Entrepreneurship, Agency Frictions and Redistributive Capital Taxation

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Abstract

Motivated by the observation that among OECD countries redistribution is negatively correlated with entrepreneurial activity, we examine the implications of entrepreneurial financial frictions for optimal linear capital taxation, in a setting where the government is concerned with redistribution. By including financial frictions, we emphasize the effect of a new channel affecting the equity-efficiency trade-off of redistribution: taxes affect the allocative efficiency of capital and, ultimately, total factor productivity. In our setting, optimal tax rates can be closely approximated by simple closed-form functions of pre-tax prices and parameters. Under plausible parameter values, we find that it is optimal to tax both consumption and wealth at relatively high positive rates and optimal to tax capital income at a negative rate. This is because capital income taxes are more inefficient than both consumption and wealth taxes in terms of their effect on aggregate total factor productivity, in addition to their well-known effect of reducing aggregate capital accumulation.

1 Introduction

How should a government concerned with redistribution tax entrepreneurial capital when entrepreneurs are subject to financial frictions? To answer this question we study optimal Ramsey taxation in a setting where the government desires to redistribute from entrepreneurs, who own capital, to workers who do not, as in Judd (1985). In contrast to Judd (1985), we assume that entrepreneurs are subject to financial frictions, so that the amount an entrepreneur can invest depends on her individual net worth. We also

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assume that entrepreneurs vary in their productivity levels. These assumptions substantially affect the analysis, because they imply that capital taxes can affect the allocation of capital between different entrepreneurs, and therefore the level of aggregate total factor productivity — a force missing from models without financial frictions. This occurs via two channels. Firstly, high taxation may affect the ability of highly productive but poor entrepreneurs to fund risky investment, thereby reducing the average level of investment efficiency. Secondly, high taxation reduces the ability of all entrepreneurs to fund risky investment, leading to inefficient under-investment in the risky projects and inefficient over-investment in the risk-free projects.

Since at least Helvétius (1769) and continuing to the present day (e.g. Piketty (2014)), taxes on wealth or capital income have been defended as a tool for redistribution from rich to poor. Many governments today tax wealth or capital or both and actively redistribute through the tax system (e.g. Chang, Chang and Kim (2016)). However, it remains unclear how far such taxes are economically costly. Recent literature has argued that financial frictions play an important role in shaping firms' investment decisions. This suggests that it may be essential to account for such frictions in order to correctly assess the effects of capital taxation. It is well known that taxes on capital may reduce capital accumulation (e.g. Atkeson, Chari and Kehoe (1999)). However, once financial frictions are considered, it emerges that capital taxes may also affect the allocation of capital, and therefore the productive efficiency of the economy. This efficiency effect of capital taxation is arguably of more practical importance than effects on capital accumulation, since it has been shown that the government can impose confiscatory levels of taxation on capital owners with little or no effect on aggregate capital accumulation if it is permitted to use investment subsidies (Abel (2007)), consumption taxes (Abel (2007), Coleman (2000)) or government saving and debt (Straub and Werning (2014)).[1]

From an empirical perspective, we are also motivated by the fact that government redistribution through the tax and welfare system appears to be negatively correlated with entrepreneurial activity.[2] This is illustrated in Figure 1 for a sample of OECD countries. This suggests a potentially important economic cost of redistribution, since a large economic literature is supportive of a connection between entrepreneurship and economic growth (Quadrini (2009)). Indeed, the perceived benefits of entrepreneurial activity lead many governments to implement tax policies to encourage it. A theoretical analysis of the optimal taxation of entrepreneurial capital may also be informative about the optimal design and effectiveness of such policies.

We carry our analysis in a framework reminiscent of Judd (1985). Our economy is populated by overlapping generations of two types of households: entrepreneurs and

[1] Proposition 12 in Straub and Werning (2014) establishes that indefinite taxation of 100% may be optimal in a setting without financial frictions when the government seeks to redistribute and is able to borrow and save.

[2] The correlation coefficient is -0.3.

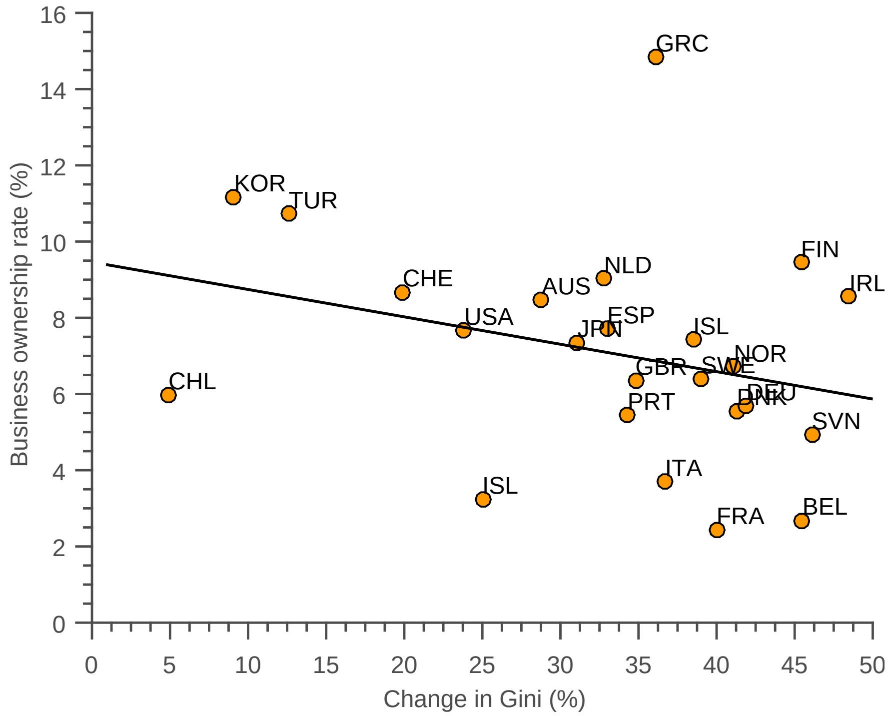

Figure 1: Redistribution and Entrepreneurial Activity

Notes: The figure shows the difference between before and after tax income Gini on the horizontal axis and the business ownership rate on the vertical axis, for a group of OECD countries in 2010. The correlation coefficient is -0.3.
Source: OECD and Global Entrepreneurship Monitor

workers. In addition, there is a continuum of competitive final goods producing firms, and a continuum of risk neutral competitive financial intermediaries (which we call banks). Entrepreneurs own capital, while workers supply labor to the final goods firms and live hand-to-mouth. Final goods firms produce output using this labor and two types of intermediate good, which we label an 'entreprenurial' intermediate good and a 'standard' intermediate good. Entrepreneurs allocate some of their capital to risky projects which each yield a stochastic amount of the entrepreneurial intermediate good. They allocate the remainder of their capital to risk-free projects that produce the standard intermediate good. Entrepreneurs may borrow from financial intermediaries. The output of entrepreneurial intermediate goods is private information to entrepreneurs. They may divert capital and intermediate goods and convert them into their own consumption or may they sell all their output to the final goods firms. The possibility of entrepreneurs diverting intermediate goods into their own consumption creates a moral hazard problem which limits the extent to which entrepreneurs can borrow and therefore limits how far capital can be reallocated to the most productive entrepreneurs. It is the government's effect on this inefficiency that we explore.

In this environment, we assume that the government chooses policies to maximize the aggregate welfare of workers in the long run steady state (an essentially Rawlsian objective) subject to the financial frictions present in the decentralized economy. In addition, the government has to finance exogenous expenditure while balancing its budget. We assume the government levies a mix of (possibly negative) linear taxes, in keeping with models in the Ramsey tradition. In particular, it taxes agents' end-of-period wealth, capital income, labor income and consumption.[3] Only entrepreneurs pay wealth and capital income taxes, since they are the only agents who hold wealth, while only workers pay labor income taxes. All agents pay consumption taxes. Since the amount of capital and intermediate goods diverted by entrepreneurs for their consumption is not observed by the government, government cannot tax this. This creates a possibility for agents to evade high taxes, which serves to limit the level of taxes that governments can raise.

While the government seeks to raise taxes on entrepreneurs in order to redistribute to workers, in doing so it faces efficiency costs of taxation. Taxation of the wealth and capital income of entrepreneurs reduces their incentives to save, as in previous models in the literatur, such as Judd (1985). However, in our setting, taxation has an additional effect on the efficiency with which capital is allocated across entrepreneurs and projects. As a consequence, taxes on entrepreneurs can affect the aggregate total factor productivity of the economy, an effect which does not occur in similar models without financial frictions. For instance, an increase in taxation can limit the resources available to highly productive but poor entrepreneurs to finance their risky projects. This can lead to inefficiently little allocation of capital to risky projects and inefficiently little allocation of capital to the most productive entrepreneurs, both of which reduce the aggregate level of total factor productivity in our setting. We follow the "sufficient statistics" approach in characterizing optimal tax rates in terms of the elasticities of agents' decisions with respect to taxes. We find that it is possible to characterize these elasticities relatively precisely in our setting – almost in closed form. Entrepreneurs' choices of how to allocate capital are relatively elastic with respect to taxes on capital income, and relatively inelastic with respect to taxes on wealth and consumption. This is because taxes on capital income affect the relative return to different uses of capital, while consumption and wealth taxes do not. Capital income taxes fall most heavily on the most efficient entrepreneurs, since these entrepreneurs obtain high rates of return to capital. As a consequence, capital income taxes tend to distort the allocation of capital towards less efficient uses, while consumption and wealth taxes distort this allocation much less relative to the revenue that they generate.

The efficiency costs of consumption and wealth taxes are low enough compared to those associated with capital income taxes that we derive sufficient conditions under which the government prefers to tax consumption at the highest possible level consistent with agents paying rather than evading this tax. Furthermore, we show that these

[3] When entrepreneurs are heterogeneous in their skills and face idiosyncratic risk, capital income and wealth taxes are not equivalent Guvenen et al. (2017).

sufficient conditions are easily satisfied in a simple calibration. Likewise, we argue that the calibration would have to be quite extreme in order to avoid a conclusion of strictly negative optimal capital income taxes. In our simple calibration we obtain that the optimal consumption tax is at least 30%, the optimal tax on wealth at least 6% per year and the optimal tax on capital income is negative and below -24%. That is, the efficiency gains from taxing capital income a negative amount are so large that it is optimal to set other taxes at a high level in order to fund negative capital income taxes.[4]

The implication of our results is not that taxing the owners of capital to redistribute to workers is necessarily a bad idea per se, but rather that capital income taxes are not an ideal tool for such redistribution. Instead, through high consumption and wealth taxes, the government is able to potentially extract a considerable revenue from entrepreneurs without serious costs to economic efficiency. This revenue can then be redistributed to workers through lower labor income taxes. While further work is certainly needed before reforming government tax policies based on our conclusions, the implication of our findings is that the current tax arrangements of many countries are far from optimal, in that capital income taxes are currently used much more often than wealth taxes as a tool to extract revenue from rich owners of capital. A shift of the tax system towards using increased wealth and consumption taxes for redistributive purposes while reducing capital income taxes could potentially increase the allocative efficiency of the economy by a substantial amount in the long run without leading to increased inequality after-tax. While the specific values of the optimal tax rates we obtain are a consequence of our particular modelling and calibration choices, we conjecture that the greater efficiency costs of capital income taxation as compared to both wealth and consumption taxation would carry through to much larger class of settings and may warrant serious consideration by policymakers.

Related literature This paper studies optimal government intervention in a setup with financial frictions. This is a relatively recent research avenue, with only a few papers that share the same broad framework. Shourideh (2014) studies the optimal design of capital tax schedules in a Mirrleesian framework in which entrepreneurs are subject to idiosyncratic capital income risk and financial frictions, and finds that a progressive tax code is optimal. We perform an exercise similar in spirit, but in a Ramsey framework, with several key differences. In Shourideh (2014), the planner redistributes among entrepreneurs (there are no workers) for insurance purposes against productivity risk. Moreover, since there are no financial intermediaries, it is left for the planner to use the tax system to allocate investment funds to their most productive uses. In our framework, on the other hand, the government wants to redistribute from entrepreneurs to workers. The role of reallocating capital to its most productive uses is left to banks, which have no scope for insuring entrepreneurs against capital risk.

[4]While a tax on wealth of 6% per year may not immediately sound like a high rate, such a tax would in practice mean that the post-tax risk-free rate of return to capital is substantially below zero.

Itskhoki and Moll (2014) analyze optimal Ramsey policies (capital and labor taxes) in a closed-economy standard growth model with financial frictions, modeled as a reduced form collateral constraint. Their objective is to determine whether governments in underdeveloped countries can accelerate economic development by market intervention. The framework of Itskhoki and Moll (2014) does not speak to the literature on capital taxation directly, since entrepreneurs are not subject to capital taxes. Policy is targeted at workers, who pay labor and savings taxes. In our model, on the other hand, entrepreneurs are key agents.

Abo-Zaid (2014) and Biljanovska (2015) both examine optimal policy in a macroeconomic model with exogenously imposed collateral constraints. While their respective setups are similar in several dimensions, they have different predictions regarding the optimal long-run capital tax. In particular, Biljanovska (2015) shows both analytically and numerically that the long-run capital tax is positive, while Abo-Zaid (2014) shows that for plausible parameter values the tax rate is negative. In both their frameworks, firms (entrepreneurs) finance their activity with debt, and their collateral constraints bind for distinct but exogenously imposed reasons. Differently from them, we choose to microfound financial frictions as an asymmetric information problem, to ensure the fact that the government cannot intervene to correct the friction itself. Crucially, we also allow for hetergeneity among entrepreneurs, ensuring that financial frictions affect the allocation of capital in a way that is economically significant.

Our paper is also related to the large literature on optimal capital taxation. The hallmark result of this literature is the Chamley (1986) and Judd (1985) claim that in the long run it is optimal to set the capital income tax is zero. A large body of work has attempted to overturn the result by relaxing the model's assumptions, or to generalize it in richer environments (see Chari and Kehoe (1999) for a survey). In a recent paper, Straub and Werning (2014) invalidate the zero tax result and show that high levels of wealth taxation may be optimal in the settings of Chamley and Judd. In a related vein, a number of authors have argued that high taxation of initial wealth is optimal in such settings and cannot easily be ruled out. For instance, Coleman (2000), points out that a uniform consumption tax implicitly imposes a pure levy on the initial stock of capital and Abel (2007) shows that the same occurs in a setup with a constant capital income tax and investment deductions. In contrast to our approach here, this body of literature does not consider the possibility that the allocation of investment is affected by financial frictions.

Lastly, our paper is related to the literature on the effect of taxation on entrepreneurial activity. For example, Meh (2005) analyzes the importance of entrepreneurship when quantifying the effects of switching from a progressive to a proportional income tax system and finds that the distributional consequences of the change in the tax regime depend a lot on the existence of entrepreneurs. Cagetti and De Nardi (2009) analyze the effect of abolishing estate taxes in a quantitative model with business investment, borrowing constraints, estate transmission, and wealth inequality. They show that eliminating estate taxes is not necessarily welfare improving. Kitao (2008) studies how changes in the capital income tax affects aggregate investment and output, and finds that the outcome depends on whether the tax targets entrepreneurial capital (business income) or non-entrepreneurial capital (capital income from risk-free savings). Unlike ours, these analyses are positive in nature and adopt a numerical approach rather than striving for analytical results. Panousi (2012) performs a similar exercise in a general equilibrium model with uninsurable idiosyncratic investment risk, and finds that a positive tax on capital yields a higher output per worker than a zero tax. Panousi and Reis (2014) use the same framework to study optimal taxation of capital. Contrary to our approach, they do not allow for agents to vary in their expected productivity of investment – thus capital taxation does not affect TFP through the allocation of capital, unlike in our setting. Guvenen et al. (2017) highlight the differences between capital income and wealth taxation in an environment where entrepreneurs are heterogeneous in their skills and find that shifting for a capital income tax system to a wealth tax system increases aggregate productivity and welfare. We complement their analysis by studying optimal capital income and wealth taxes in a related framework.

The rest of the paper is organized as follows. Section 2 outlines the assumptions of the model. Section 3 discusses properties of the equilibrium of this model. Section 4 presents the government's optimization problem and the optimal tax policy that results. Section 5 concludes.

2 Model

In this section we describe the assumptions of the model in detail.

Environment Time is discrete: $t = 0, 1, ...\infty$. There are two types of household: entrepreneurs and workers. There is a measure one of of entrepreneurs and measure N of workers. In addition, there is a continuum of competitive final goods producers, and a continuum of competitive financial intermediaries, which we refer to as banks. Entrepreneurs own capital, while workers supply labor and live hand to mouth. Entrepreneurs use their capital to produce intermediate goods, which they sell to the final goods firms. In particular, each entrepreneur is the owner of two different investment projects, which produce two different types of intermediate good: she owns a risky project, which produces 'entrepreneurial' intermediate goods denoted by y_E, and she owns a risk-free project, which produces 'standard' intermediate goods denoted by y_F. Both types of intermediate goods are sold to final goods firms.[5] Workers supply labor

[5]The device of having two separate types of intermediate goods is a simple way to allow entrepreneurs to choose between allocating capital in a risky way or risk-free way. This is designed to capture the idea that some investment projects are more risky than others and that capital owners must take into account the risks associated with different projects when making investment decisions.

to the final goods firms. The final goods firms use this labor and intermediate goods to produce a final good. The government levies (possibly negative) taxes on the agents and funds the fixed (exogenously given) level of government spending $\overline{G}$.

Timing Each period t is divided into three sub-periods: morning, afternoon and evening. In the morning, entrepreneurs buy and sell capital amongst themselves and each entrepreneur freely divides her capital between her risky and her risk-free investment projects. In the afternoon, each entrepreneur draws an idiosyncratic shock which affects the quantity of entrepreneurial intermediate goods produced by her risky project, and her two projects produce intermediate goods. Entrepreneurs sell the intermediate goods they produce to final goods firms. In the evening, the final goods firms use intermediate goods and labour to produce output of the final good, which is sold to households. Workers consume all of the final goods that they purchase, while entrepreneurs divide their final goods between consumption and investment for the next period. Existing capital depreciates at rate $\delta \in (0,1)$. At the end of the period, some agents die and new agents are born.

Technology: Entrepreneurs At the beginning of each period t, each entrepreneur i is endowed with $k_{i,t} > 0$ units of capital. In the morning, before capital is traded, each entrepreneur draws an idiosyncratic ability $\theta_{i,t}$ from the interval $(\underline{\theta}, \overline{\theta})$, which affects the productivity of her risky project. The draw of $\theta_{i,t}$ is independent across entrepreneurs and across periods and all entrepreneurs draw from an identical distribution, with cdf $G(\theta) = A_0 - \frac{A_1}{\theta}$ for $\theta \in [\underline{\theta}, \overline{\theta}]$. We assume that $\frac{\overline{\theta}}{1+\overline{\theta}} > \underline{\theta}$.[6] The constants A_0 and A_1 are set so that $G(\underline{\theta}) = 0$ and $G(\overline{\theta}) = 1$.[7] Let $g(\theta)$ denote the pdf associated with $G(\theta)$. We refer to θ as the entrepreneur's 'type'. If an entrepreneur with type $\theta_{i,t}$ allocates $k_{E,i,t}$ to her risky project in the morning of period t, then in the afternoon the risky project yields a number of units of entrepreneurial intermediate goods equal to $y_{E,i,t} = \epsilon_{i,t}\theta_{i,t}k_{E,i,t}$, where $\epsilon_{i,t}$ is an idiosyncratic productivity shock which is independent across time and across entrepreneurs. $\epsilon_{i,t}$ is distributed according to the cumulative distribution function $H(\epsilon)$, with probability distribution function $h(\epsilon)$. We assume that $h(0) > 0$, $h(\epsilon) = 0$ for all $\epsilon < 0$, $h(\cdot)$ is continuously differentiable for $\epsilon > 0$, $\mathbb{E}(\epsilon) = 1$ and, for all $x \geq 0$, $\frac{\partial^2}{\partial x^2}\left(\frac{\int_\epsilon (1+x\epsilon)^{-1} \mathrm{d}H(\epsilon)}{\int_\epsilon (1+x\epsilon)^{-1} \epsilon \mathrm{d}H(\epsilon)}\right) \geq 0$.[8]

After allocating capital to her risky project, the entrepreneur is able to allocate any remaining capital to her risk-free project. Let $k_{F,i,t}$ be the amount of capital entrepreneur

[6] This assumption guarantees that in a steady state at least some entrepreneurs do not put resources into risky projects.

[7] The distributional assumptions on G imply that the distribution of entrepreneurial ability resembles a Pareto distribution, except for the upper bound $\overline{\theta}$. That ability has an upper bound is essential in our setting, because it can readily be shown that the financial friction will cease to be relevant if there are types with sufficiently high θ.

[8] Numerically, we find that the latter assumption holds for many distributions with non-negative support: exponential, lognormal, Pareto, generalized Pareto, gamma and chi-square.

i allocates to her risk-free project. In the afternoon, her risk-free project produces an output of $y_{F,i,t} = k_{F,i,t}$ standard intermediate goods, so that each unit of capital in this project produces one unit of intermediate goods.

In addition to allocating capital to her projects and selling intermediate goods to the final goods firms, entrepreneurs are also able to hide capital and intermediate goods and convert them directly into units of consumption. In particular, at the start of each period, the entrepreneur can choose to hide any number of units of capital instead of allocating them to her projects. In the evening, each unit of hidden capital is converted directly into $\rho_K \in (0, 1-\delta)$ units of consumption. Furthermore, even if the entrepreneur allocates all her units of capital to her projects, she may also hide intermediate goods produced by her projects, instead of selling them to the final goods firms. In the evening, each unit of hidden intermediate goods is converted into $\rho_Y \in (0, \delta)$ units of consumption. It will be shown that, when taxes are set optimally, entrepreneurs will not choose to hide any units of capital or intermediate goods in equilibrium. However, the ability of entrepreneurs to hide units of capital or intermediate goods nevertheless affects allocations and optimal tax rates by creating frictions in financial markets in the economy and providing a vehicle for entreprenerus to avoid taxes, which prevents the government from setting high tax rates.

The capital held by the entrepreneur i evolves from one period to the next according to the usual law of motion:

$$k_{i,t+1} = (1-\delta)(k_{E,i,t} + k_{F,i,t}) + I_{i,t} \tag{1}$$

where $I_{i,t}$ denotes the quantity of investment by the entrepreneur at the end of the period.

Technology: Final Goods Firms We assume that the final goods producer buys entrepreneurial intermediate goods from the entrepreneur at price $r_{E,t}$ per unit, and buys standard intermediate goods from the entrepreneur at price $r_{F,t}$ per unit. Since an entrepreneur's output of standard intermediate goods is $y_{F,i,t} = k_{F,i,t}$, a consequence of this is that $r_{F,t}$ is the market rate of return to capital in risk-free projects.

The final goods producer also hires workers at wage rate w_t. The representative final goods producer produces final output according to the production function:

$$Y_t = F\left(Y^S_{E,t}, Y^S_{F,t}, N\right)$$

where Y_t is the aggregate final output, N is aggregate labor, $Y^S_{E,t}$ is aggregate input of the entrepreneurial intermediate good and $Y^S_{F,t}$ is aggregate input of the standard intermediate good. We assume that F is concave and strictly increasing in all arguments, provided the quantity of each factor is strictly positive, and that for each factor $i \in \{N, Y^S_E, Y^S_F\}$: $\lim_{i\to 0} F'_i = \infty$ and $F = 0$ at $i = 0$. Furthermore, we assume that $\lim_{N\to\infty} F'_N = 0$. However, in contrast to the usual Inada conditions, we assume that $\delta > \lim_{Y_E^S \to \infty} F'_{Y_E^S} > \rho_Y > 0$ and $\delta > \lim_{Y_F^S \to \infty} F'_{Y_F^S} > \rho_Y$, provided that $F > 0$.[9]

Demographics At the end of each period fraction $\gamma \in (0,1)$ of entrepreneurs and workers die. They are replaced by an equal measure of newborn entrepreneurs and newborn workers. An agent's occupation as entrepreneur or worker is fixed exogenously from birth and never changes. The capital of dead entrepreneurs is redistributed equally between newborn entrepreneurs, so that each newborn entrepreneur i starts period t with capital $k_{i,t} = K_t$, where K_t is the aggregate capital stock at the start of period t, which is also equal to the average capital per entrepreneur. Workers are born with no wealth and live hand-to-mouth every period.[10]

Preferences Each worker has a constant labor endowment $n = 1$ which he supplies inelastically. Entrepreneurs do not work. The consumption of an entrepreneur i is denoted $c_{i,t}$ and the consumption of a worker is denoted $c_{N,t}$. Workers are identical and so each will have the same consumption. A worker born in period t values his future consumption stream according to: $\sum_{j=0}^{\infty} \beta^j (1-\gamma)^j u(c_{N,t+j})$, where u is some strictly increasing function. An entrepreneur born in period t values her future consumption stream according to $\sum_{j=0}^{\infty} \beta^j (1-\gamma)^j U(c_{i,t+j})$ and we assume that the entrepreneur's utility function satisfies $U(c) \equiv \log(c)$.

Government The government sets four different types of tax: a consumption tax $\tau_{C,t}$, a labor income tax $\tau_{N,t}$, a capital income tax $\tau_{K,t}$ and a wealth tax $\tau_{W,t}$. Any of these tax rates can be negative. The government has to finance exogenous expenditure $\overline{G}$, and must balance its budget every period. Taxes are paid in the evening and government spending also takes place in the evening. The government is not allowed to trade in financial assets at any time. The government's budget constraint each period is:

$$\overline{G} = \tau_{N,t} w_t N + \tau_{K,t}(r_{E,t} Y_{E,t}^S + r_{F,t} Y_{F,t}^S - \delta K_t) + \tau_{W,t}(K_t - K_{H,t}) + \tau_{C,t}(C_t - C_{H,t}) \tag{2}$$

It is instructive to examine the right hand side term by term. The labor income tax revenue is simply $\tau_{N,t} w_t N$. The total gross capital income earned by entrepreneurs each period is equal to the total revenue they earn by selling intermediate goods: $r_{E,t} Y_{E,t}^S + r_{F,t} Y_{F,t}^S$. The government taxes their capital income at rate $\tau_{K,t}$ after deducting for depreciation. The total private wealth of the economy at the start of each period is the total

[9]This assumption is to guarantee that entrepreneurs do not hide intermediate goods in the first-best allocation.

[10]Provided that workers are born with no wealth, the assumption that they live hand-to-mouth is not critical for the formal results. If we instead allowed workers to hold risk-free bonds earning the risk-free interest rate, they would always choose to hold zero wealth in the steady state, because the steady state risk-free rate is lower than the discount rate $\frac{1}{\beta(1-\gamma)} - 1$. Therefore, all the steady state results derived below under the assumption that workers live hand-to-mouth would carry through to this alternative case, and steady state optimal taxes would be unchanged.

capital held by entrepreneurs: K_t. The government taxes this at rate $\tau_{W,t}$. At the same time, it is assumed that capital and intermediate goods that are hidden by entrepreneurs cannot be taxed by the government. The consequence is that government revenue from the wealth tax is proportional to aggregate capital K_t, net of aggregate capital hidden by the entreprneurs $K_{H,t}$. Finally, all agents pay the tax $\tau_{C,t}$ in proportion to their consumption. As such, government revenue from the consumption tax is equal to aggregate consumption C_t, net of the consumption $C_{H,t}$ which is generated by entrepreneurs from hiding capital and intermediate goods, since this consumption cannot be taxed. We show below that $C_{H,t}$ is always zero in the steady state if the tax policy is set optimally.

Financial Markets Entrepreneurs may fund capital purchases each morning by writing one-period state-contingent financial contracts with banks. We assume that banks are risk neutral and perfectly competitive and live for only one period each, so they have no interest in multi-period financial contracts. New banks are created at the start of each period. The financial market opens immediately after each entrepreneur's ability $\theta_{i,t}$ is revealed. If she writes a financial contract with the bank, the entrepreneur receives from the bank some quantity $b_{i,t}$ (possibly negative) in the morning and in exchange she agrees to return to the bank the quantity $\hat{b}_{i,t}$ (possibly negative) at the end of the period, where $\hat{b}_{i,t}$ may depend on the realization of the entrepreneur's shock $\epsilon_{i,t}$. In this way, financial contracts function as a within-period loan for entrepreneurs, and entrepreneurs can also use them to insure themselves against the idiosyncratic risk associated with the shock $\epsilon_{i,t}$. We refer to an entrepreneur as a borrower if she chooses $b_{i,t} > 0$ and a saver if she chooses $b_{i,t} < 0$.

It is convenient to write the entrepreneur's choices of $b_{i,t}$ and $\hat{b}_{i,t}$ as policy functions of the relevant variables. In general, an entrepreneur's choice of $b_{i,t}$ will depend on her ability $\theta_{i,t}$, her start of period capital $k_{i,t}$ and the aggregate state of the economy, which we label X_t. Therefore, abusing notation slightly, we write $b_{i,t} \equiv b(\theta_{i,t}, k_{i,t}, X_t)$. Likewise, we write $\hat{b}_{i,t} \equiv \hat{b}(\theta_{i,t}, k_{i,t}, \epsilon_{i,t}, X_t)$, since $\hat{b}_{i,t}$ will depend on the variables $\theta_{i,t}, k_{i,t}, X_t$, as well as the shock $\epsilon_{i,t}$.

Since banks are risk-neutral, perfectly competitive and profit maximizing, a bank will agree to a financial contract written by an entrepreneur if and only if the financial contract delivers it non-negative profits in expectation at the end of the period. As such, banks will only lend to entrepreneurs in the morning if the expected return on the loan in the evening is equal to the market risk-free rate. This implies the following constraint:

$$\int_{\epsilon} \hat{b}\left(k_{i,t}, \theta_{i,t}, \epsilon, X_t\right) \mathrm{d}H\left(\epsilon\right) \geq R_{F,t} b\left(k_{i,t}, \theta_{i,t}, X_t\right) \tag{3}$$

where $R_{F,t}$ denotes the gross market risk-free rate of interest. Since entrepreneurs have no desire to pay the banks more than is necessary, the inequality (3) will be satisfied with equality. The consequence is that banks make exactly zero profits in equilibrium.[11]

Budget Constraints Workers live hand to mouth. Thus, the consumption of each worker satisfies:

$$c_{N,t}(1+\tau_{C,t}) = w_t(1-\tau_{N,t}) \quad (4)$$

In the morning of each period the entrepreneur may buy and sell capital, divide her capital between a risky and risk-free project, hide some capital and write a financial contract with a bank. Her choices in the morning must satisfy the budget constraint:

$$k_{E,i,t}+k_{F,i,t}+k_{H,i,t} = k_{i,t}+b_{i,t} \quad (5)$$

where $k_{H,i,t}$ denotes the quantity of capital the entrepreneur hides. After receiving the $\epsilon_{i,t}$ shock, the entrepreneur chooses how many (if any units) of intermediate goods to hide. We let $y_{EH,i,t}$ and $y_{FH,i,t}$ denote, respectively the quantity of entrepreneurial and standard intermediate goods that she hides. In the evening, the entrepreneur chooses how much to consume, which we denote by $c_{i,t}$, and how much to invest, $I_{i,t}$. Hidden intermediate goods and hidden capital are transformed into $c_{H,i,t}$ units of consumption. Finally, in the evening the entrepreneur repays the bank $\hat{b}_{i,t}$ (or is paid by the bank if $\hat{b}_{i,t} < 0$) and pays her taxes to the government. Consequently, in the evening the entrepreneur's budget constraint is as follows.

$$\begin{aligned}(1+\tau_{C,t})(c_{i,t}-c_{H,i,t})+I_{i,t}+\hat{b}_{i,t} &= r_{E,t}(y_{E,i,t}-y_{EH,i,t})+r_{F,i,t}(y_{F,i,t}-y_{FH,i,t}) \\ &- \tau_{K,t}(r_{E,t}(y_{E,i,t}-y_{EH,i,t})+r_{F,i,t}(y_{F,i,t}-y_{FH,i,t})) \\ &+ \tau_{K,t}(\delta k_{E,i,t}+\delta k_{F,i,t})-\tau_{W,t}(k_{E,i,t}+k_{F,i,t}) \quad (6)\end{aligned}$$

where $c_{H,i,t}$ and $y_{E,i,t}$ and $y_{F,i,t}$ satisfy:

$$\begin{aligned}c_{H,i,t} &= \rho_K k_{H,i,t}+\rho_Y(y_{EH,i,t}+y_{FH,i,t}) \quad (7)\\ y_{E,t} &= \theta_{i,t}\epsilon_{i,t}k_{E,i,t} \quad (8)\\ y_{F,i,t} &= k_{F,i,t} \quad (9)\end{aligned}$$

It is convenient to define the entrepreneur's end-of-period resources $\omega_{i,t}$ as:

$$\begin{aligned}(1+\tau_{C,t})\omega_{i,t} &= (1+\tau_{C,t})c_{H,i,t}+(1-\delta(1-\tau_{K,t})-\tau_{W,t})(k_{E,i,t}+k_{F,i,t}) \\ &+ (1-\tau_{K,t})(r_{E,t}(y_{E,i,t}-y_{EH,i,t})+r_{F,i,t}(y_{F,i,t}-y_{FH,i,t}))-\hat{b}_{i,t} \quad (10)\end{aligned}$$

Then, using the capital accumulation equation (1), the entrepreneur's evening budget

[11] Since banks make zero profits, it makes no difference to the equilibrium behavior of the economy who owns the banks. We may assume that they are owned either by workers or by entrepreneurs.

constraint can be re-written as:

$$c_{i,t} + \frac{k_{i,t,+1}}{1+\tau_{C,t}} = \omega_{i,t} \tag{11}$$

We can write the entrepreneur's decisions as policy functions of the relevant variables, as was done in the previous section for $b_{i,t}$ and $\hat{b}_{i,t}$. That is, we define the functions: $k_E(k_{i,t}, \theta_{i,t}, X_t) \equiv k_{E,i,t}$; $k_F(k_{i,t}, \theta_{i,t}, X_t) \equiv k_{F,i,t}$; $k_H(k_{i,t}, \theta_{i,t}, X_t) \equiv k_{H,i,t}$; $c(k_{i,t}, \theta_{i,t}, \epsilon_{i,t}, X_t) \equiv c_{i,t}$; $c_H(k_{i,t}, \theta_{i,t}, \epsilon_{i,t}, X_t) \equiv c_{H,i,t}$; $I(k_{i,t}, \theta_{i,t}, \epsilon_{i,t}, X_t) \equiv I_{i,t}$; $y_E(k_{i,t}, \theta_{i,t}, \epsilon_{i,t}, X_t) \equiv y_{E,i,t}$; $y_F(k_{i,t}, \theta_{i,t}, \epsilon_{i,t}, X_t) \equiv y_{F,i,t}$; $y_{EH}(k_{i,t}, \theta_{i,t}, \epsilon_{i,t}, X_t) \equiv y_{EH,i,t}$; $y_{FH}(k_{i,t}, \theta_{i,t}, \epsilon_{i,t}, X_t) \equiv y_{FH,i,t}$; $\omega(k_{i,t}, \theta_{i,t}, \epsilon_{i,t}, X_t) \equiv \omega_{i,t}$. Furthermore we let $k'(k_{i,t}, \theta_{i,t}, \epsilon_{i,t}, X_t)$ denote the choice of $k_{i,t+1}$.

Agency Friction During the period, an entrepreneur's realization of ϵ, her output of intermediate goods, the quantity of intermediate goods she hides, her end-of-period investment and the consumption she obtains from converting hidden capital and hidden intermediate goods are all private information. In particular, after observing the shock ϵ, an entrepreneur can choose to honestly report her output of intermediate goods, but she can also lie by under-reporting the quantity of intermediate goods she produces and hiding more intermediate goods than she admits to. However, the quantity of capital allocated to the entrepreneur's projects, and the quantity of intermediate goods she sells to the final goods producer are assumed to be public information.[12]

When an entrepreneur writes a financial contract in the morning, the market will expect the entrepreneur to repay $\hat{b}(k_{i,t}, \theta_{i,t}, \epsilon, X_t)$ in the evening, given her realization of ϵ. In equilibrium, the market must be correct in expecting this, and so the entrepreneur must have an incentive to repay this amount, rather than lying about ϵ and repaying too little. Therefore, it is without loss of generality to restrict attention to contracts where the entrepreneur honestly reports her ϵ, and pays the promised amount $\hat{b}(k_{i,t}, \theta_{i,t}, \epsilon, X_t)$. Such a contract is only incentive compatible if it is optimal for the entrepreneur to report ϵ honestly, rather than lying by reporting some $\hat{\epsilon} \neq \epsilon$ and hiding more (or fewer) intermediate goods than she admits to. The entrepreneur will be tempted to lie about ϵ only if doing so increases her available resources for consumption and/or her next period capital, which will be true if and only if by lying she is able to increase her end of period $\omega_{i,t}$. This gives rise to the following incentive compatibility constraint:

$$\omega(k, \theta, \epsilon, X) \geq \omega(k, \theta, \hat{\epsilon}, X) + \rho_Y \cdot k_E(k, \theta, X) \cdot \theta(\epsilon - \hat{\epsilon}) \tag{12}$$

for any ϵ and $\hat{\epsilon} > 0$ satisfying:

$$y_E(k, \theta, \hat{\epsilon}, X) - y_{E,H}(k, \theta, \hat{\epsilon}, X) \leq y_E(k, \theta, \epsilon, X) \tag{13}$$

[12] In the extreme case $\rho_Y = \rho_K = 0$ there would be no informational friction, since the entrepreneur has no incentive to hide capital or intermediate goods.

This constraint arises from the fact that an entrepreneur who reports $\hat{\epsilon} \neq \epsilon$ will find herself with $k_E(k, \theta, X)\theta(\epsilon - \hat{\epsilon})$ more entrepreneurial intermediate goods than she claimed to have, which she can hide and then transform into $\rho_Y k_E(k, \theta, X)\theta(\epsilon - \hat{\epsilon})$ units of consumption. In that case her end of period cash on hand will be the amount $\omega(k, \theta, \hat{\epsilon}, X)$ prescribed by the contract if she drew $\hat{\epsilon}$, plus the value of the extra consumption she produces from the hidden entrepreneurial intermediate goods. For truth telling to be optimal, this must be less than the end of period cash on hand she would obtain from reporting truthfully, $\omega(k, \theta, \epsilon, X)$. This incentive compatibility constraint need only be satisfied for $\hat{\epsilon}$ consistent with equation (13) because the entrepreneur cannot convincingly claim to have an $\hat{\epsilon}$ so high that the contract would mandate her delivering more intermediate goods to the final goods producer than she has actually produced. She cannot do this because sales to the final goods producer are publicly observed.

Entrepreneur's Optimization Problem We write the entrepreneur's optimization problem recursively. Let $V(k, \theta, X)$ denote the continuation value of the entrepreneur who starts the period with capital k and draws ability θ, when the aggregate state is X. Then, the entrepreneur's optimization problem is to choose functions $k_E(\cdot)$, $k_F(\cdot)$, $k_H(\cdot)$, $b(\cdot)$, $\hat{b}(\cdot)$, $c(\cdot)$, $c_H(\cdot)$, $I(\cdot)$, $y_E(\cdot)$, $y_F(\cdot)$, $y_{EH}(\cdot)$, $y_{FH}(\cdot)$, $\omega(\cdot)$ and $k'(\cdot)$ to solve:

$$V(k, \theta, X) = \sup \int_{\epsilon > 0} \left(\log(c(k, \theta, \epsilon, X)) + \beta(1-\gamma)E\left[V(k'(k, \theta, \epsilon, X), \theta', X') \middle| \epsilon \right] \right) \mathrm{d}H(\epsilon), \tag{14}$$

subject to: the budget constraints (5) and (6); the law of motion for capital (1); the production functions for c_H in (7), y_E in (8) and y_F in (9); the definition of ω in (10); the incentive compatibility constraint (12); the break-even condition for the banks (3), and the non-negativity conditions $k_E(\cdot) \geq 0$, $k_F(\cdot) \geq 0$, $k_H(\cdot) \geq 0$, $c(\cdot) \geq 0$, $c_H(\cdot) \geq 0$, $I(\cdot) \geq 0$, $y_E(\cdot) \geq y_{EH}(\cdot) \geq 0$, $y_F(\cdot) \geq y_{FH}(\cdot) \geq 0$, $\omega(\cdot) \geq 0$ and $k'(\cdot) \geq 0$.[13]

Here, by having the entrepreneur choose the functions $b(\cdot)$ and $\hat{b}(\cdot)$ subject to the incentive compatibility constraint and break-even condition for the bank, we are assuming that the entrepreneur designs a financial contract and proposes it to a bank. The bank accepts provided that the contract is incentive compatible and the bank breaks even in expectation.

Aggregation and Market Clearing The aggregate level of consumption C_t and of $C_{H,t}$

[13]We do not impose the constraint $c_H(\cdot) \leq c(\cdot)$ for the entrepreneur, although we do impose $C_{H,t} \leq C_t$ in the aggregate in equation (16). To not impose $c_H(\cdot) \leq c(\cdot)$ is justified if the entrepreneur is able to covertly sell the consumption goods generated from hidden capital and intermediate goods to other entrepreneurs and/or workers, which would allow the entrepreneur to choose $c_H(\cdot) > c(\cdot)$. Regardless, we conjecture that this assumption is of no consequence in practice, because the constraint $c_H(\cdot) \leq c(\cdot)$ would not bind at realistic parameter values even if it were imposed.

satisfy:

$$C_t = Nc_{N,t} + \int_i c_{i,t} \mathrm{d}i \tag{15}$$

$$C_{H,t} = \int_i c_{H,i,t} \mathrm{d}i \leq C_t \tag{16}$$

The aggregate levels of capital devoted to each use satisfy:

$$K_t = \int_i k_{i,t} \mathrm{d}i \tag{17}$$

$$K_{E,t} = \int_i k_{E,i,t} \mathrm{d}i \tag{18}$$

$$K_{F,t} = \int_i k_{F,i,t} \mathrm{d}i \tag{19}$$

$$K_{H,t} = \int_i k_{H,i,t} \mathrm{d}i \tag{20}$$

In each period, the asset market must clear. This requires that the net amount banks lend to entrepreneurs must equal zero:

$$\int_i b_{i,t} \mathrm{d}i = 0 \tag{21}$$

The market for intermediate goods of each type must clear each period:

$$Y^S_{E,t} = \int_i (y_{E,i,t} - y_{EH,i,t}) \mathrm{d}i \tag{22}$$

$$Y^S_{F,t} = \int_i (y_{F,i,t} - y_{FH,i,t}) \mathrm{d}i \tag{23}$$

The first order conditions of the representative final goods producer imply that the (before tax) returns on capital and wage rate are given by:

$$r_{E,t} = F_1 \left(Y^S_{E,t}, Y^S_{F,t}, N \right) \tag{24}$$

$$r_{F,t} = F_2 \left(Y^S_{E,t}, Y^S_{F,t}, N \right) \tag{25}$$

$$w_t = F_3 \left(Y^S_{E,t}, Y^S_{F,t}, N \right) \tag{26}$$

The final goods market clearing condition then follows by Walras' law:[14]

$$C_t + K_{t+1} + \overline{G} = F \left(Y^S_{E,t}, Y^F_{E,t}, N \right) + (1-\delta) \left(K_{E,t} + K_{F,t} \right) + C_{H,t} \tag{27}$$

[14] In particular, the goods market clearing condition can be obtained by summing the budget constraints of workers, entrepreneurs and government, substituting the other market clearing conditions, aggregation conditions and first order conditions of the final goods firm and using that F displays constant returns to scale.

Equilibrium We define an equilibrium as follows:

Definition 1. *For a given sequence of tax rates* $\{\tau_{W,t}, \tau_{K,t}, \tau_{C,t}, \tau_{N,t}\}_{t=0}^{\infty}$, *an* ***equilibrium*** $\mathcal{E}$ *of this economy is a sequence of prices* $\{r_{E,t}, r_{F,t}, w_t\}_{t=0}^{\infty}$, *policy functions giving entrepreneurs' decisions, a sequence of worker consumption* $\{c_{N,t}\}_{t=0}^{\infty}$, *and a sequence of aggregate variables* $\{C_t, C_{H,t}, K_t, K_{E,t}, K_{F,t}, K_{H,t}, Y_t, Y_{E,t}^S, Y_{F,t}^S\}_{t=0}^{\infty}$ *such that:*

1. *The Government's budget constraint* (2) *is balanced every period.*[15]

2. *Worker consumption satisfies* (4).

3. *Entrepreneurs' decision rules are given by the solution to the entrepreneur's problem* (14).

4. $\{C_t, C_{H,t}, K_t, K_{E,t}, K_{F,t}, K_{H,t}\}_{t=0}^{\infty}$ *represent the aggregate of agents' decisions given by equations* (15)-(20).

5. *The asset market clears, according to equation* (21).

6. *The markets for intermediate goods clear, according to equations* (22) *and .*

7. *Prices of intermediate goods* $r_{E,t}$ *and* $r_{F,t}$, *and wages* w_t *are determined by the first order conditions of the final goods firms* (24)-(26).

3 Properties of the Model Equilibrium

In this section, we characterize the optimal decisions of entrepreneurs in the model and derive comparative statics for how entrepreneurs' choices change in response to changes in taxes and prices. We also characterize an aggregate steady state and derive comparative static results for how aggregate steady state variables change in response to changes in taxes.

3.1 Entrepreneur's Optimal Decisions

We now solve the entrepreneur's optimization problem in (14). To simplify the problem, note first that all the entrepreneur ultimately cares about this period is her level of consumption c and her level of capital for the next period, k'. These are the only variables over which she has influence that enter directly into the entrepreneur's Bellman equation in (14). Now, the level of c and k' that the entrepreneur can afford at the end of the period depend solely on her total resources ω at the end of the period, according to the equation

[15] Naturally only some sequences of tax rates $\{\tau_{W,t}, \tau_{E,t}, \tau_{F,t}, \tau_{N,t}\}_{t=0}^{\infty}$ will be consistent with this condition and, therefore, with existence of an equilibrium.

(11). That is, her within-period choices of how much capital to put into each project, how much to borrow and how much capital and intermediate goods to hide only impact on the level of c and k' she can afford insofar as they affect the ω she will have at the end of the period. Therefore, we can split the entrepreneur's problem into a within-period choice of trying to achieve a high value of ω, and a between period choice of how to divide her resources ω between consumption and next-period capital. To this end, let $\tilde{V}(\omega, X)$ denote the value in the evening of a period of an entrepreneur with resources ω, who is yet to divide her resources between consumption and next period capital. Then, we can write the entrepreneur's between period problem recursively as:

$$\tilde{V}(\omega, X) = \sup_{c \geq 0; k' \geq 0} \left(\log(c) + \beta(1-\gamma) E\left[V(k', \theta', X') \right] \right), \tag{28}$$

$$\text{s.t.} \quad c + \frac{k'}{1+\tau_C} = \omega. \tag{29}$$

Writing the entrepreneur's within period problem recursively, it is to choose non-negative functions $k_E(\cdot)$, $k_F(\cdot)$, $k_H(\cdot)$, $y_{EH}(\cdot)$, $y_{FH}(\cdot)$, $\omega(\cdot)$ and functions $b(\cdot)$ and $\hat{b}(\cdot)$ to solve:

$$V(k, \theta, X) = \sup \int_{\epsilon > 0} \tilde{V}(\omega(k, \theta, \epsilon, X), X) \mathrm{d}H(\epsilon), \tag{30}$$

subject to the constraints:

$$\begin{aligned}
k_E(k,\theta,X) + k_F(k,\theta,X) + k_H(k,\theta,X) &= k + b(k,\theta,X) && (31)\\
\int_\epsilon \hat{b}\left(k,\theta,\epsilon,X\right) \mathrm{d}H\left(\epsilon\right) &= R_F b\left(k,\theta,X\right) && (32)\\
(1+\tau_C)\omega &= (1+\tau_C)[\rho_K k_H + \rho_Y(y_{EH}+y_{FH})] - \hat{b} \\
&+ (1-\tau_K)[r_E(\theta\epsilon k_E - y_{EH}) + r_F(k_F - y_{FH})] \\
&+ [1-\delta(1-\tau_K)-\tau_W](k_E+k_F) && (33)\\
\theta\epsilon k_E(k,\theta,X) &\geq y_{EH}(k,\theta,\epsilon,X) && (34)\\
k_F(k,\theta,X) &\geq y_{FH}(k,\theta,\epsilon,X), && (35)\\
\text{and} \qquad \omega(k,\theta,\epsilon,X) &\geq \omega(k,\theta,\hat{\epsilon},X) + \rho_Y \theta(\epsilon-\hat{\epsilon})k_E(k,\theta,X), && (36)\\
\text{for any } \epsilon \text{ and } \hat{\epsilon} > 0 \text{ satisfying:} & \\
\theta\hat{\epsilon}k_E(k,\theta,X) - y_{E,H}(k,\theta,\hat{\epsilon},X) &\leq \theta\epsilon k_E(k,\theta,X) && (37)
\end{aligned}$$

Here, for compactness, we suppressed the arguments of the functions in equation (33).

The constraints (31), (32), (33) and (36) are simply the equations (3), (5), (10) and (12), where we substituted in the equations (7), (8) and(9) to eliminate C_H, y_E and y_F.

The constant returns to scale assumptions on the entrepreneur's technology for producing intermediate goods means that the value function must take a particular form, as shown in the following lemma. This considerably simplifies the solution to the entrepreneur's problem.

Lemma 1. *There exists a function $\overline{V}(X)$ such that, for any k, and X,*

$$E\left[V(k,\theta,X)\middle|k,X\right] = \overline{V}(X) + \frac{\log(k)}{1-\beta(1-\gamma)}.$$

Proof. See Appendix A. □

Using Lemma 1, the solution to the between period problem can be found immediately by taking the first order condition:

$$\frac{1}{c} = \left(\frac{\beta(1-\gamma)}{1-\beta(1-\gamma)}\right)\frac{1}{\omega - c}$$

Rearranging and combining this with equation (29) we conclude that the entrepreneur chooses:

$$c = (1-\beta(1-\gamma))\omega \tag{38}$$

$$k' = (1+\tau_C)\beta(1-\gamma)\omega \tag{39}$$

Substituting these choices into the Bellman equation (28), we have that

$$\tilde{V}(\omega,X) = \frac{\log(\omega)}{1-\beta(1-\gamma)} + \log(1-\beta(1-\gamma)) + \frac{\beta(1-\gamma)\log((1+\tau_C)\beta(1-\gamma))}{1-\beta(1-\gamma)} + E[\overline{V}(X')] \tag{40}$$

This completes the solution of the between period problem. To solve the within period problem, we first take the first order approach to the incentive compatibility constraint.[16] In particular, we can replace the equation (36) with the associated first order condition for the choice of $\hat{\epsilon}$:

$$\frac{\partial\omega(k,\theta,\hat{\epsilon},X)}{\partial\hat{\epsilon}} - \rho_Y\theta k_E(k,\theta,X) = 0$$

Evaluating this at the honest report $\hat{\epsilon} = \epsilon$ and integrating with respect to ϵ, it follows that there must exist some function $\underline{\omega}(\cdot)$ such that:

$$\omega(k,\theta,\epsilon,X) \equiv \underline{\omega}(k,\theta,X) + \rho_Y\theta k_E(k,\theta,X)\epsilon \tag{41}$$

Thus, the entrepreneur's end-of-period resources ω must linearly increase with ϵ, so that it is optimal for her to report her ϵ honestly, rather than under-reporting ϵ, hiding some additional intermediate goods, and converting them into units of consumption. Note

[16] A proof that the first order approach provides the correct solution to this problem is available upon request.

that the more an entrepreneur's end-of-period resources are sensitive to ϵ, the more risk the entrepreneur faces. Since the entrepreneur has log utility, she is risk averse. On the other hand, the bank is risk neutral. Therefore, in the absence of agency frictions, the entrepreneur and bank would prefer a contract in which the bank took all the risk and the entrepreneur's ω was independent of ϵ. The agency friction prevents this, leading the entrepreneur to face the level of risk implied by the equation (41).

To simplify the within period problem further, note that during the period t the entrepreneur can do four different activities each of which yields a riskless return to her capital. First, she can sell capital and lend to a bank, setting $b < 0$. The bank's break even condition (32) implies that the bank would be willing to borrow from her and pay back the risk-free gross interest rate of R_F at the end of the period, for each unit lent to the bank. Second, the entrepreneur can allocate capital to her risk-free project and sell the output of this to the final goods firms. Third, the entrepreneur can allocate a unit of capital to her risk-free project, and then hide the standard intermediate goods that result. Fourth, the entrepreneur can hide her capital at the start of the period. Lemma 2 below establishes that, for the asset market and market for standard intermediate goods to be in equilibrium, the entrepreneur's return from lending to the bank must be identical to the return she obtains from allocating capital to her risk-free project and must be the same or greater than the return from hiding standard intermediate goods or hiding capital. The consequence is that the entrepreneur will weakly prefer to lend to the bank over all other risk-free activities.

Lemma 2. *In an equilibrium of the economy, it must hold every period that*

$$\begin{aligned} R_F &= (1-\tau_K)r_F + 1 - \delta(1-\tau_K) - \tau_W \\ &\geq \max\left\{\rho_Y(1+\tau_C) + 1 - \delta(1-\tau_K) - \tau_W; \rho_K(1+\tau_C)\right\}. \end{aligned} \tag{42}$$

In equilibrium, all entrepreneurs are indifferent over their choices of the level of k_F. If the inequality in (42) is strict, all entrepreneurs strictly prefer to set $y_{FH} = 0$ and $k_H = 0$. If the inequality in (42) holds with equality, entrepreneurs will either be indifferent over the level they choose of k_H or indifferent over the level they choose of y_{FH}.

Proof. See Appendix B. □

Equation (42) establishes that in a equilibrium of the economy each entrepreneur is indifferent between allocating capital to her risk-free project (and selling the intermediate goods that result) and lending to a bank. If ρ_K or ρ_Y are sufficiently high, an entrepreneur may also be indifferent between these and hiding capital or intermediate goods produced by the risk-free project. On the other hand, if the inequality in (B.2) is strict, the entrepreneur will choose to hide no capital and hide no intermediate goods produced by her risk-free project.

Given that the entrepreneur weakly prefers to lend to the bank than the other risk-free activities, we can solve her optimization problem under the assumption that she chooses $k_F = y_{FH} = k_H = 0$, with the understanding that in equilibrium some entrepreneurs may borrow from banks to engage in these other risk-free activities, to the extent needed to clear markets. Combining the rewritten incentive compatibility constraint (41) with the definition of ω in (33), and integrating with respect to ϵ using that $k_F = y_{FH} = k_H = 0$ and $E[\epsilon] = 1$, reveals that $\underline{\omega}(\cdot)$ must satisfy:

$$\begin{aligned}(1+\tau_C)(\underline{\omega}+\rho_Y\theta k_E) &= [(1+\tau_C)\rho_Y-(1-\tau_K)r_E]\int_\epsilon y_{EH}\mathrm{d}H(\epsilon)+(1-\tau_K)r_E\theta k_E \\ &+ [1-\delta(1-\tau_K)-\tau_W]k_E-\int_\epsilon \hat{b}(k,\theta,\epsilon,X)]\mathrm{d}H(\epsilon)\end{aligned}$$

Combining this with the bank zero profit condition (32) and the budget constraint (31) and rearranging gives:

$$\begin{aligned}(1+\tau_C)\underline{\omega} &= R_F(k-k_E)+[(1+\tau_C)\rho_Y-(1-\tau_K)r_E]\int_\epsilon y_{EH}\mathrm{d}H(\epsilon) \\ &+ [1-\delta(1-\tau_K)-\tau_W+(1-\tau_K)r_E\theta-\rho_Y(1+\tau_C)\theta]k_E \qquad (43)\end{aligned}$$

Setting $k_F = y_{FH} = k_H = 0$ and using equations (40), (43) and (41), we can rewrite the entrepreneur's within period problem more compactly. The entrepreneur seeks to choose non-negative functions $k_E(\cdot)$, $y_{EH}(\cdot)$ and $\underline{\omega}(\cdot)$ to solve:

$$\sup\int_{\epsilon>0}\left(\frac{\log(\underline{\omega}+\rho_Y\theta\epsilon k_E)}{1-\beta(1-\gamma)}\right)\mathrm{d}H(\epsilon), \qquad (44)$$

subject to the constraints:

$$\begin{aligned}(1+\tau_C)\underline{\omega} &= R_Fk+[(1+\tau_C)\rho_Y-(1-\tau_K)r_E]\int_\epsilon y_{EH}\mathrm{d}H(\epsilon) \\ &+ [1-\delta(1-\tau_K)-\tau_W+(1-\tau_K)r_E\theta-\rho_Y(1+\tau_C)\theta-R_F]k_E \qquad (45) \\ \theta k_E &\geq \int_\epsilon y_{EH}\mathrm{d}H(\epsilon) \qquad (46)\end{aligned}$$

Here, we used that the only part of $\tilde{V}$ in (40) which depends on the entrepreneur's decisions is the term $\frac{\log(\omega)}{1-\beta(1-\gamma)}$. Therefore, maximizing the expected value of $\tilde{V}$ amounts to maximizing the expected value of this term.

Now, we note that, in an equilibrium of this economy, it must be that the price of entrepreneurial intermediate goods is sufficiently high that entrepreneurs weakly prefer to sell these goods rather than hide them, but the price of entrepreneurial intermediate goods must also be sufficiently low that entrepreneurs do not wish to produce and sell infinite quantities of entrepreneurial intermediate goods. These conditions are contained in the following lemma.

Lemma 3. *In an equilibrium of the economy, it must be the case every period that:*

$$(1+\tau_C)\rho_Y - (1-\tau_K)r_E \leq 0 \tag{47}$$

$$1-\delta(1-\tau_K) - \tau_W + (1-\tau_K)\overline{\theta} r_E - \rho_Y(1+\tau_C)\overline{\theta} - R_F \leq 0 \tag{48}$$

If the inequality in (47) is strict, then all entrepreneurs choose $y_{EH} = 0$. If the inequality in (47) holds with equality, then entrepreneurs are indifferent about their choice of y_{EH}.

Proof. See Appendix C □

Lemma 3 establises that in equilibrium all entrepreneurs weakly prefer to choose $y_{EH} = 0$. Then, we can solve the entrepreneur's problem on the assumption that the entrepreneur chooses $y_{EH} = 0$, subject to the proviso that if the inequality in equation (47) is an equality, the entrepreneur will be happy to choose any level of $y_{EH} = 0$ that is feasible for her and consistent with market clearing. Inspecting the entrepreneur's optimization problem (44), it remains only to determine the entrepreneur's optimal choice of k_E. This is simply a problem of a tradeoff between risk and return. Choosing higher k_E increases the variance of ω, since $\omega = \underline{\omega} + \rho_Y \theta \epsilon k_E$ and decreases the value of $\underline{\omega}$ but higher k_E may carry a higher expected return. The optimal choice of k_E can be found by taking the first order condition for k_E in the problem (44), and rearranging.

The optimal decision rule that results is summarized in Proposition 1 below. The decision rule can be written more neatly in terms of after-tax prices and a function $S : \mathbf{R}_{++} \to (1, \infty)$, which we define as follows:

$$\tilde{r}_F \equiv \frac{r_F\,(1-\tau_K)}{1+\tau_C} \tag{49}$$

$$\tilde{r}_E \equiv \frac{r_E\,(1-\tau_K)}{1+\tau_C} \tag{50}$$

$$\tilde{p} \equiv \frac{1-\delta(1-\tau_K)-\tau_W}{1+\tau_C} = \frac{R_F}{1+\tau_C} - \tilde{r}_F \tag{51}$$

$$S(x) \equiv \frac{\int_{\epsilon>0}(1+x\epsilon)^{-1}\mathrm{d}H\,(\epsilon)\,\epsilon}{\int_{\epsilon>0}\epsilon(1+x\epsilon)^{-1}\mathrm{d}H\,(\epsilon)\,\epsilon} \tag{52}$$

Here the second equality in equation (51) follows from equation (42).

We let $S^{-1}(\cdot)$ denote the inverse of $S(\cdot)$. The following lemma summarizes some useful properties of $S(\cdot)$ and $S^{-1}(\cdot)$.

Lemma 4. *$S(\cdot)$ is differentiable and monotonically increasing. $\frac{y}{S^{-1}(y)}$ is decreasing in y. Let $\mathscr{F}(y) \equiv \frac{1}{1+\frac{y}{S^{-1}(y)}}$. Then $\mathscr{F}(\cdot)$ is increasing and concave.*

Proof. Omitted. Available upon request. □

Using these definitions and equation (45), the entrepreneur's end of period resources given $y_H = 0$ can be written as:

$$\underline{\omega} = (\tilde{p} + \tilde{r}_F)k + (\tilde{r}_E\theta - \rho_Y\theta - \tilde{r}_F)k_E \quad (53)$$

$$\omega = \underline{\omega} + \rho_Y\theta k_E\epsilon \quad (54)$$

It follows that the after-tax risk-free (gross) rate of return in the economy is $\tilde{r}_F + \tilde{p}$, since an entrepreneur who has one higher unit of k, but keeps k_E constant (thereby taking on the same risk) can increase her ω by $\tilde{r}_F + \tilde{p}$ units. The expected after-tax rate of return to the entrepreneur's risky project $\theta\tilde{r}_E + \tilde{p}$, since by holding one higher unit of k and raising k_E by one unit (thereby keeping the same amount of capital in the risk-free project) an entrepreneur can increase her ω by $\theta\tilde{r}_E + \tilde{p}$ in expectation.

Using these definitions, the entrepreneur's optimal decisions are summarized in Proposition 1.

Proposition 1. *In equilibrium, the entrepreneur's problem has a unique solution for $c(k,\theta,\epsilon,X)$, $k'(k,\theta,\epsilon,X)$, $\omega(k,\theta,\epsilon,X)$ and $k_E(k,\theta,X)$ which depends continuously on the parameters. If $\theta\tilde{r}_E \leq \tilde{r}_F$ then the entrepreneur's optimal choice entails: $k_E = 0$, $\omega = k\left(\tilde{r}_F + \tilde{p}\right)$. If $\theta\tilde{r}_E > \tilde{r}_F$ then the entrepreneur's optimal choice entails:*

$$k_E = \frac{k(\tilde{r}_F + \tilde{p})}{\rho_Y\theta - \theta\tilde{r}_E + \tilde{r}_F + \frac{\rho_Y\theta}{S^{-1}\left(\frac{\rho_Y\theta}{\rho_Y\theta - \theta\tilde{r}_E + \tilde{r}_F}\right)}} \quad (55)$$

$$\omega = k\left(\tilde{r}_F + \tilde{p}\right) + k_E\left(\rho_Y\theta\left(\epsilon - 1\right) + \theta\tilde{r}_E - \tilde{r}_F\right) \quad (56)$$

For any equilibrium values of $\tilde{r}_E, \tilde{r}_F$, the entrepreneur chooses:

$$c = (1 - \beta(1-\gamma))\omega \quad (57)$$

$$k' = (1 + \tau_C)\beta(1-\gamma)\omega \quad (58)$$

Proof. See Appendix D. □

As such, the solution to the entrepreneur's optimization problem implies that if the after-tax expected return on the risky project $\tilde{p} + \theta\tilde{r}_E$ is lower than the after-tax risk-free net return $\tilde{p} + \tilde{r}_F$, then it is optimal for the entrepreneur to set $k_E = 0$ and either allocate all her capital to her risk-free project, or sell it and use the revenue from the sale to lend to a bank. If the after-tax expected return on the risky project is higher than the after-tax risk-free return, then the entrepreneur allocates an amount of capital to her risky project which is proportional to her initial capital $k_{i,t}$. As in many other models with financial market frictions, it therefore follows that the allocation of capital in the economy depends on the wealth distribution across entrepreneurs – capital is not necessarily allocated to its most productive uses.

Comparative Statics Differentiating the entrepreneur's optimal policy function in Proposition 1 yields a number of comparative static results that are informative for the effect of capital taxes on entrepreneurs' optimal choices. We summarize these comparativer static results in Proposition 2:

Proposition 2. *Provided the conditions* (42), (47) *and* (48) *are satisfied, in each period the entrepreneur's optimal allocation of capital into her risky project* k_E *is:*

(i) increasing in k *and* θ*,*

(ii) increasing in $\tilde{r}_E$*, with elasticity greater than* 1*,*

(iii) decreasing in $\tilde{r}_F$*, provided* $\tilde{p} > 0$*.*

(iv) increasing in $\tilde{p}$*.*

Proof. See Appendix E. □

Unsurprisingly, all else equal, a richer and more productive entrepreneur invests more in her risky project. Furthermore, entrepreneurs invest more in risky projects and less in risk-free projects when the after-tax return to risky projects is relatively higher (i.e. higher $\tilde{r}_E$) and the after-tax return to risk-free projects is relatively lower (lower $\tilde{r}_F$). A consequence of agency frictions is that entrepreneurs also invest relatively more in risky projects when $\tilde{p}$ is higher, which occurs if the tax rate on wealth is lower or if the tax rate on consumption is lower. Higher $\tilde{p}$ shifts resources to risky projects even though an increase of $\tilde{p}$ by one unit increases the after-tax return to both risky and risk-free projects by one unit and so does not affect the relative rates of return of the two types of project. The reason that higher $\tilde{p}$ shifts resources to risky projects is that it makes entrepreneurs wealthier. Since entrepreneurs allocate more funds to their risky projects when wealthier (due to the presence of agency frictions), an increase in their after-tax wealth shifts resources to their risky projects. Evidently, by affecting $\tilde{r}_E$, $\tilde{r}_F$ and $\tilde{p}$, the taxation of capital income, wealth and consumption can affect the allocation of capital between risky and risk-free projects.

3.2 Aggregate Steady State

In this section, we formally characterize a steady state of the model. We derive comparative static results for how aggregate steady state variables change in response to changes in taxes. In section 4 below, we use these results to derive the government's optimal steady state tax policy.

We define a steady state as follows:

Definition 2. *A **steady state** $\mathscr{S}$ of the economy is a set of values of tax rates $\{\tau_W^*, \tau_K^*, \tau_C^*, \tau_N^*\}$, prices $\{r_E^*, r_F^*, w^*\}$, aggregate variables $\{C^*, C_H^*, K^*, K_E^*, K_F^*, K_H^*, Y^*, Y_E^{S*}, Y_F^{S*}\}$ and an equilibrium $\mathcal{E}$ in which all tax rates, prices and aggregate variables are equal to these steady state values in every period.*

Using the conditions for an equilibrium in Definition 1, and the solution to the entrepreneur's problem in Proposition 1, we can infer the values that aggregate variables must take in a steady state. We focus on steady states in which no capital or intermediate goods are hidden because, as argued below, the government will never design a tax policy to select a steady state in which capital and intermediate goods are hidden. The full set of conditions that must be satisfied in a steady state are summarized in Proposition 3 below. To write the conditions compactly, we express them in post-tax prices. The steady state post tax prices $\tilde{r}_E^*$, $\tilde{r}_F^*$ and $\tilde{p}^*$ are defined according to equations (49)-(51) above. We define the steady state post-tax wage as:

$$\tilde{w}^* \equiv \frac{w^*(1-\tau_N^*)}{1+\tau_C^*} \tag{59}$$

Using these definitions, Proposition 3 summarizes the necessary and sufficient conditions for a steady state:

Proposition 3. *There exists a steady state $\mathscr{S}$ which is consistent with the particular values of aggregate variables $\{K^*, K_E^*, C^*, Y_E^{S*}, Y_F^{S*}\}$, post-tax prices $\{\tilde{r}_E^*, \tilde{r}_F^*, \tilde{w}^*, \tilde{p}^*\}$ and consumption tax rate τ_C^* and in which no entrepreneurs hide capital or intermediate goods, if and only if the following conditions hold:*

$$\frac{\tilde{r}_F^*}{\tilde{r}_E^*} = \frac{F'_{Y_F^{S*}}}{F'_{Y_E^{S*}}} \tag{60}$$

$$Y_F^{S*} = K^* - K_E^* \tag{61}$$

$$\frac{K^*}{1+\tau_C^*} = \beta(1-\gamma)\left(\tilde{r}_E^* Y_E^{S*} + \tilde{r}_F^* Y_F^{S*} + \tilde{p}^* K^*\right) \tag{62}$$

$$C^* = \tilde{w}^* N + (1-\beta(1-\gamma))(\tilde{r}_E^* Y_E^{S*} + \tilde{r}_F^* Y_F^{S*} + \tilde{p}^* K^*) \tag{63}$$

$$C^* + \delta K^* + \overline{G} = F(Y_E^{S*}, Y_F^{S*}, N) \tag{64}$$

$$Y_E^{S*} = \frac{A_1 K^* (\tilde{r}_F^* + \tilde{p}^*)}{\tilde{r}_F^*} \int_1^{\overline{x}} \left(\frac{1}{x + \frac{x^2}{S^{-1}(x)}}\right) dx \tag{65}$$

$$K_E^* - Y_E^{S*}\left(\frac{\tilde{r}_E^* - \rho_Y}{\tilde{r}_F^*}\right) = \left(\frac{\rho_Y}{\tilde{r}_F^*}\right) \frac{A_1 K^* (\tilde{r}_F^* + \tilde{p}^*)}{\tilde{r}_F^*} \int_1^{\overline{x}} \frac{1}{x} \left(\frac{1}{x + \frac{x^2}{S^{-1}(x)}}\right) dx \tag{66}$$

where $A_1 = \frac{\underline{\theta}}{1 - \frac{\underline{\theta}}{\overline{\theta}}}$, $\overline{x} = \frac{\rho_Y \overline{\theta}}{\rho_Y \overline{\theta} - \tilde{r}_E^* \overline{\theta} + \tilde{r}_F^*} \in (1, \infty)$,
and $\tilde{r}_F^* \geq \rho_Y$; $\tilde{r}_E^* \geq \rho_Y$; $\tilde{r}_F^* + \tilde{p}^* \geq \rho_K$, $K_E^* < K^*$.

Proof. See Appendix F. □

These conditions are simply aggregated steady state analogues of the conditions for equilibrium found in Section 2 and Section 3.1. Equation (60) states that the ratio of prices of entrepreneurial and standard intermediate goods post-tax is the same as the ratio of marginal products of these intermediate goods. This follows from the first order conditions of the final goods firms. Equation (61) is simply the consequence of the fact that each unit of capital not placed in risky projects is placed in risk-free projects (since no capital is hidden) and each unit of capital in risk-free projects yields one unit of standard intermediate goods. Equations (62), (63), (65) and (66) come from integrating the entrepreneur's optimal decision rules in Proposition 1 across entrepreneurs, while noting that workers live hand-to-mouth. Equation (64) is the goods market clearing condition. Finally the inequalities $\overline{x} \in (1, \infty)$, $\tilde{r}_F^* \geq \rho_Y$, $\tilde{r}_E^* \geq \rho_Y$ and $\tilde{r}_F^* + \tilde{p}^* \geq \rho_K$ are the same inequalities as were shown to hold in equilibrium in Lemma 2 and Lemma 3, restated in terms of post-tax prices.

Comparative Statics of Capital Allocation Using the results in Proposition 3, this section analyses how the allocation of capital in the economy changes in response to changes in post-tax prices, treating post-tax prices and the aggregate stock of capital K^* as exogenous for the moment. We take this approach because treating post-tax prices and K^* as exogenous is relevant for the government's optimization problem below, since the government can essentially choose the level of post-tax prices and K^* by varying taxes.[17]

We define the elasticity $e_K^\star$ as the percentage change in aggregate steady state output Y^*, on the margin, when the post-tax prices $\tilde{r}_E^*$ and $\tilde{r}_F^*$ are each simultaenously decreased by 1%, holding K^* and other post-tax prices constant. We define the elasticity $e_P^\star$ as the percentage change in Y^*, on the margin, when the post-tax price $\tilde{p}^*$ is decreased enough to decrease $\tilde{r}_F^* + \tilde{p}^*$ by exactly 1%. That is, we define

$$\begin{aligned} e_K^\star &= \frac{\tilde{r}_E^*}{Y^*}\frac{\partial Y^*}{\partial \tilde{r}_E^*} + \frac{\tilde{r}_F^*}{Y^*}\frac{\partial Y^*}{\partial \tilde{r}_F^*} \\ e_P^\star &= \frac{\tilde{r}_F^* + \tilde{p}^*}{Y^*}\frac{\partial Y^*}{\partial \tilde{p}^*}, \end{aligned}$$

where all partial derivatives are evaluated on the assumption that K^* and other post-tax prices are held constant. It turns out that these $e_K^\star$ and $e_P^\star$ are the crucial elasticities that determine the effect of taxes on how efficiently capital is allocated in the economy. Since tax changes affect entrepreneurs' choices of where to allocate capital solely via the effect of taxes on post-tax prices, the effect of tax changes on aggregate output, holding K^* fixed, can be directly expressed in terms of these elasticities, using the definitions of post-tax prices in equations (49)-(51). That is, for instance:

$$\begin{aligned} \frac{\partial Y^*}{\partial \tau_K^*} &= -\frac{\partial Y^*}{\partial \tilde{r}_E^*} \cdot \frac{\partial \tilde{r}_E^*}{\partial (1-\tau_K^*)} - \frac{\partial Y^*}{\partial \tilde{r}_F^*} \cdot \frac{\partial \tilde{r}_F^*}{\partial (1-\tau_K^*)} - \frac{\partial Y^*}{\partial \tilde{p}^*} \cdot \frac{\partial \tilde{p}^*}{\partial (1-\tau_K^*)} \\ &= \frac{Y^*}{1-\tau_K^*}\left(\frac{\delta(1-\tau_K^*)e_P^\star}{R_F} - e_K^\star\right) \simeq -\frac{Y^* e_K^\star}{1-\tau_K^*}, \end{aligned}$$

where the approximate equality is because δ is much closer to zero than to one in practice. Then, the effect of capital income tax changes on aggregate output is roughly proportional to $e_K^\star$. By similar reasoning, we can express the response of output to changes in other taxes, holding constant K^*, in terms of $e_K^\star$ and $e_P^\star$:

$$\begin{aligned} \frac{\partial Y^*}{\partial \tau_W^*} &= -\frac{Y^* e_P^\star}{1-\delta(1-\tau_K^*)-\tau_W^*} \simeq -\frac{Y^* e_P^\star}{1-\tau_W^*} \\ \frac{\partial Y^*}{\partial \tau_C^*} &= -\frac{Y^*}{1+\tau_C^*}(e_P^* + e_K^*) \end{aligned}$$

[17] The government can effectively choose the level of K^* because, for given post-tax prices $\tilde{r}_E^*, \tilde{r}_F^*$ and $\tilde{p}^*$, the level of K^* is infinitely elastic with respect to τ_C^*, according to equation (62).

Then, it follows that $e_P^\star$ is the key determinant the sensitivity of aggregate output to wealth taxes, holding $K^\star$ constant, and $e_K^\star + e_P^\star$ describes the sensitivity of aggregate output to consumption taxes, holding $K^\star$ constant. Using Proposition 3, it is possible to characterize these elasticities almost as closed form functions of prices and parameters in our setting. To derive these, we differentiate (64) with respect to Y_E^{*S} and Y_F^{*S}, and differentiate (F.6) and (F.7) with respect to $\tilde{r}_E^*$, $\tilde{r}_F^*$ and $\tilde{p}^*$. We obtain, after some algebra, the following characterization:

Proposition 4. *Suppose that in the steady state $\mathscr{S}$ there is a minimal amount of allocative inefficiency, in that the least productive entrepreneur who chooses $k_E > 0$ has a productivity $\tilde{\theta} < \frac{\overline{\theta}}{1+10^{-6}}$. Then the elasticities $e_K^\star$ and $e_W^\star$ satisfy:*

$$e_K^\star = \left(\frac{A_1 K^* \hat{k}_E(\overline{\theta}) r_F^*}{\overline{\theta} Y^*}\right)\left(\frac{\overline{\theta} r_E^*}{r_F^*} - 1\right)^2 - \left(\frac{\tilde{r}_F + 2\tilde{p}}{\tilde{r}_F + \tilde{p}}\right)\left(\frac{Y_E^{S*} r_E^* - r_F^* K_E^*}{Y^*}\right) > 0 \quad (67)$$

$$e_W^\star = \frac{r_E^* Y_E^{S*} - r_F^* K_E^*}{Y^*} > 0 \quad (68)$$

$$\frac{e_K^\star}{e_W^\star} \geq \frac{Y^* e_W^\star}{r_F^* K_E^*} + \frac{\tilde{r}_F^*}{\tilde{r}_F^* + \tilde{p}^*} > e_W^\star \quad (69)$$

Proof. Omitted. Available upon request. □

Since $e_K^\star$ and $e_W^\star$ are both positive, we may conclude that an increase in capital, wealth or consumption taxes serves to decrease the aggregate output of the economy, even holding fixed K^*. The reason is that an increase in taxes distorts the allocation of capital in the economy by affecting both the fraction of aggregate capital allocated to entrepreneurs' risky projects and risk-free projects, and also by affecting the fraction of capital that is allocated to the projects of the most high θ entrepreneurs. The general effect of tax increases is to reduce aggregate output, by reducing the proportion of capital allocated to risky projects, which on average carry a higher expected return than risk-free projects. Since tax increases can decrease aggregate output in this economy even conditional on the value of aggregate capital K^* and aggregate labor N, an economist conducting a growth accounting exercise would attribute the resulting decrease in output following a tax rise to a decrease in measured aggregate total factor productivity. The elasticity of measured steady state total factor productivity with respect to a change in post-tax prices would therefore be exactly equal to the elasticity of aggregate steady state output Y^* with respect to a change in post-tax prices, holding fixed K^*. This effect of tax increases of reducing the total factor productivity of the economy by distorting the allocation of capital is a distinct effect of taxation on capital owners which has not been emphasized in earlier literature on optimal taxation. The literature has instead focused on the effect of capital taxation on aggregate investment. In our model, taxation of capital owners affects aggregate output via both channels.

4 Government Optimization

In this section, we formulate and solve the government's problem of choosing optimal taxes. We focus on an optimal steady state tax policy. In particular, we assume that the government chooses steady state tax rates $\{\tau_K^*, \tau_C^*, \tau_W^*, \tau_N^*\}$, and an aggregate steady state $\mathscr{S}$ of the economy consistent with these tax rates, in order to maximize the present discounted utility of a newborn worker in the steady state.[18]

To solve the govenrment's optimization problem, we first remark that the assumptions made on the final goods production function in Section 2 ensure that it is inefficient for the aggregate economy if entrepreneurs hide capital or intermediate goods. Furthermore, Lemma 2 and Lemma 3 ensure that, in any steady state, entrepreneurs weakly prefer not to hide capital or intermediate goods. Then, the government will always prefer to select a steady state in which entrepreneurs do not hide capital or intermediate goods. Thus, we have the following lemma:

Lemma 5. *Suppose that in the steady state $\mathscr{S}$ that entrepreneurs hide a strictly positive amount of capital or intermediate goods. Then the steady state $\mathscr{S}$ cannot be part of a solution to the government's optimization problem.*

Proof. See Appendix G. □

Since entrepreneurs do not hide capital or intermediate goods in the government's choice of steady state, we can solve the government's optimization problem making use of Proposition 3 above. Note that since workers live hand-to-mouth, the present discounted utility of a newborn worker in the steady state is simply $\frac{u(\tilde{w}^*)}{1-\beta}$ and so the government's objective amounts to simply maximizing the steady state value of $\tilde{w}^*$. By Proposition 3, any particular value of $\tilde{w}^*$ is consistent with a steady state in which entrepreneurs do not hide capital or intermediate goods if and only if there exist aggregate variables $\{K^*, K_E^*, C^*, Y_E^{S*}, Y_F^{S*}\}$, post-tax prices $\{\tilde{r}_E^*, \tilde{r}_F^*, \tilde{p}^*\}$ and consumption tax rate τ_C^* which, along with $\tilde{w}^*$, satisfy the conditions of Proposition 3. Then, we can recast the government's problem as choosing $\{\tilde{w}^*, K^*, K_E^*, C^*, Y_E^{S*}, Y_F^{S*}\}$, post-tax prices $\{\tilde{r}_E^*, \tilde{r}_F^*, \tilde{p}^*\}$ and consumption tax rate τ_C^* to maximize $\tilde{w}^*$ subject to the constraint that the values of all these variables satisfy the conditions of Proposition 3. Given the government's optimal choices of these variables, we can then back out the optimal tax rates from the values of post-tax prices, using the definitions of post-tax prices in equations (49)-(51) and (59).

[18]Provided that newborn entrepreneurs are better off than newborn workers (which will be the case provided the number of entrepreneurs is small relative to K^*), this amounts to maximizing the steady state value of a Rawlsian social welfare function. Equally, provided the number of entrepreneurs is small relative to K^* and relative to the number of workers, this is also a close approximation to maximizing a utilitarian social welfare function, since entrepreneurs' consumption will then be relatively high compared to the consumption of workers, and so the marginal utility of consumption of entrepreneurs multiplied by the mass of entrepreneurs will be relatively low compared to the marginal utility of workers multiplied by their mass.

The conditions on the final goods production function imply that there is a maximum level of capital and output that the economy could feasibly produce. This implies upper bounds on the feasible values of post-tax prices and aggregate variables. In combination with the constraints in Proposition 3, this implies that all feasible values of aggregate variables must be contained in a compact set. Moreover, since all the functions in Proposition 3 are differentiable, a solution to the government's optimization problem must exist, and must satisfy the following first order conditions:[19]

$$\begin{aligned}(1-\beta(1-\gamma))(Y_E^{S*}\tilde{r}_E^*K_E^*+\tilde{r}_F^*K_F) &= \frac{\tau_C+\tau_K}{1+\tau_C}Y^*e_K^\star \\ &+ \tilde{r}_E^*\lambda_E+\left(\frac{\tilde{r}_F^*\lambda_P}{\tilde{p}^*+\tilde{r}_F^*}\right)+\lambda_F\tilde{r}_F^* \quad (70)\\ (1-\beta(1-\gamma))(K\tilde{p}^*+K\tilde{r}_F^*) &= \frac{\tau_C+\tau_K}{1+\tau_C}Y^*e_W^\star+\lambda_P \quad (71)\\ (1-\beta(1-\gamma))\left(\frac{\tilde{r}_E^*Y_E^{S*}+\tilde{r}_F^*Y_F^{S*}+\tilde{p}^*K}{K}\right) &= \frac{Y_E^{S*}r_E+Y_F^{S*}r_F}{K}-\delta \quad (72)\end{aligned}$$

where λ_E, λ_F and λ_P are, respectively, Lagrange multipliers on the constraints $\tilde{r}_E^*\geq\rho_Y$, $\tilde{r}_F^*\geq\rho_Y$ and $\tilde{r}_E^*+\tilde{p}^*\geq\rho_K$.

Intuitively, these three first order conditions should be viewed as first order conditions for, respectively τ_K^*, τ_W^* and τ_C^*. The left hand side of the first two first order conditions is proportional to the marginal decrease in entrepreneurs' consumption if the government raises taxes enough to reduce post tax prices by 1%, holding all agents' behaviour constant. This represent a roughly one percentage point increase in taxes. For instance, the left hand side of equation (70) states that if τ_K^* rises enough to reduce post-tax $\tilde{r}_E^*$ and $\tilde{r}_F^*$ by 1%, then entrepreneurs' post-tax income from selling intermediate goods falls by 0.01 times $Y_E^{S*}\tilde{r}_E^*K_E^*+\tilde{r}_F^*K_F$ units and so their consumption falls by 0.01 times $(1-\beta(1-\gamma))(Y_E^{S*}\tilde{r}_E^*K_E^*+\tilde{r}_F^*K_F)$ units. This is the marginal benefit for the government in raising taxes this way since, holding all agents' behaviour fixed, one unit less of entrepreneurs' consumption means one more unit of resources for workers' consumption. The right hand side of equation (70) represents the marginal cost to the government in raising taxes enough to reduce post-tax prices by 1%. In particular, the first term on the right hand side represents the decrease in government revenue from a tax rise due to entrepreneuers' behavioral changes in response, holding fixed K^*. Holding fixed K^*, a decrease by 1% of $\tilde{r}_E^*$ and $\tilde{r}_F^*$ reduces Y^* by $Y^*e_K^\star$ units. This in turn reduces government revenue by $\frac{\tau_C^*+\tau_K^*}{1-\tau_K^*}Y^*e_K^\star$ units. The other right hand side terms are proportional to the Lagrange multipliers and represent the fact that the more entrepreneurs are taxed, the more likely it is that the constraints that they do not hide capital or intermediate goods

[19] Since most of the constraints in Proposition 3 are linear, it is trivial to show that the constraint qualification must be satisfied at the government's optimal allocation.

will bind.

The second first order condition (71) is analogous to the first, but for wealth taxes. The left hand side is proportional to the decrease in entrepreneurs consumption if wealth taxes are raised enough to reduce $\tilde{p}^* + \tilde{r}_F^*$ by 1%. The right hand side is proportional to the loss in revenue associated with entrepreneurs' behavioral response to this, holding fixed K^*, plus a Lagrange multiplier to reflect the fact that a tax rise makes it more likely that the constraint that entrepreneurs do not wish to hide capital will bind.

It may seem surprising that these first two first order conditions seem to ignore the effect of a tax change on K^*. This is because, the third first order condition (72) is that the consumption tax τ_C^* is set to ensure that K^* is at the level consistent with maximizing workers' steady state consumption.[20] Then, a small change in K^* has no marginal effect on workers' steady state consumption. The left hand side of (72) signifies the marginal increase in aggregate national income, net of depreciation, associated with an increase in K^*. The right hand side denotes the marginal increase in entrepreneur's steady state consumption associated with an increase in K^*. Then, the left hand side minus the right hand side is the marginal increase in workers' consumption associated with an increase in K^*. It is natural that this should be zero at an optimum.

Substituting the sharp characterizations of elasticities in Proposition 4 into the first order conditions (70)-(72) and rearranging, it is possible to characterize optimal taxes relatively precisely in terms of pre-tax prices and parameters. This is done in Propostion 5:

Proposition 5. *Let the steady state $\mathscr{S}$ be a solution to the government's problem. Suppose that in the steady state $\mathscr{S}$ there is a minimal amount of allocative inefficiency, in that the least productive entrepreneur who chooses $k_E > 0$ has a productivity $\tilde{\theta} < \frac{\bar{\theta}}{1+10^{-6}}$. Suppose further than in steady state $\mathscr{S}$, $r_F^* < \rho_K$. Then, the steady state $\mathscr{S}$ must entail:*

$$\tau_C^* = \frac{1}{\beta\left((\bar{r}_E - r_F^*)(1-\chi) + \rho_K\right)} - 1 \tag{73}$$

$$\tau_K^* = 1 - (1-\chi)(1+\tau_C^*) \tag{74}$$

$$\tau_W^* = (\bar{r}_E - \delta)(1-\tau_K^*) - \left(\frac{1}{\beta(1-\gamma)} - 1\right) \tag{75}$$

[20]The reason that the consumption tax is set so that K^* is at the optimal steady state level is because K^* is infinitely elastic with respect to the consumption tax.

where

$$\bar{r}_E = \frac{r_E^* Y_E^{S*}}{K_E^*}, \tag{76}$$

is the average return to capital in risky projects gross of taxes and depreciation, and

$$\chi = \min\left\{1 - \frac{\rho_Y}{\min\{r_E; r_F\}}; \frac{(1-\beta(1-\gamma))e_W^\star}{e_K^\star - \frac{(1-\chi)r_F e_W^\star}{\rho_K}}\right\} \text{ and} \tag{77}$$

$$\chi \in \left(0, \frac{(1-\beta(1-\gamma))r_F^*}{\bar{r}_E - \beta(1-\gamma)r_F^*}\right] \tag{78}$$

Proof. See Appendix H. □

To interpret these results, first note that the conditions under which they apply are likely to be fulfilled in any practically relevant case. The minimal amount of allocative inefficiency condition, $\tilde{\theta} < \frac{\bar{\theta}}{1+10^{-6}}$, is simply that not all entrepreneurs who put resources into risky projects have exactly the same productivity. The condition $r_F^* < \rho_K$ will be satisfied provided that capital can be hidden without losing close to 100% of its value. To see this, consider an annual frequency and take r_F to be the pre-tax risk-free rate of return to capital, gross of depreciation. Then, it seems appropriate to calibrate r_F to be in the region of 8%. We justify the choice of 8% in the discussion of the numerical calibration below. Then, the condition $r_F^* < \rho_K$ will be satisfied provided $\rho_K > 0.08$, that is, capital can be hidden from the tax collector without losing more than 92% of its value within a year. This seems almost certain to hold in practice, given modern possibilities for evading taxes by storing wealth overseas.

Assuming that the conditions of Proposition 5 hold, the proposition states that the optimal τ_C^* is given by equation (73). With a little rearrangement, this condition can be shown to imply that τ_C^* is set at exactly the level at which entrepreneurs are indifferent between hiding capital and not hiding capital. This implies that the government should tax consumption at the highest rate consistent with agents not evading taxes by concealing their wealth. The reason for this result is that, insofar as taxes distort the economy and reduce aggregate output, taxes on consumption entail relatively less distortion for each unit of revenue they extract than do capital income taxes. The net effect of this is that the government seeks to cut capital income taxes and raise consumption taxes where possible. It is optimal for the government to do this until consumption taxes are at the highest possible level consistent with entrepreneurs not hiding their capital. That is, taxes on consumption are set at the highest possible level consistent with agents not evading these high consumption taxes.

There are two reasons that consumption taxes reduce aggregate output less for each unit of revenue extracted than do capital income taxes. The first reason is that consumption taxes, unlike capital income taxes, do not affect the relative desirability of consuming today versus consuming tomorrow. Therefore they do not distort entrepreneurs' choices of how much to save. This shows up in equation (62) where it is evident that a decrease in post-tax prices does not reduce K^* if, and only if, it occurs simultaneously with a rise in consumption taxes. The second reason is that, for each unit of revenue extracted, consumption taxes tend to reduce aggregate total factor productivity less than do capital income taxes. Intuitively, this is because consumption taxes do not not affect the relative return of risky and risk-free projects – in terms of post-tax prices, a rise in consumption taxes affects $\tilde{r}_E$, $\tilde{r}_F$ and $\tilde{p}$ all proportionately. This means that consumption taxes do not enormously affect agents' incentives as to how best to use their capital. By contrast, capital income taxes reduce the rate of return to capital and therefore reduce $\tilde{r}_E$ and $\tilde{r}_F$ relative to $\tilde{p}$. The effect of this is that capital income taxes fall much more heavily in the model on entrepreneurs with higher θ: since these entrepreneurs earn a very high rate of return from their risky projects, capital income taxes are particularly costly to them. Consequently, higher capital income taxes both reduce the proportion of capital allocated to high θ entrepreneurs and reduce the proportion of capital allocated to risky projects. The effect of this is that capital income taxes tend to distort the allocation of capital more than do consumption taxes. Wealth taxes have an intermediate effect: they distort aggregate capital accumulation to a similar level to capital income taxes, but tend to distort the allocation of capital less than either consumption or capital income taxes. In our calibration below, we find that optimal wealth taxes are therefore positive while optimal capital income taxes are negative.

The greater negative effect of capital taxes on total factor productivity, relative to the revenue they extract, can be seen using the results of Proposition 4. Recall that, holding K^* constant, a one percentage point increase in capital income taxes reduces Y^* by, roughly, $Y^* e^{\star}_K$ units, a one percentage point increase in consumption taxes Y^* by, roughly, $Y^*(e^{\star}_K + e^{\star}_P)$ units and a one percentage point increase in wealth taxes reduces Y^* by, roughly, $Y^* e^{\star}_P$ units. The inequality (69) in Proposition 4 implies that $e^{\star}_K$ is at least as large as $e^{\star 2}_W$. Then, it must be the case that either $e^{\star}_W$ and $e^{\star}_K$ are both low, or $e^{\star}_K$ is reasonably large compared to $e^{\star}_W$. In the former case, the effects of any taxation on total factor productivity are small and the government might as well set consumption taxes at the highest level consistent with agents not evading consumption taxes. In the latter case, a one percentage point increase in capital income taxes produces a similar drop in total factor productivity to a one percentage point increase in consumption taxes. However, a one percentage point increase in consumption taxes implies a much larger drop in entrepreneurs' consumption (given K^*) than a one percentage point increase in capital income taxes, since entrepreneurs' consumption depends more on their wealth than on their income. Therefore, a rise in consumption taxes large enough to reduce entrepreneurs' consumption by 1 unit (thereby making room for workers to consume 1 unit more) entails a significantly smaller drop in aggregate total factor productivity than the rise in capital income taxes needed to reduce entrepreneurs' consumption by one unit. This motivates the government to make greater use of consumption taxes rather than capital income taxes.[21]

Further insight can be gleaned by noting that equation (74) implies that the optimal capital income tax depends on a function of parameters and prices χ, which is itself bounded above by $\frac{(1-\beta(1-\gamma))r_F^*}{\bar{r}_E-\beta(1-\gamma)r_F^*}$. This upper bound on χ is highly dependent on the degree of financial market frictions. For instance, if there were no frictions in financial markets, then the return to capital in risky and risk-free projects would be identical, in which case we would have $\bar{r}_E = \tilde{r}_F^*$. In that case, the upper bound on χ would be equal to 1, and so equation (74) implies that it could be optimal to tax capital income at 100%. This is intuitive: taxing capital income extracts resources most from the most high ability entrepreneurs, who are the entrepreneurs who can most afford to pay such taxes. In the absence of any financial market frictions, capital income taxes do not cause allocative inefficiency and so high capital income taxes may be optimal. On the other hand, if there is significant allocative inefficiency and entrepreneurs' risky projects earn a much higher return on average than risk-free projects, then the upper bound on χ will be close to zero. In that case, equations (73) and (74) imply that, approximately $\tau_K^* = -\tau_C^* < 0$, and so optimal capital income taxes will be negative. Negative capital income taxes may be optimal because they increase the relative return earned by high ability entrepreneurs in their risky projects, thereby causing entrepreneurs to shift resources to these high return projects.

A Numerical Calibration To interpret the magnitudes of these optimal taxes in practice, we may undertake a rough numerical calibration. Calibrating at annual frequency, we set the depreciation rate δ to 7%, approximately the average depreciation rate in the US fixed asset tables. We set γ to 1% (i.e. agents have a life expectancy of 100 years). We set β to 0.97, which implies that a (non-optimal) steady state would exist given an average rate of return to capital net of depreciation of 4%, and a capital income tax of 25%, comparable to current US levels of taxes on dividend income (McGrattan and Prescott, 2005). We set the average return to capital in risky projects gross of depreciation, $\bar{r}_E$, at 15% so that their return net of depreciation is 8%, approximately the annual rate of return to equities in the US over the twentieth century (Mehra and Prescott, 2003). We set the risk-free return gross of depreciation, r_F^*, to 8%, consistent with a risk-free interest rate of 1% which is close to the average return of relatively riskless securities in the US over the twentieth century (Mehra and Prescott, 2003). Using these parameter values, we obtain an upper bound for χ of 0.04, which is close to zero. Indeed, unless we were to calibrate $\bar{r}_E$ to be very close to r_F^*, the upper bound of χ would inevitably be relatively close to zero, since if $\bar{r}_E - r_F^*$ is of a similar magnitude to $1-\beta(1-\gamma)$, then the formula for the upper bound of χ implies that this will be of a similar magnitude to r_F^*.

[21] When calibrating the model, below, we find that it is the latter of the two cases, where $e_K^\star$ is large relative to $e_W^\star$, which is more relevant. Neverthless, it is optimal for the government to set τ_C^* at the maximum level consistent with agents not hiding capital in either case.

If χ is small, then equation (74) implies that the optimal tax on capital income will be close to χ minus τ_C^*. As discussed above, Proposition 4 implies that τ_C^* will be set at the highest level consistent with agents not evading the consumption tax by hiding their wealth. The value of this maximum level in practice is extremely difficult to infer. Nevertheless, since many European countries tax consumption at rates between 20% and 30%, we may infer that consumption taxes can rise to at least 30% without widespread evasion. Then, with an upper bound on χ of 0.04, equation (74) implies that the tax on capital income must be no higher than minus 26%. Since, as discussed above, the gap between risky and risk-free return would have to be calibrated at a very low level to infer an upper bound of χ much larger than 0.04, it follows that optimal capital income taxes are negative in any plausible calibration.

Inputting an optimal capital tax of -24% along with the other calibrated parameters into the formula for the optimal wealth tax (75), we obtain approximately 6%. That is, the model, with these parameters, implies that agents should pay a tax on their wealth at a rate of around 6% per year. Since the value we used for the optimal capital tax relied on the upper bound for χ and a maximum consumption tax of only 30%, it is arguable that this represents an upper bound on the optimal capital tax and, therefore, a lower bound on the optimal taxes on wealth and consumption. Evidently, the model strongly favors taxing wealth and consumption rather than capital income. This is unsurprising because of the highly distortionary nature of capital income taxes discussed above. Neither wealth nor consumption taxes fall relatively more heavily on the most efficient entrepreneurs so it is not surprising that these taxes are much less costly for the allocative efficiency of the economy relative to the revenue that they extract.

5 Conclusion

We examine the implications of entrepreneurial financial frictions for optimal linear capital taxation, in a setting where the government desires to redistribute from entrepreneurs, who own capital, to workers who do not, as in Judd (1985). Allowing for financial frictions implies that capital taxation can affect the efficiency of the allocation of capital – a force missing from models without financial frictions. In our framework, entrepreneurs can invest capital in projects which yield risky return or in risk-free projects, while workers supply labor and live hand to mouth. The government seeks to redistribute from entrepreneurs to workers using a mixture of (possibly negative) taxes on wealth, consumption, capital income and on labor, and has to finance exogenous government expenditure while balancing its budget.

We follow the "sufficient statistics" approach in characterizing optimal tax rates in terms of the elasticities of agents decisions with respect to taxes. We find that for parameter values that appear defensible, the optimal tax on capital income is negative –

less than -24%, while consumption taxes and wealth taxes are both positive and large, with the optimal consumption tax being at least 30% and the optimal wealth tax at least 6% per year. This suggests that a significant amount of redistribution away from capital owners is possible, but that it should take place through consumption and wealth taxes rather than through capital income taxes. Capital income taxes are much more distortionary than wealth and consumption taxes because not only do they reduce entrepreneurs' incentives to save (unlike consumption taxes) they also reduce the fraction of capital used in risky projects by the most efficient entrepreneurs, by particularly taxing these individuals (more than either wealth or consumption taxes).

To isolate the consequences of taxation on the efficiency of capital allocation, we abstract from a number of important features of relevance for capital taxation. For instance, we abstract away from entry and exit into entrepreneurship, which may be affected by taxes. We also abstract from the savings of workers and from the possibility of nonlinear taxation. These assumptions certainly affect the precise optimal tax results we obtain. Nevertheless, we conjecture that the characterization we obtain for optimal taxes in terms of elasticities, and the characterization of these elasticities, would remain relevant and useful even when these extensions are considered. As such, our finding that it is typically optimal to tax wealth and consumption rather than capital income may well survive in a larger class of models than the model we consider here. We plan to consider some of these extensions in future work.

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Appendices

A Proof of Lemma 1

Note that the technology associated with an entrepreneur's risky and risk-free projects displays constant returns to scale. Consider an entrepreneur Alice, who at some period t has $a > 0$ times as much capital as an entrepreneur Bob. The constant returns to scale properties imply that Alice can make the same decisions as Bob, for each ϵ and θ, except do everything a times as much: she can consume a times as much each period, devote a times as much capital to each project, hide a times as many units of intermediate goods etc. If Alice does this she can consume a times as much as Bob each period, for the same draws of θ and ϵ. Since, for any c, $\log(ac) \equiv \log(a) + \log(c)$, Alice's present discounted utility from these choices would then be the same as Bob's plus an additional $\sum_{j=0}^{\infty} \beta^j (1-\gamma)^j \log(a) = \frac{\log(a)}{1-\beta(1-\gamma)}$. Since these choices are possible for Alice, it must be that $V(ak, \theta, X) \geq \frac{\log(a)}{1-\beta(1-\gamma)} + V(k, \theta, X)$. However, on the other hand, Bob can choose to do everything that Alice does only $\frac{1}{a}$ times as much. By the same logic as before, doing so would yield Bob a present discounted utility equal to Alice's minus $\frac{\log(a)}{1-\beta(1-\gamma)}$. Therefore, it must be the case that $V(k, \theta, X) \geq V(ak, \theta, X) - \frac{\log(a)}{1-\beta(1-\gamma)}$. Comparing these two inequalities that V must fulfil, it is immediate that it cannot satisfy both unless $V(ak, \theta, X) = \frac{\log(a)}{1-\beta(1-\gamma)} + V(k, \theta, X)$. In that case, it must be that $V(k, \theta, X) \equiv \frac{\log(k)}{1-\beta(1-\gamma)} + V(1, \theta, X)$. Let $\overline{V}(X)$ denote $E[V(1, \theta, X)|X]$. Then it follows that $E\left[V(k, \theta, X)\middle| k, X\right] = \overline{V}(X) + \frac{\log(k)}{1-\beta(1-\gamma)}$. □

B Proof of Lemma 2

During the period t the entrepreneur can do four different activities each of which yields a riskless return to her capital. First, she can sell capital and lend to a bank, setting $b < 0$. The bank's break even condition (32) implies that the bank would be willing to borrow from her and pay back the risk-free gross interest rate of R_F at the end of the period, for each unit lent to the bank. Then, equation (33) implies that to do this would increase the entrepreneur's ω at the end of the period by $\frac{R_F}{1+\tau_C}$. Second, the entrepreneur can allocate capital to her risk-free project and sell the output of this to the final goods firms. Examining the equation (33) reveals that for each additional unit the entrepreneur can allocate to k_F she is able to increase her ω by $\frac{(1-\tau_K)r_F+1-\delta(1-\tau_K)-\tau_W}{1+\tau_C}$. Third, the entrepreneur can allocate a unit of capital to her risk-free project, and then hide the intermediate goods that result, setting $y_{HF} + k_F$. Equation (33) reveals that for each additional unit of k_F the entrepreneur can allocate in this way, she can increase her ω by $\frac{\rho_Y(1+\tau_C)+1-\delta(1-\tau_K)-\tau_W}{1+\tau_C}$. Fourth, the entrepreneur can hide her capital at the start of the period. Each additional unit of hidden capital allows her to increase her ω by ρ_K.

Since these four activities are risk-free, that is they increase ω by the same amount regardless of the realization of ϵ, they do not affect $\frac{\partial \omega(k,\theta,\epsilon,X)}{\partial \epsilon}$. Consequently, which of these activites the entrepreneur participates in will have no bearing on the incentive compatibility constraint (41), for a given choice of k_E by the entrpreneur. Therefore, agency frictions will not prevent the entrepreneur from allocating her capital to whichever of these four activities offers the highest return.

Since the entrepreneur optimally allocates her funds between these four activities, it must be the case in equilibrium that lending to the bank offers at least as high a return as the other three activities:

$$R_F \geq \max\left\{(1-\tau_K)r_F + 1 - \delta(1-\tau_K) - \tau_W; \rho_Y(1+\tau_C) + 1 - \delta(1-\tau_K) - \tau_W; \rho_K(1+\tau_C)\right\} \tag{B.1}$$

To verify this, suppose otherwise. Then, the entrepreneur would be able to borrow without limit at the start of the period, buy capital which she allocates to one of the other three activities that offers higher return, repay her debts at the end of the period, and obtain an arbitrarily high ω at the end of the period. This cannot be a equilibrium of the economy.

Equally, in equilibrium, it must be the case that the entrepreneur's risk-free project offers at least as high a return as lending to a bank. Suppose otherwise, then no entrepreneurs would allocate capital to their risk-free projects, instead they would sell and lend to banks. The consequence of this would be that the economy's output of standard intermediate goods would fall to zero, driving the price of standard intermediate goods to infinity, because of the Inada conditions on the final goods production function. This also cannot be a competitive equilibrium. As such, we may conclude that, in equilibrium:

$$\begin{aligned} R_F &= (1-\tau_K)r_F + 1 - \delta(1-\tau_K) - \tau_W \\ &\geq \max\left\{\rho_Y(1+\tau_C) + 1 - \delta(1-\tau_K) - \tau_W; \rho_K(1+\tau_C)\right\} \end{aligned} \tag{B.2}$$

Equation (B.2) establishes that in equilibrium each entrepreneur is indifferent between allocating capital to her risk-free project (and selling the intermediate goods that result) and lending to a bank. If the inequality in (B.2) holds with equality then the entrepreneur obtains an equally high return by either hiding capital or hiding standard intermediate goods, in which case she is indifferent about the level of these that she chooses. On the other hand, if the inequality in is strict, the entrepreneur obtains a strictly lower return by hiding capital or hiding standard intermediate goods and so will not do so. □

C Proof of Lemma 3

In an equilibrium of the economy, it must be the case every period that:

$$ \tag{C.1} $$

$$1 - \delta(1-\tau_K) - \tau_W + (1-\tau_K)\overline{\theta} r_E - \rho_Y(1+\tau_C)\overline{\theta} - R_F < 0 \tag{C.2}$$

If the inequality in (47) is strict, then all entrepreneurs choose $y_{EH} = 0$. If the inequality in (47) holds with equality, then entrepreneurs are indifferent about their choice of y_{EH}.

First we show that in an equlibrium it must hold that

$$(1+\tau_C)\rho_Y - (1-\tau_K) r_E \leq 0 \tag{C.3}$$

If this were not the case, then it would be optimal for each entrepreneur to set $y_{EH} = \theta\epsilon k_E$, so that $\theta k_E = \int_\epsilon y_{EH} \mathrm{d}H(\epsilon)$ because, by doing so, the entrepreneur would be able to achieve a higher ω according to equations (45) and (41) for every realization of ϵ. This choice amounts to hiding all the intermediate goods produced by the entrepreneur's risky project. If it is optimal for entrepreneurs to do this, then no entrepreneurial intermediate goods would be sold to the final goods firms. The Inada conditions on the final goods production function then imply that the price of entrepreneurial intermediate goods would be driven to infinity. This cannot be an equilibrium.

If the inequality in (C.3) is strict, then equations (45) and (41) imply that ω is strictly decreasing in y_{EH} for any realization of ϵ. Then, the entrepreneur will choose $y_{EH} = 0$. If the inequality in (C.3) holds with equality, then the entrepreneur's ω does not depend on y_{EH} and so the entrepreneur will be indifferent about the value of y_{EH}.

Next we show that in an equilibrium it must hold that

$$1 - \delta(1-\tau_K) - \tau_W + (1-\tau_K)\overline{\theta} r_E - \rho_Y(1+\tau_C)\overline{\theta} - R_F \leq 0 \tag{C.4}$$

If this were not the case, then equations (45) and (41) imply that ω is strictly increasing in k_E for all $\epsilon > 0$ for entrepreneurs with θ close enough to $\overline{\theta}$. Then, by choosing an arbitrarily high k_E, such entrepreneurs will be able to achieve arbitrarily high consumption and therefore utility. Therefore, if this condition held, some entrepreneurs would desire to allocate an infinite amount of capital to their risky projects, which cannot be an equilibrium, since the capital stock is finite each period. □

D Proof of Proposition 1

Using the definitions (49)-(51) and fixing $y_E = 0$, the entrepreneur's within period problem of choosing k_E and $\underline{\omega}$ can be simplified to:

$$\sup_{k_E \geq 0;\ \underline{\omega} \geq 0} \quad \int_{\epsilon>0} \log(\underline{\omega} + \rho_Y \theta \epsilon k_E) \mathrm{d}H(\epsilon) \tag{D.1}$$

s.t.

$$\underline{\omega} = \quad (\tilde{p} + \tilde{r}_F)k + (\tilde{r}_E \theta - \rho_Y \theta - \tilde{r}_F)k_E \tag{D.2}$$

Note that

$$\int_{\epsilon>0} \underline{\omega} + \rho_Y \theta \epsilon k_E \mathrm{d}H(\epsilon) = (\tilde{p} + \tilde{r}_F)k + (\tilde{r}_E \theta - \tilde{r}_F)k_E$$

If $\tilde{r}_E \theta \leq \tilde{r}_F$, then it follows immediately that $\int_{\epsilon>0} \underline{\omega} + \rho_Y \theta \epsilon k_E \mathrm{d}H(\epsilon)$ is decreasing in k_E. By Jensen's inequality, it follows that $\int_{\epsilon>0} \log(\underline{\omega} + \rho_Y \theta \epsilon k_E) \mathrm{d}H(\epsilon)$ is strictly decreasing in k_E and so the entrepreneur will optimally choose $k_E = 0$.

In the case $\tilde{r}_E \theta > \tilde{r}_F$ we take the first order condition. Given the strict concavity of the objective function, the first order condition is a sufficient condition for a unique optimum. We guess that the constraints $k_E \geq 0$ and $\underline{\omega} \geq 0$ will not bind, and will verify that this guess is correct by showing that the first order condition has a solution where they do not bind. In this case, the first order condition is:

$$\int_\epsilon \left(k\left(\tilde{r}_F + \tilde{p}\right) + k_E\left(k,\theta\right)\left(\rho_Y \theta \left(\epsilon - 1\right) + \theta \tilde{r}_E - \tilde{r}_F\right) \right)^{-1} \left(\rho_Y \theta \left(\epsilon - 1\right) + \theta \tilde{r}_E - \tilde{r}_F\right) h\left(\epsilon\right) \mathrm{d}\epsilon = 0$$

After some rearrangement, this first order condition can be written:

$$\frac{\rho_Y \theta}{\rho_Y \theta - \theta \tilde{r}_E + \tilde{r}_F} = \frac{\int_\epsilon \left(1 + x\epsilon\right)^{-1} h\left(\epsilon\right) \mathrm{d}\epsilon}{\int_\epsilon \left(1 + x\epsilon\right)^{-1} \epsilon h\left(\epsilon\right) \mathrm{d}\epsilon} \tag{D.3}$$

where

$$x = \frac{k_E\left(k,\theta\right) \rho_Y \theta}{k\left(\tilde{r}_F + \tilde{p}\right) + k_E\left(k,\theta\right)\left(-\rho_Y \theta + \theta \tilde{r}_E - \tilde{r}_F\right)}$$

Using the definition of $S(\cdot)$ in equation (52), we can rewrite the first order condition as:

$$\frac{\rho_Y \theta}{\rho_Y \theta - \theta \tilde{r}_E + \tilde{r}_F} = S(x)$$

In Lemma 4, $S(x)$ is shown to be monotonically increasing in x, and therefore is an invertible function. Consequently, we can re-write the first order condition as:

$$\frac{k_E(k,\theta)\,\rho_Y\theta}{k(\tilde{r}_F+\tilde{p})+k_E(k,\theta)(-\rho_Y\theta+\theta\tilde{r}_E-\tilde{r}_F)} = S^{-1}\left(\frac{\rho_Y\theta}{\rho_Y\theta-\theta\tilde{r}_E+\tilde{r}_F}\right) \tag{D.4}$$

Rearranging equation (D.4) yields the unique optimal solution for k_E :

$$k_E(k,\theta,X) = \frac{k(\tilde{r}_F+\tilde{p})}{\rho_Y\theta-\theta\tilde{r}_E+\tilde{r}_F+\frac{\rho_Y\theta}{S^{-1}\left(\frac{\rho_Y\theta}{\rho_Y\theta-\theta\tilde{r}_E+\tilde{r}_F}\right)}} \tag{D.5}$$

Note that we already assumed that $\tilde{r}_E\theta > \tilde{r}_F$. Furthermore, the results of Lemma 3 imply that $\tilde{r}_E\theta-\rho_Y\theta-\tilde{r}_F < 0$, since $\theta < \overline{\theta}$. It follows that $\frac{\rho_Y\theta}{\rho_Y\theta-\theta\tilde{r}_E+\tilde{r}_F} > 1$. Since $S^{-1}:(1,\infty)\to\mathbf{R}_{++}$, it follows that the value of $S^{-1}\left(\frac{\rho_Y\theta}{\rho_Y\theta-\theta\tilde{r}_E+\tilde{r}_F}\right)$ is strictly positive. Then, equation (D.5) has a unique solution $k_E > 0$, which must be the unique solution to the entrepreneur's problem. The differentiability of $S(\cdot)$ implies that the entrepreneur's choice of k_E is differentiable with respect to the parameters.

Combining these results with equations (38), (39), (45) and (41) yields all the results of the proposition. □

E Proof of Proposition 2

To derive the results, it is convenient to define $x = \frac{\rho_Y\theta}{\rho_Y\theta-\theta\tilde{r}_E+\tilde{r}_F}$ and $\tilde{m} = \tilde{r}_F+\tilde{p}$, so that $k_E(k,\theta,X) = \frac{k\tilde{m}}{\rho_Y\theta}\frac{x}{1+\frac{x}{S^{-1}(x)}}$ when $x > 1$ and $k_E(k,\theta,X) = 0$ when $x < 1$. Furthermore, we define, for any variable v, that $\frac{\partial v}{\partial\tilde{m}} \equiv \frac{\partial v}{\partial\tilde{p}}$. Note that the conditions (42), (47) and (48), combined with the definitions (49)-(51) imply the following inequalities:

$$\tilde{r}_F \geq \rho_Y > 0 \tag{E.1}$$
$$\tilde{m} \geq \rho_K > 0 \tag{E.2}$$
$$\tilde{r}_E \geq \rho_Y > 0 \tag{E.3}$$
$$\tilde{r}_E\theta - \rho_Y\theta - \tilde{r}_F < 0 \tag{E.4}$$

We now proceed to the comparative statics.

Effect of k : Consider, first of all, the effect of a change of k on $k_E(k,\theta,X)$. Since x

does not depend on k, we have that $k_E(k,\theta,X)$ is proportional to k. Thus :

$$\frac{\partial k_E}{\partial k} = \frac{k_E}{k} \geq 0$$

Effect of $\tilde{p}$: Similarly, since x does not depend on $\tilde{m}$, we have that $k_E(k,\theta,X)$ is proportional to $\tilde{m}$. Thus:

$$\frac{\partial k_E}{\partial \tilde{m}} = \frac{k_E}{\tilde{m}} \geq 0$$

Since $\frac{\partial k_E}{\partial \tilde{m}} \equiv \frac{\partial k_E}{\partial \tilde{p}}$ it follows that k_E is increasing in $\tilde{p}$.

Effect of $\tilde{r}_E$: It is immediately apparent that x is strictly increasing in $\tilde{r}_E$. Since $\frac{x}{S^{-1}(x)}$ is strictly decreasing in x by Lemma 4, it follows that $\frac{1}{1+\frac{x}{S^{-1}(x)}}$ is strictly increasing in x, and so $k_E(k,\theta,X)$ is strictly increasing in $\tilde{r}_E$ provided $x > 1$. It is also weakly increasing for $x < 1$ since $k_E(k,\theta,X) = 0$ in that case. By continuity of $k_E(k,\theta,X)$, it follows then that $k_E(k,\theta,X)$ must be weakly increasing in $\tilde{r}_E$ everywhere. Furthermore, we can also show that $k_E(k,\theta,X)$ is relatively elastic with respect to $\tilde{r}_E$ whenever $k_E(k,\theta,X) > 0$. By Proposition 1 it follows that in that case $x > 1$. Then we have that:

$$\begin{aligned}
\frac{\tilde{r}_E}{k_E}\frac{\partial k_E}{\partial \tilde{r}_E} &\equiv \frac{\tilde{r}_E}{k_E}\left(\frac{k(\tilde{r}_F+\tilde{p})}{\rho_Y\theta}\right)\left[\frac{\partial}{\partial x}\left(\frac{1}{1+\frac{x}{S^{-1}(x)}}\right)x\frac{\partial x}{\partial \tilde{r}_E} + \left(\frac{1}{1+\frac{x}{S^{-1}(x)}}\right)\frac{\partial x}{\partial \tilde{r}_E}\right] \\
&= \left[x\left(1+\frac{x}{S^{-1}(x)}\right)\frac{\partial}{\partial x}\left(\frac{1}{1+\frac{x}{S^{-1}(x)}}\right)+1\right]\frac{\tilde{r}_E}{x}\frac{\partial x}{\partial \tilde{r}_E}
\end{aligned}$$

Now,

$$\frac{\tilde{r}_E}{x}\frac{\partial x}{\partial \tilde{r}_E} = \frac{\tilde{r}_E}{x}\frac{\rho_Y\theta^2}{(\rho_Y\theta-\theta\tilde{r}_E+\tilde{r}_F)^2} = \frac{\tilde{r}_E x}{\rho}$$

which is larger than 1 when $\tilde{r}_E \geq \rho$ and $x > 1$. Since $\frac{\partial}{\partial x}\left(\frac{1}{1+\frac{x}{S^{-1}(x)}}\right) > 0$ (as argued above), we can conclude that $\frac{\tilde{r}_E}{k_E}\frac{\partial k_E}{\partial \tilde{r}_E} > 1$ whenever $k_E > 0$. That is, the elasticity of entrepreneurial capital with respect to the $\tilde{r}_E$ has to be greater than 1.

Effect of $\tilde{r}_F$: We note that $\frac{\partial k_E}{\partial \tilde{r}_F} = \frac{\partial k_E}{\partial x}\frac{\partial x}{\partial \tilde{r}_F} + \frac{\partial k_E}{\partial \tilde{m}}\frac{\partial \tilde{m}}{\partial \tilde{r}_F}$. Using the approach taken above for $\tilde{m}$ and $\tilde{r}_E$ to evaluate these, we have, whenever $k_E > 0$, that :

$$\frac{\partial k_E}{\partial \tilde{r}_F} = \frac{k_E}{\tilde{r}_F}\left(\left[x\left(1+\frac{x}{S^{-1}(x)}\right)\frac{\partial}{\partial x}\left(\frac{1}{1+\frac{x}{S^{-1}(x)}}\right)+1\right]\frac{\tilde{r}_F}{x}\frac{\partial x}{\partial \tilde{r}_F} + \frac{\tilde{r}_F}{\tilde{m}}\right)$$

where $\left[x\left(1+\frac{x}{S^{-1}(x)}\right)\frac{\partial}{\partial x}\left(\frac{1}{1+\frac{x}{S^{-1}(x)}}\right)+1\right]>0$ and

$$\frac{\tilde{r}_F}{x}\frac{\partial x}{\partial \tilde{r}_F}=\frac{-\tilde{r}_F x}{\rho_Y\theta}=\frac{-\tilde{r}_F}{\rho_Y\theta-\theta\tilde{r}_E+\tilde{r}_F}\leq -1$$

since $\tilde{r}_E \geq \rho_Y$. Now, provided $\tilde{p}>0$, it follows that $\frac{\tilde{r}_F}{\tilde{m}}=\frac{\tilde{r}_F}{\tilde{r}_F+\tilde{p}}<1$. Consequently, it follows that $\frac{\partial k_E}{\partial \tilde{r}_F}<0$ in that case.

Effect of θ: Since $\frac{1}{1+\frac{x}{S^{-1}(x)}}$ is increasing in x, and x is increasing in θ (since $\tilde{r}_E \geq \rho$) it follows that $\frac{1}{1+\frac{x}{S^{-1}(x)}}$ is increasing in θ. Furthermore, $\frac{x}{\rho_Y\theta}\equiv\frac{1}{\rho_Y\theta-\tilde{r}_E\theta+\tilde{r}_F}$ is increasing in θ (since $\tilde{r}_E \geq \rho$). Therefore, $k_E(k,\theta,X)$ is increasing in θ whenever $x=1$. Furthermore, $k_E(k,\theta,X)=0$ when $x<1$ so it is obviously (weakly) increasing in θ then. Since $k_E(k,\theta,X)$ is continous in θ, it follows that it is (weakly) increasing in θ everywhere. □

F Proof of Proposition 3

For convenience, we restate here the necessary and sufficient conditions for a steady state that are described in Proposition 3. They are:

$$\frac{\tilde{r}_F^*}{\tilde{r}_E^*}=\frac{F'_{Y_F^{S*}}}{F'_{Y_E^{S*}}} \tag{F.1}$$

$$Y_F^{S*}=K^*-K_E^* \tag{F.2}$$

$$\frac{K^*}{1+\tau_C^*}=\beta(1-\gamma)\left(\tilde{r}_E^*Y_E^{S*}+\tilde{r}_F^*Y_F^{S*}+\tilde{p}^*K^*\right) \tag{F.3}$$

$$C^*=\tilde{w}^*N+(1-\beta(1-\gamma))(\tilde{r}_E^*Y_E^{S*}+\tilde{r}_F^*Y_F^{S*}+\tilde{p}^*K^*) \tag{F.4}$$

$$C^*+\delta K^*+\overline{G}=F(Y_E^{S*},Y_F^{S*},N) \tag{F.5}$$

$$Y_E^{S*}=\frac{A_1K^*(\tilde{r}_F^*+\tilde{p}^*)}{\tilde{r}_F^*}\int_1^{\overline{x}}\left(\frac{1}{x+\frac{x^2}{S^{-1}(x)}}\right)\mathrm{d}x \tag{F.6}$$

$$K_E^*-Y_E^{S*}\left(\frac{\tilde{r}_E^*-\rho_Y}{\tilde{r}_F^*}\right)=\left(\frac{\rho_Y}{\tilde{r}_F^*}\right)\frac{A_1K^*(\tilde{r}_F^*+\tilde{p}^*)}{\tilde{r}_F^*}\int_1^{\overline{x}}\frac{1}{x}\left(\frac{1}{x+\frac{x^2}{S^{-1}(x)}}\right)\mathrm{d}x \tag{F.7}$$

where $\overline{x}=\frac{\rho_Y\overline{\theta}}{\rho_Y\overline{\theta}-\tilde{r}_E^*\overline{\theta}+\tilde{r}_F^*}\in(1,\infty)$, and $\tilde{r}_F^*\geq\rho_Y$; $\tilde{r}_E^*\geq\rho_Y$; $\tilde{r}_F^*+\tilde{p}^*\geq\rho_K$, $K_E^*<K^*$.

First we prove that these conditions are necessary for a steady state. We do this equation by equation.

For equation (F.1) note that $r_F^* = F'_{Y_F^{S*}}$ and $r_E^* = F'_{Y_E^{S*}}$ because of the first order conditions of the final goods firms. Now, by definition:

$$\begin{aligned} \tilde{r}_F^* &= \frac{(1-\tau_K^*) r_F^*}{1+\tau_C^*} \\ \tilde{r}_E^* &= \frac{(1-\tau_K^*) r_E^*}{1+\tau_C^*} \end{aligned}$$

Dividing the first of these equations by the second yields (F.1).

Equation (F.2) follows from the fact that each unit of capital in risk-free projects produces 1 unit of standard intermediate goods. Since, by assumption, no intermediate goods are hidden, if follows that $Y_F^{S*} = K_F^*$. Since, by assumption, no units of capital are hidden, all capital must either find its way into entrepreneurs' risky projects or risk-free projects. Therefore $K = K_F^* + K_E^*$ and (F.2) follows immediately.

To derive equation (F.3) note that, from equation (39) each entrepreneur chooses $\frac{k'}{1+\tau_C} = \beta(1-\gamma)\omega$. Since, in a steady state $K_t = K_{t+1}$, equation (F.3) follows immediately as long as:

$$\int_i \omega_i di = \tilde{r}_E^* Y_E^{S*} + \tilde{r}_F^* Y_F^{S*} + \tilde{p}^* K^* \tag{F.8}$$

Each entrepreneur's ω is given by equations (53) and (54). Integrating these equations across entrepreneurs and using $E[\epsilon] = 1$, we obtain (F.8) .

To derive equation (F.4) note that each worker consumes $c_W = \tilde{w}^*$ (this can be shown by combining equations (4) and (59)) and so aggregate worker consumption is $\tilde{w}^* N$. Each entrepreneur i consumes $(1-\beta(1-\gamma))\omega_i$. Then, (F.8) implies that aggregate entrepreneurial steady state consumption is equal to $(1-\beta(1-\gamma))(\tilde{r}_E^* Y_E^{S*} + \tilde{r}_F^* Y_F^{S*} + \tilde{p}^* K^*)$, whence (F.4) follows.

Equation (F.5) is just the steady state form of the goods market clearing condition (27), where we impose that $K_E^* + K_F^* = K_{t+1} = K^*$, since by assumption no capital is hidden, and impose $C_H^* = 0$, since by assumption no capital or intermediate goods are hidden. This immediately yields (F.5).

To show (F.6) and (F.7) , note that Proposition 1 implies that $k_E(k, \theta, X)$ is proportional to k. For the proof, it is convenient to define $\hat{k}(\theta) = \frac{k_E(k,\theta,X)}{k}$. Thus, two entrepreneurs with the same θ will always choose the same $\hat{k}(\theta)$.

Now, in equilibrium aggregate $K_{E,t}$ and $Y_{E,t}^S$ are given by equations (18) and (22). Since $\theta_{i,t}$ and $\epsilon_{i,t}$ are drawn independently of $k_{i,t}$ each period, and by assumption no intermediate goods are hidden, we use the standard abuse of the law of large numbers to infer that:

$$K_E^* = K^* \int_i \hat{k}_E(\theta)\mathrm{d}i = K^* \int_{\underline{\theta}}^{\overline{\theta}} \hat{k}_E(\theta)\mathrm{d}G(\theta) \tag{F.9}$$

$$Y_E^{S*} = K^* \int_{\underline{\theta}}^{\overline{\theta}} \theta\hat{k}_E(\theta)\mathrm{d}G(\theta) \tag{F.10}$$

Using that $\hat{k}_E(\theta)$ is given by Proposition 1 and that $G(\theta) = A_0 - \frac{A_1}{\theta}$ for $\theta \in [\underline{\theta}, \overline{\theta}]$, it is possible to integrate $\int_{\underline{\theta}}^{\overline{\theta}} \theta\hat{k}_E(\theta)\mathrm{d}G(\theta)$ and $\int_{\underline{\theta}}^{\overline{\theta}} \hat{k}_E(\theta)\mathrm{d}G(\theta)$ using the substitution $x = \frac{\rho_Y\theta}{\rho_Y\theta - \tilde{r}_E^*\theta + \tilde{r}_F^*}$ to transform the integral into an integral with respect to x. After some rearrangement, we obtain:

$$Y_E^{S*} = \frac{A_1K^*(\tilde{r}_F^* + \tilde{p}^*)}{\tilde{r}_F^*} \int_{\underline{x}}^{\overline{x}} \left(\frac{1}{x + \frac{x^2}{S^{-1}(x)}}\right) \mathrm{d}x \tag{F.11}$$

$$K_E^* - Y_E^{S*}\left(\frac{\tilde{r}_E^* - \rho_Y}{\tilde{r}_F^*}\right) = \left(\frac{\rho_Y}{\tilde{r}_F^*}\right) \frac{A_1K^*(\tilde{r}_F^* + \tilde{p}^*)}{\tilde{r}_F^*} \int_{\underline{x}}^{\overline{x}} \frac{1}{x} \left(\frac{1}{x + \frac{x^2}{S^{-1}(x)}}\right) \mathrm{d}x \tag{F.12}$$

where $\overline{x} = \frac{\rho_Y\overline{\theta}}{\rho_Y\overline{\theta} - \tilde{r}_E^*\overline{\theta} + \tilde{r}_F^*} \in (1, \infty)$, and $\underline{x} = \inf\{\theta \geq \underline{\theta} : \hat{k}_E(\theta) > 0\}$. Note that there must be at least some θ for which $\hat{k}_E(\theta) > 0$ since otherwise there would be no output of entrepreneurial intermediate goods, the Inada conditions would mean that r_E would go to infinity, and this could not be an equilibrium. Equations (18) and (22) follow immediately as long as $\underline{x} = 1$. To show that $\underline{x} = 1$, note that, by Proposition 1, an entrepreneur chooses $k_E > 0$ if and only if $x = \frac{\rho_Y\theta}{\rho_Y\theta - \tilde{r}_E^*\theta + \tilde{r}_F^*} > 1$. Then, it must be that $\underline{x} = 1$ provided at least some entrepreneurs choose $k_E = 0$ in the steady state. To show the latter, it suffices, given Proposition 1, to show that $\tilde{r}_F^* > \tilde{r}_E^*\underline{\theta}$. To this end, note that Proposition 3 requires that $\tilde{r}_F^* \geq \rho_Y$ and $\overline{x} \in (1, \infty)$, which itself requires that $\rho_Y\overline{\theta} - \tilde{r}_E^*\overline{\theta} + \tilde{r}_F^* > 0$. Combining these two inequalities, it follows that $\tilde{r}_F^*(\overline{\theta} + 1) - \tilde{r}_E^*\overline{\theta} > 0$. Recall that, by assumption, $\underline{\theta} < \frac{\overline{\theta}}{1+\overline{\theta}}$. Then, it follows that $\tilde{r}_F^* > \frac{\overline{\theta}\tilde{r}_E^*}{1+\overline{\theta}} > \tilde{r}_E^*\underline{\theta}$, as desired.

To show that the conditions of Proposition 1 are necessary for a steady state, it remains only to show that the conditions $\overline{x} \in (1, \infty)$ and $\tilde{r}_F^* \geq \rho_Y$, $\tilde{r}_E^* \geq \rho_Y$, $\tilde{r}_F^* + \tilde{p}^* \geq \rho_K$ and $K_E^* < K^*$ are necessary.To show that $\overline{x} \in (1, \infty)$ is necessary, note that this follows immediately provided $\rho_Y\overline{\theta} - \tilde{r}_E^*\overline{\theta} + \tilde{r}_F^* > 0$ and $\tilde{r}_E^*\overline{\theta} > \tilde{r}_F^*$. The condition $\rho_Y\overline{\theta} - \tilde{r}_E^*\overline{\theta} + \tilde{r}_F^* > 0$ is the same as the inequality (48), which was shown in Lemma 3 to be necessary for any equilibrium. The only change is that we have rewritten this inequality in terms of post-tax prices using the definitions (49)-(51). Hence, this inequality must be necessary for a steady state. The inequality $\tilde{r}_E^*\overline{\theta} > \tilde{r}_F^*$ must hold since otherwise Proposition 1 implies that all entrepreneurs will set $k_E = 0$. In that case, the Inada conditions on the final goods production function imply that r_E^* goes to infinity, which cannot be an equilibrium. Similarly, the inequality $K_E^* < K^*$ must hold in equilibrium since otherwise $K_F^* \leq 0$, in which case the Inada conditions imply that r_F^* goes to infinity (or is undefined), which also cannot be an equilibrium.

It remains to show that $\tilde{r}_F^* \geq \rho_Y$, $\tilde{r}_E^* \geq \rho_Y$ and $\tilde{r}_F^* + \tilde{p}^* \geq \rho_K$ are necessary for a steady state. These three inequalities are simply the same inequalities as (42) and (47), which were shown in Lemma 2 and Lemma 3 to be necessary for any equilibrium. The only change is that we have rewritten these inequalities in post-tax prices using the definitions (49)-(51). Hence, they must be necessary for a steady state.

This completes the proof that the conditions in Proposition 3 are necessary for a steady state. To show that these conditions are sufficient for a steady state, we assume that they hold and use them to construct an equilibrium in which aggregate variables are stable over time, thereby proving sufficiency by construction. In particular, we set $K_F^* = K^* - K_E^*$ and $Y^* = F(Y_E^{S*}, Y_F^{S*}, N)$ and we set pre-tax prices r_E^*, r_F^* and w^* to be consistent with the final goods firms' first order conditions (24)-(26). We set the tax rates τ_N^*, τ_K^* and τ_W^* to be consistent with these values of pre-tax prices, and the already assumed values of post-tax prices, according to equations (49)-(51) and (59). Since these aggregate variables are, by assumption, all stable over time, it follows that they form a steady state provided that these values are all consistent with equilibrium.

Therefore, it remains to show only that these values of aggregate variables and prices form an equilibrium in the sense of Definition 1. This requires that the following conditions be satisfied:

1. The Government's budget constraint (2) is balanced every period.
2. Worker consumption satisfies (4).
3. Entrepreneurs' decision rules are given by the solution to the entrepreneur's problem (14).
4. $\{C^*, K^*, K_E^*, K_F^*\}$ represent the aggregate of agents' decisions given by equations (15)-(20).
5. The asset market clears, according to equation (21).
6. The markets for intermediate goods clear, according to equations (22) and .
7. Prices of intermediate goods r_E^* and r_F^*, and wages w^* are determined by the first order conditions of the final goods firms (24)-(26).

Government budget balance can be shown by combining the condition (F.5) with the conditions (F.3) and (F.4), to eliminate C^* and $\beta(1-\gamma)$, using the final goods firms' first order conditions (24)-(26) and definitions of post-tax prices (49)-(51) and (59) to eliminate post-tax prices from the resulting equation, and using that constant returns to scale of final goods firms imply that $Y = r_E Y_E^S r_F Y_F^S + wN$ to eliminate $F(Y_E^{S*}, Y_F^{S*}, N)$. After a little rearrangement of the resulting equation, we obtain the government budget balance condition (2).

That workers and entrepreneurs are optimizing in the proposed steady state allocation follows because for the necessity section of this proof, we showed that equations (F.3) and (F.4) are consistent with workers' and entrepreneurs' optimization. Likewise for the necessity section of this proof we showed that the equations (F.3)- (F.7) come directly from integrating entrepreneurs' and workers' decision rules, and therefore must be consistent with the aggregation and market clearing conditions (15)-(20), (22) and , which were also direct integration and summation of the decision rules of individual agents.

The pre-tax prices are consistent with the final goods frims' first order conditions, since we explicitly defined the pre-tax prices so that these conditions would hold.

Finally, the definitions (49)-(51) imply that entrepreneurs are indifferent between lending to banks and putting resources into their risk-free projects. Therefore, we may assume that they lend to banks to exactly the level needed to ensure that asset markets clear. For instance, we can consider an equilibrium in which only one entrepreneur, whose wealth wealth is negligible compared to aggregate wealth, puts any resources into a risk-free project, and all the remaining entrepreneurs either put funds into risky projects or lend to the bank. Then, the total net quantity that the other entrepreneurs will wish to lend to the bank is equal to their total wealth, minus the amount they allocate to their risky projects, that is $K^* - K_E^*$. This will be precisely the amount that the one entrepreneur who manages a risk-free project will want to borrow, since $K^* - K_E^* = K_F^*$. Therefore, this is consistent with asset market clearing. □

G Proof of Lemma 5

By the conditions of Lemma 2 and Lemma 3, in any steady state entrepreneurs weakly prefer to hide no capital or intermediate goods. Therefore, for any feasible level of steady state post-tax prices the government chooses, entrepreneurs will weakly prefer to not hide capital or intermediate goods in the steady state. Consider a steady state $\mathscr{S}$ in which entrepreneurs hide a strictly positive amount of capital or intermediate goods. Now consider the alternative steady state $\mathscr{S}'$ with the same value of all post-tax prices facing entrepreneurs as $\mathscr{S}$, but in which entrepreneurs do not hide any capital or intermediate goods. Then, it must be that $\mathscr{S}'$ involves a higher value of aggregate output than $\mathscr{S}$, but entrepreneurs' consumption must be the same in $\mathscr{S}$ and $\mathscr{S}'$, since they are indifferent over these choices. Then, by the economy's aggregate resource constraint, $\mathscr{S}'$ must involve a higher level of worker consumption than $\mathscr{S}$ and so the government must strictly prefer $\mathscr{S}'$. Therefore, $\mathscr{S}$ cannot be a solution to the government's optimization problem. □

H Proof of Proposition 5

First we define

$$\chi = 1 - \frac{\tilde{r}_E^*}{r_E^*} \equiv \frac{\tau_C + \tau_K}{1 + \tau_C}$$

where the equivalence follows from the definition of $\tilde{r}_E^*$.

Next, we seek to characterize χ. To this end, subtract $\frac{\tilde{r}_F^*}{\tilde{r}_F^* + \tilde{p}^*}$ times the first order condition (71) from the first order condition (70). We get:

$$\begin{aligned}(1-\beta(1-\gamma))\left((Y_E^{S*}\tilde{r}_E^*K_E^* + \tilde{r}_F^*K_F - \frac{\tilde{r}_F}{\tilde{r}_F + \tilde{p}}(K\tilde{p}^* + K\tilde{r}_F^*)\right) &= \frac{\tau_C + \tau_K}{1+\tau_C}Y^*\left(e_K^\star - \frac{\tilde{r}_F^*}{\tilde{r}_F^* + \tilde{p}^*}e_W^\star\right) \\ &+ \tilde{r}_E^*\lambda_E + \lambda_F\tilde{r}_F^* \end{aligned} \tag{H.1}$$

whichi immediately simplifies to:

$$\begin{aligned}(1-\beta(1-\gamma))\left((Y_E^{S*}\tilde{r}_E^*K_E^* - \tilde{r}_F^*K_E^*\right) &= \frac{\tau_C + \tau_K}{1+\tau_C}Y^*\left(e_K^\star - \frac{\tilde{r}_F^*}{\tilde{r}_F^* + \tilde{p}^*}e_W^\star\right) \\ &+ \tilde{r}_E^*\lambda_E + \lambda_F\tilde{r}_F^* \end{aligned} \tag{H.2}$$

We can rewrite this as:

$$\begin{aligned}(1-\beta(1-\gamma))e_W^\star &= \chi\left(e_K^\star - \frac{\tilde{r}_F^*}{\tilde{r}_F^* + \tilde{p}^*}e_W^\star\right) \\ &+ \frac{\tilde{r}_E^*\lambda_E + \lambda_F\tilde{r}_F^*}{Y^*} \end{aligned} \tag{H.3}$$

Now, the assumptions of the proposition imply that we can use (68) and (69) above which characterize the elasticities. These imply that

$$e_K^\star - \frac{\tilde{r}_F^*}{\tilde{r}_F^* + \tilde{p}^*}e_W^\star \geq \frac{Y_E^{*S}r_E^*e_W^\star}{r_F^*K_E^*} > 0$$

Then it is immediate that $\chi > 0$ unless a constraint binds, since the bracketed term multiplying χ is strictly positive, and the left hand side is strictly positive. If a constraint binds it must be that either $\tilde{r}_E^* = \rho_Y$ or $\tilde{r}_F^* = \rho_Y$, in which case it must be the case that either $r_E^* > \tilde{r}_E^*$ or $r_F^* > \tilde{r}_F^*$ since $r_E^*, r_F^* > \rho_Y$ by the assumptions on the production function, and in that case it must be that $\chi > 0$ from the definition of χ. So χ is greater than zero. We can write χ as the solution to:

$$\chi = \min\left\{1 - \frac{\rho_Y}{\min\{r_E; r_F\}}; \frac{(1-\beta(1-\gamma))e_W^\star}{e_K^\star - \frac{(1-\chi)r_F e_W^\star}{\rho_K}}\right\} \tag{H.4}$$

Here the first half of the "min" is what χ must be by definition if a constraint binds and the second half of the "min" is the solution to (H.3) in the case where no constraint binds. We can infer that χ must be the minimum of these two since it must be weakly less than each side and must be either one or the other, given the definition of χ and equation (H.3).

Substituting (68) and (69) into (H.4) in place of $e_K^\star$ and $e_W^\star$ and rearranging, we obtain

$$\chi \leq \frac{(1-\beta(1-\gamma))r_F^*}{\bar{r}_E - \beta(1-\gamma)r_F^*}$$

This completes the proof of the characterization of χ given in 5.

Now, we show that optimal consumption taxes hit the highest level consistent with entrepreneurs choosing to hide no capital. That is, we prove that, in the government's choice of steady state, it must be the case that $\tilde{r}_F^* + \tilde{p}^* = \rho_K$. To this end, combine the two first order conditions (70) and (71) to get:

$$e_W^\star(Y_E^{S*}\tilde{r}_E^* + \tilde{r}_F^* K_F) \geq e_K^\star(K\tilde{p}^* + K\tilde{r}_F^*) - \tilde{\lambda}_P e_K^\star$$

where $\tilde{\lambda}_P = \frac{\lambda_P}{1-\beta(1-\gamma)}$. Then, using (68) and (69), we have:

$$\frac{Y_E^{S*}\tilde{r}_E^* + \tilde{r}_F^* K_F}{K\tilde{p}^* + K\tilde{r}_F^* - \tilde{\lambda}_P} \geq \frac{e_K^\star}{e_W^\star} \geq \frac{\frac{Y_E^{*S} r_E^*}{K_E^*} - r_F^*}{r_F^*} = \frac{Y_E^{*S} r_E^*}{r_F^* K_E^*} - \frac{\tilde{p}^*}{\tilde{r}_F^* + \tilde{p}^*}$$

That is, we can write:

$$\frac{Y_E^{S*}\tilde{r}_E^* + \tilde{r}_F^* K_F}{K\tilde{p}^* + K\tilde{r}_F^* - \tilde{\lambda}_P} \geq \frac{e_K^\star}{e_W^\star} \geq \frac{Y_E^{*S} r_E^*}{r_F^* K_E^*} - \frac{\tilde{p}^*}{\tilde{r}_F^* + \tilde{p}^*}$$

This can be rearranged to give:

$$\frac{\tilde{r}_F^*}{\tilde{r}_F^* + \tilde{p}^*} \left(\frac{K_E^* + \left(\frac{\tilde{r}_F^* K_E^*}{\tilde{r}_E^* Y_E^{*S} - \tilde{r}_F^* K_E^*} \right) \tilde{\lambda}_P}{K - \tilde{\lambda}_P} \right) \geq 1$$

We now prove by contradiction that $\tilde{\lambda}_P > 0$. To this end, suppose that $\tilde{\lambda}_P = 0$. Then, the round-bracketed term must be less than 1. In that case, the inequality cannot be satisfied unless

$$\frac{\tilde{r}_F^*}{\tilde{r}_F^* + \tilde{p}^*} > 1$$

However, $\tilde{r}_F^* + \tilde{p}^* \geq \rho_K$ by the constraints on the government's problem. Furthermore, $\tilde{r}_F^* \equiv (1-\chi) r_F^* < r_F^*$ since we showed above that $\chi > 0$. In that case, for $\tilde{\lambda}_P = 0$ to hold, it would need to be the case that $\frac{r_F^*}{\rho_K} > 1$. This is directly ruled out by the assumptions of the proposition. Therefore, under the conditions of the proposition, we can infer that $\tilde{\lambda}_P \neq 0$.

Now, since $\tilde{\lambda}_P = \frac{\lambda_P}{1-\beta(1-\gamma)}$ and λ_P was the multiplier on the constraint $\tilde{r}_F^* + \tilde{p}^* \geq \rho_K$ we conclude that it must be the case that $\lambda_P > 0$ and $\tilde{r}_F^* + \tilde{p}^* = \rho_K$ as desired.

Now, substituting $\tilde{r}_E^* = (1-\chi) r_E^*$, $\tilde{r}_F^* = (1-\chi) r_F^*$ and $\tilde{r}_F^* + \tilde{p}^* = \rho_K$, to eliminate the post-tax prices from equations (49)-(51) and rearranging the resulting equations, we obtain the results of the proposition. □