in \left\{ \frac{\lambda_\theta + m}{\lambda_\theta \pi + m}; \sqrt{\frac{\lambda_\theta + m}{\lambda_\theta \pi + m} \frac{\varphi^2}{m}} \right\}$$

and therefore that

$$v \leq \max \left\{ \frac{\lambda_\theta + m}{\lambda_\theta \pi + m}; \sqrt{\frac{\lambda_\theta + m}{\lambda_\theta \pi + m} \frac{\varphi^2}{m}} \right\} \tag{B.61}$$

Furthermore, (B.60) implies that:

$$1 \leq v^2 \frac{\lambda_\theta \pi + m}{\lambda_\theta + m} \frac{m}{\varphi^2} \quad \text{and} \quad 1 \leq v \frac{\lambda_\theta \pi + m}{\lambda_\theta + m},$$

which in turn rearrange to:

$$v \geq \sqrt{\frac{\lambda_\theta + m}{\lambda_\theta \pi + m} \frac{\varphi^2}{m}} \quad \text{and} \quad v \geq \frac{\lambda_\theta + m}{\lambda_\theta \pi + m}$$

and therefore imply that

$$v \geq \max \left\{ \frac{\lambda_\theta + m}{\lambda_\theta \pi + m}; \sqrt{\frac{\lambda_\theta + m}{\lambda_\theta \pi + m} \frac{\varphi^2}{m}} \right\}. \tag{B.62}$$

Combining equations (B.61) and (B.62), we get:

$$v = \max \left\{ \frac{\lambda_\theta + m}{\lambda_\theta \pi + m}; \sqrt{\frac{\lambda_\theta + m}{\lambda_\theta \pi + m} \frac{\varphi^2}{m}} \right\}.$$

Then, given the definition of v, we can write this as:

$$\frac{1}{(1+\tau_C)\phi(1-\underline{\epsilon})} \frac{K^*}{K_E^*} = \max \left\{ 1 + \frac{\lambda_\theta (1-\pi)}{\lambda_\theta \pi + m}; \sqrt{\left(1 + \frac{\lambda_\theta (1-\pi)}{\lambda_\theta \pi + m}\right) \frac{\varphi^2}{m}} \right\}.$$

which rearranges to equation (70).

B.6 Proof of Lemma 4

Formally, we define the government's optimization problem as seeking taxes $\tau_K^\star, \tau_C^\star, \tau_N^\star, \tau_W^\star$ and an allocation to achieve the supremum of worker steady state utility subject to the constraint that all aggregate variables must be consistent with the equations and inequalities in Proposition 2 – i.e. the allocation must be a steady state.

Since we are seeking a supremum to the government's problem, all the strict inequalities in Proposition 2 can be replaced with weak inequalities, since all the inequalities are continuous functions of the aggregate variables. After this replacement, all the constraints of the government's problem are either equalities or weak inequalities.

Consider a compact neighborhood of combinations of feasible taxes and allocations around the optimal taxes $\tau_K^\star, \tau_C^\star, \tau_N^\star, \tau_W^\star$. Such a compact set exists, since all the constraints of the government's problem are equalities and weak inequalities. Since the government's objective function is continuous over this compact set, it follows that, within this compact set, the maxium of the government's objective function must be attained at the tax rates $\tau_K^\star, \tau_C^\star, \tau_N^\star, \tau_W^\star$, by the Weierstrass theorem.

At the optimal tax rates $\tau_K^\star, \tau_C^\star, \tau_N^\star, \tau_W^\star$ and optimal allocation, it must be the case that $r_E^* > r_F^*$, since, by the Inada conditions on the production function, $r_E^* = \infty$, if $K_E = 0$, and if $K_E > 0$ then equation (70) implies that it must be the case that $r_E^* > r_F^*$, since $\tau_K^\star < 1$. Since $r_E^* > r_F^*$, it must also be the case that $K_F^* > 0$, since the Inada conditions imply $r_F^* = \infty$ otherwise.

Then, the only inequality constraints on the government's problem that may bind at the optimal allocation are that $\tilde{R}_F = (1 + \tau_C^*)(\tilde{r}_F^* - \tilde{p}^*) \geq \underline{r}$ and $\frac{1}{1+\tau_C^*} \geq \phi$. Since all the relevant functions are continuously differentiable, we may apply the Kuhn-Tucker theorem to deduce that the optimum must satisfy the Kuhn-Tucker first order conditions, with Lagrange multipliers on these latter two inequality constraints that may bind. Combining the steady state conditions in Proposition 2 to get an expression for worker's post tax utility as a function of K_E and K_F and differentiating yields the first order conditions in the Lemma.

□

C Data

To calibrate the entrepreneur's stake in the business, we use data from two sources: the Survey of Consumer Finances (SCF) and the (National) Survey of Small Business Finances (SSBF). Both surveys contain information regarding business ownership, with the difference that the first is a household survey, while the second is a survey of small businesses. We can identify in each of them groups of respondents that are in line with our notion of entrepreneurship. We use both sources as validation for our results.

The Survey of Consumer Finances is a a triennial cross-sectional survey of U.S. families which provides information on individual household portfolio composition, including investment in private firms. While the SCF was initially administered in 1983, it was not until 1989 that questions about business ownership were introduced. Therefore, we use all survey waves from 1989 until 2013. We restrict the sample to households who report owning a business in which they have an active management interest, and are between 25 and 65 years old. This represents, on average, 14.3% of the sample. If a household is an active participant in multiple businesses, we examine the average share across businesses.[27]

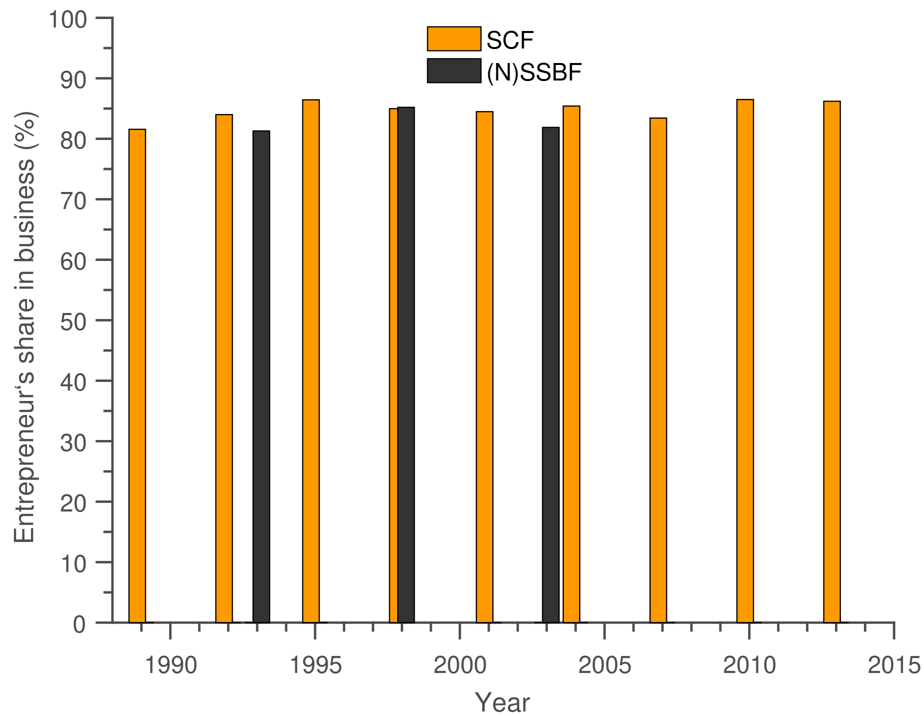
The (National) Survey of Small Business Finances collects information on private, non-financial, non-agricultural businesses in the U.S., with fewer than 500 employees. There are four surveys to date, but only the last three (1993, 1998 and 2003) collect ownership share information and are useful for our purposes. The surveys detail the demographic and financial characteristics of the firms and their principal shareholder.[28] Approximately 90% of these firms are managed by the principal shareholder. We apply the same sample restrictions as in the SCF.

Figure 4 displays the evolution of the ownership share over time. Both surveys indicate that ownership is highly concentrated, entrepreneurs holding, on average, 84% of their firm's equity. In particular, the average share is 85% in SCF and 83% in (N)SSBF. Ownership rates are very stable not only across surveys, but also across the time horizon we consider. For this reason, for our calibration exercise we work with their average over time and surveys, which is 84%.

[27]We obtain similar results if we only focus on the business in which the household has the largest investment.

[28]In 2003 information was collected for up to three owners. We only focus on the main owner, i.e. the one with the largest ownership in the business.

Figure 4: Ownership Share in the U.S.



Notes: The orange bars show the average share that entrepreneurs in SCF own in their business. The black bars show the average share of small businesses in the (N)SSBF that is owned by the principal shareholder.