The Nash Wage Elasticity and its Business Cycle Implications

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Abstract

We develop a new measure of wage rigidity, the Nash wage elasticity (NWE). The NWE is the percentage change in the actual wage rate when the wage that would occur under Nash bargaining changes by 1%. We show that the NWE can be measured from aggregate data under relatively weak assumptions which hold across a large class of search and matching models. The empirical value can then be compared with the values predicted by specific models in this class. In the US data, our estimates of the NWE are generally between 0 and 0.1, indicating that, for both continuing workers and new hires, (a) there is a high degree of wage rigidity and (b) Nash bargaining provides a poor description of wage setting. We show that our estimates imply that wage rigidity greatly amplifies business cycles: A simple SAM model suggests that, if workers were paid Nash-bargained wages rather than actual wages, then the cyclical volatility of unemployment would decrease to less than one seventh of what it is in the data. We compare our results to various models of rigid and flexible wages in the literature.

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1 Introduction

The existing literature that seeks to estimate the degree of wage rigidity and to assess its importance in business cycles has so far failed to reach consensus, in part due to a fundamental obstacle to inference. The obstacle is that it has been difficult to argue that particular measures and estimation approaches correctly evaluate the rigidity of the marginal cost of labor except under strong and controversial assumptions about which theoretical model describes the labor market. As we discuss below, this obstacle has been a significant barrier not only for structurally estimated models featuring wage rigidity, but also for the reduced form literature on wage cyclicality. As such, without agreement on the preferred model of the labor market overall, it has been not been possible to agree on the level of wage rigidity supported by the data.

In an effort to surmount this obstacle, this paper develops and applies a new semi-structural method to estimate real wage rigidity and assess its business cycle implications. In doing so, we provide new evidence that wage rigidity is highly quantitatively important in US data, and plays a dominant role in explaining the volatility of unemployment over the business cycle. Our approach relies on an equation we derive relating the wage to other aggregate variables, which we show commonly holds across a very large class of different search and matching (SAM) models with Nash bargaining. By estimating specifications that regress actual wages on the Nash bargained wage implied by this equation, we are able to directly and straightforwardly compare US wage data to what would be implied by this large class of SAM models. The large class of models we study includes, for instance, models with many different shocks, rich firm and match heterogeneity, job-to-job transitions and various frictions in goods and financial markets. Our results indicate that models within this large class can only be made consistent with wage data (under conventional calibrations of other parameters) if the wages of both newly hired workers and job stayers are far more rigid than implied by Nash bargaining.

Our approach also allows us to infer the likely contribution of wage rigidity to the business cycle volatility of unemployment and to assess how far the data supports different models of rigid wages. For instance, in a simple SAM model with productivity shocks, we find that our estimated level of wage rigidity increases the volatility of unemployment more than sevenfold compared to what would occur under Nash bargaining, and can account for around half the unemployment volatility in the data. We show that our NWE estimates suggest that wages in the data are at least as rigid as in an alternating offer bargaining model based on Christiano, Eichenbaum and Trabandt (2016).

Throughout this paper, we use the term 'wage rigidity' to represent the notion that wages do not vary with macroeconomic conditions to the extent that would be expected if they were set by Nash bargaining. We propose a new measure of wage rigidity, the Nash wage elasticity (NWE). The NWE represents the percentage increase in the cost of labor when the wage rate implied by the Nash bargaining solution increases by 1%.[1] By construction, if wages are indeed set by Nash bargaining, the NWE is equal to 1. On the other hand, if wages are very rigid compared to Nash bargaining, the NWE will be closer to zero.

Measuring wage rigidity against the benchmark of Nash bargaining is desirable, we argue, because some flexible wage benchmark is required to meaningfully assess whether or not wages are rigid. That is, wages can only meaningfully be called rigid if their behavior deviates from what would be considered a flexible wage. We provide four reasons why Nash bargaining represents a logical flexible wage benchmark. First, Nash bargaining is perhaps the most common assumption used in the recent literature on unemployment over the business cycle, and so it is useful to know how far this is consistent with actual wage setting. Second, Nash bargaining is constrained efficient in the labor market in important cases, and so the NWE provides a useful yardstick of how flexible or rigid wages are likely to be compared to what would be constrained efficient. Third, as we discuss below, we show that the NWE is a strong predictor of the effects of wage rigidity on the cyclical volatility of unemployment, regardless of whether or not wages are actually set by Nash bargaining. Fourth, we show that different models without Nash bargaining, such as various rigid wage models, imply significantly different values for the NWE, and the implied NWE is less sensitive to other model assumptions aside from those about wage setting. Therefore, the NWE allows us to adjudicate which of these models are more consistent with wage data.

Our paper begins by developing and applying a semi-structural approach to estimate the NWE in US data. We derive a common wage equation that holds across a large class of models with Nash bargaining. We use this equation to impute a time series for the Nash wage from US data, without needing to adjudicate over which model in this large class corresponds to the true data generating process. We obtain estimates of the NWE by regressing measures of the actual cost of labor on the Nash wage. Across 180 regressions using various series for the cost of labor, various (or no) instruments for the Nash wage and various assumptions about the opportunity cost of employment and hiring costs, we mainly obtain NWE estimates between 0 and 0.1, indicating that wages of both job stayers and new hires are highly rigid in comparison to Nash bargaining. The data consistently favors an NWE below 0.65, except in the most extreme specifications, which use *both* the most procyclical wage series we consider (the 'user cost of labor' from the NLSY) *and* also assume high values of the opportunity cost of employment and/or very large fixed hiring costs. Intuitively, our consistently small estimates of the NWE are a consequence of the fact

[1] Pissarides (2009) uses the phrase Nash wage elasticity on occasion to mean the elasticity of Nash wages with respect to productivity. To avoid confusion, we stress that we use this term to mean something completely different.

that the Nash wage is much more procyclical than measures of actual wages, across these many different specifications.

Next, we provide novel analytical and simulation results to show that the NWE is a strong predictor of the cyclical volatility of unemployment, across a large class of models with shocks affecting the marginal revenue product of labor (e.g. productivity or markup shocks). We derive a tight mathematical relationship between the NWE and the Fundamental Surplus, which Ljungqvist and Sargent (2017) have shown is a valuable predictor of the cyclical volatility of unemployment in many search models. When the NWE is as low as most of our empirical estimates, we show that wage rigidity amplifies unemployment fluctuations in a simple SAM model with productivity shocks more than sevenfold compared to the case of Nash bargaining, and that such a model can easily account for around half of the empirical volatility of unemployment over the business cycle.

Lastly, we investigate how far our results are consistent with various other models of wage setting in the literature. We first examine models in which the wage is consistent with constrained efficiency in the labor market, such as many models of directed search. We find that these models would imply values of the NWE equal to or greater than 1, which is inconsistent with our empirical findings. We then examine a model of staggered wage bargaining similar to Gertler and Trigari (2009) and a model of alternating offer bargaining similar to Christiano, Eichenbaum and Trabandt (2016). We find that wages in the data are perhaps less rigid than implied by the calibrated staggered wage bargaining model but are more rigid than implied by the alternating offer bargaining model.

Overall, this paper makes five main contributions relative to the literature. These contributions are, first, to show that a large class of search and matching models imply a common equation for the Nash bargained wage. Second, to develop the concept of the Nash wage elasticity, which can be estimated for this large class of models without having to specify which model in this class corresponds to the true data generating process. Third, to provide a range of empirical estimates of the Nash wage elasticity, which overwhelmingly imply extremely rigid wages. Fourth, to show that, across a class of models with or without wage rigidity (or Nash bargaining), the Nash wage elasticity is a strong predictor of the contribution of wage rigidity to the volatility of unemployment over the business cycle. Fifth, to use our Nash wage elasticity estimates to make inferences about how far different models of non-Nash wage setting are consistent with wage data, such as models with constrained efficient wages or rigid wages.

Finally, while our approach is motivated by a desire to estimate wage rigidity and its business cycle implications, we anticipate that the general methodology that we develop could be useful in other contexts. For instance, it may be possible to use similar approaches to estimate price rigidity in goods markets, to estimate the elasticity of asset prices to fundamentals, or to estimate the elasticity of nominal exchange rates to differences in relative goods prices across countries.

The remainder of the paper is structured as follows. Section 1.1 discusses the related literature and compares our method and findings to this literature. Section 2 discusses the intuition for our approach, develops the modelling framework, derives equations to calculate the Nash wage and formally defines the Nash wage elasticity. Section 3 outlines the data sources and calibration used to calculate the Nash wage. Section 4 presents our empirical results and discusses the intuition behind our findings. Section 5 discusses the implications of our NWE estimates for business cycles and for models with non-Nash wage setting. Section 6 concludes.

1.1 Related Literature

In this section, we compare our approach to the large existing literature that seeks to estimate the level of wage rigidity and to infer its importance for business cycles. We also outline why our findings differ substantially from some of the work in this literature. While the literature has contributed greatly to our understanding of wage dynamics and business cycle propagation mechanisms, it is, as of yet, still far from consensus on the key questions of how far wages are rigid, and how far wage rigidity matters for business cycles.[2] The literature has been dominated by two broad approaches which differ quite significantly from ours: fully structural models and reduced form estimation.

The first of these two approaches taken by the literature has been to build structural SAM or other DSGE models and either calibrate them and compare to data or structurally estimate them against the data.[3] Since the wage setting process is modelled explicitly, the resulting parameters that are found to best fit the data are directly informative about the presence and/or nature of wage rigidity. Nevertheless, the conclusions drawn about the degree and importance of wage rigidity differ substantially across the different studies in this literature. A major reason is that the level of wage rigidity implied by any such calibrated or structurally estimated model may be very model specific, in that a different model may fit the same data equally well with a very different level of wage rigidity. As such, it is

[2]For instance, see Christiano, Eichenbaum and Trabandt (2021), Dupraz, Nakamura and Steinsson (2019), Gertler, Huckfeldt and Trigari (2020), Pissarides (2009), Basu and House (2016), and Bellou and Kaymak (2021) for recent contrasting views.

[3]Examples from this literature using SAM models include Christiano, Eichenbaum and Trabandt (2016), Gertler and Trigari (2009), Hagedorn and Manovskii (2008), Hagedorn and Manovskii (2013), Hall and Milgrom (2008) and Pissarides (2009). A number of studies in this literature, such as Merkl and Stüber (2017), Hagedorn and Manovskii (2013), and Gertler, Huckfeldt and Trigari (2020) provide new reduced-form estimates of wage cyclicality using an approach along the lines of the reduced form literature discussed below, and use the results of these regressions to calibrate or empirically evaluate a structural model. Prominent recent studies from the large literature assessing the importance of wage rigidity in New Keynesian DSGE models include Auclert, Bardóczy and Rognlie (2021) and Broer et al. (2020).

unsurprising that the findings of this literature regarding wage rigidity differ across models, and it is not usually clear how far the findings from any particular model are robust to model misspecification.

The principal difference between our approach and this literature is that we rely on a wage equation for the Nash bargained wage that we show holds commonly across a very large class of SAM models. Therefore, our conclusions about wage rigidity do not depend on which model in this class is the correct one, and so are arguably less sensitive to model misspecification.

Our findings also differ substantially from many (although certainly not all) of the SAM models in this literature in that we find a high level of wage rigidity. A key reason for this difference is that our approach is based on a Nash wage equation that applies under models with many different shocks. Therefore, our NWE estimates may be valid even if multiple shocks are important influences on labor demand. In contrast, many models in this literature assume that the only shocks driving fluctuations in labor demand are productivity shocks.[4] Under this assumption, Nash bargaining implies a tight link between wages and productivity, and so much of this literature considers the elasticity of wages with respect to productivity in the data to be very informative about wage rigidity, and empirically evaluates models accordingly.[5] However, given the lack of a strong correlation between unemployment and productivity in the data, it seems implausible that productivity shocks are the only driver of unemployment fluctuations. It is not clear then whether the elasticity of wages with respect to productivity is very informative about rigidity once we allow for other shocks. On the contrary, in our framework, which is consistent with multiple shocks, we find that the Nash wage is practically uncorrelated with productivity in the data, and so the elasticity of wages with respect to productivity is not informative about the NWE.

The second approach taken by the prior literature to estimate wage rigidity has been to estimate the cyclicality of real wages via reduced-form regressions. This approach originates with Dunlop (1938) and Tarshis (1939), Following Bils (1985), this literature has typically regressed a measure of wages (at the individual or aggregate level) on a cyclical indicator, such as the unemployment rate or productivity.[6]. An advantage of this approach, relative to the structural approach above, is that, to estimate the cyclicality of some wage measure,

[4]Examples of this include Hagedorn and Manovskii (2008), Hall and Milgrom (2008), Pissarides (2009) and Malcomson and Mavroeidis (2017).

[5]Thus, Hagedorn and Manovskii (2008), for instance, calibrate their model to match this elasticity, while Pissarides (2009) suggests that Nash bargaining is supported in the data if the elasticity of new hire wages with respect to productivity is close to 1.

[6]Recent examples of work in this vein includes Haefke, Sonntag and Van Rens (2013), Martins, Solon and Thomas (2012), Carneiro, Guimarães and Portugal (2012), Kudlyak (2014), Basu and House (2016), Gertler, Huckfeldt and Trigari (2020) Grigsby, Hurst and Yildirmaz (2021), Hazell and Taska (2020) and Schaefer and Singleton (2021). Much of the earlier literature that studies the wage cyclicality of job stayers and new hires in this way is surveyed by Pissarides (2009)

it is not necessary to write down a structural model of the labor market. This might seem to avoid the concerns of model misspecification inherent in the structural approach.

However, in practice, the reduced form literature has faced the same barrier to inference as the structural approach – the interpretability of its conclusions often rely on strong assumptions about the underlying theoretical model of the labor market. This is because estimates of wage cyclicality found by the reduced form literature are often highly sensitive to the choices of wage measure (e.g. the wage of all workers, new hires or new hires out of unemployment) and of cyclical indicator (e.g. unemployment or productivity). Which of these choices seems most justified depends on the theoretical model that the researcher has in mind. For instance, as discussed above, models in which productivity is the only shock suggest that productivity is the natural cyclical indicator on which to regress wages, but this conclusion does not immediately follow if there are other shocks. Equally, whether the wage measure that best captures the marginal cost of labor is the average wage of all workers, the wage of newly hired workers, the 'user cost of labor' developed by Kudlyak (2014) or none of these varies across theoretical models depending on whether a worker's current wages in the model are influenced by conditions when they hired, and depending on whether average match quality may vary over time (Kudlyak, 2014; Gertler, Huckfeldt and Trigari, 2020). Moreover, for a given wage series and cyclical indicator it is regressed on, it is impossible to infer whether an estimated wage cyclicality of e.g. 2% signifies a sticky or flexible wage without knowing how a flexible wage should behave. This is hard to ascertain without a theoretical model. Consequently, the literature has overwhelmingly interpreted the results of reduced form regressions using specific calibrated models; conclusions from this literature regarding whether the data supports a flexible or rigid wage then depend on the particular theoretical model and calibration strategy chosen.

Our approach differs from this literature in that it delivers an estimate of the Nash wage elasticity that can immediately be interpreted, for instance as evidence in favor or against Nash bargaining, without the need to commit to a particular theoretical model. Furthermore, our theoretical derivation of the Nash wage equation makes clear that the ideal measure of actual wages to estimate the NWE is the user cost of labor based on workers newly hired out of unemployment, and the cyclical indicator it should be regressed on is the Nash wage. This is true across a large class of models, since the same Nash wage equation holds across a large class of models. Therefore, we are able to answer questions of which wage measure and which cyclical regressor should be used, without needing to commit to a particular theoretical model. Finally, we find that estimates of the Nash wage elasticity are ultimately far below one in many specifications for *all* the measures of actual wages we consider, because all these wage measures are much less cyclical than the Nash wage. Therefore it turns out that the question of how to measure actual wages is relatively less important for estimating the

NWE.

In addition to the literature discussed above, our approach relates closely to recent work by Malcomson and Mavroeidis (2017), Bils, Klenow and Malin (2018), Koenig, Manning and Petrongolo (2021) and Ljungqvist and Sargent (2017). Malcomson and Mavroeidis (2017), like us, seek to estimate a wage-setting equation while imposing weak assumptions on the data generating process. Unlike us, they find that the data is consistent with new hire wages being set by Nash bargaining. We conjecture that the difference in results arises because they do not allow for markups and implicitly assume that fluctuations in labor demand are driven entirely by productivity shocks, which could lead to a bias against finding wage rigidity for the reasons discussed above. Koenig, Manning and Petrongolo (2021) show that a canonical DMP model implies a wage elasticity to unemployment far higher than the data, consistent with our finding of a low NWE. They also suggest informally that the elasticity of wages with respect to unemployment is informative across a class of models, and develop a model of reference-dependent wages to account for rigidity. Bils, Klenow and Malin (2018) study the cyclicality of the labor market wedge under search models, finding that much of this cyclicality can be accounted for by the product market wedge. Their findings provide additional evidence that labor market dynamics are affected by time-varying markups, as allowed for in our approach. Ljungqvist and Sargent (2017) show that in many different search and matching models, the determinants of unemployment fluctuations is driven by a common factor they call the 'fundamental surplus'.

More broadly, our work relates to the literature that studies the implications of search and matching models with wage rigidity for business cycle fluctuations. Hall (2005), Hall and Milgrom (2008), Christiano, Eichenbaum and Trabandt (2016) and Gertler and Trigari (2009), among others, develop models of rigid wages and show that these can help explain the volatility of unemployment over the business cycle. Dupraz, Nakamura and Steinsson (2019) find that downward wage rigidity can help account for business cycle asymmetries.

Finally, our approach of developing a measure of wage rigidity, the NWE, that is useful across different models has significant similarities to the literature on estimable sufficient statistics originating with Chetty (2009). Analogous to this literature, the Nash wage elasticity is a rough sufficient statistic that is highly informative about, for instance, the contribution of wage rigidity to business cycle fluctuations, across many different models.

2 Modelling Framework

In this section, we formally derive an equation for the Nash bargained wage that holds across a large class of search and matching models, incorporating rich firm and match heterogeneity, a wide variety of different shocks, possible frictions in goods and financial markets, job-to-job transitions and varying labor force participation.

We proceed in stages. First, to provide intuition for how it can be possible to derive a wage equation that holds under such broad conditions, we briefly discuss the case of perfectly competitive labor markets in Section 2.1. In Section 2.2 we derive the equation for the Nash wage in a framework featuring no firm or match heterogeneity. In Section 2.3 we expand our approach to show that virtually the same equation for the average Nash wage arises in a model which is much more general on a number of dimensions, including (but not limited to) firm and match heterogeneity, job-to-job transitions and time-varying labor force participation. Our aim is to derive an equation for the Nash wage which holds in as broad a class of SAM models as possible. Finally, in Section 2.5, we discuss how our Nash wage equation can be used to estimate the Nash Wage Elasticity using data on wages and labor market flows. Since our Nash wage equation holds across a very large class of models, it is possible to estimate the NWE without needing to make assumptions about which model in this class accurately describes the data generating process.

2.1 Intuition From Perfectly Competitive Labor Markets

Assume that identical households seek work in a single perfectly competitive spot labor market. As is well known, the resultant equilibrium wage rate must be on the household's labor supply curve, which means that it must equal the household's marginal rate of substitution between consumption and leisure, or the MRS for short.

The essence of our approach is to note that the wage rate will equal the MRS under a competitive spot labor market regardless of the determinants of labor demand. For instance, if firms have sticky prices in goods markets, or their ability to hire is affected by working capital constraints, or their capital investment is affected by financial frictions, all of these things will affect their labor demand and affect equilibrium employment and wages but the wage will continue to equal the MRS in all these cases. Likewise, if firms have heterogeneous productivities or markups, this will affect the aggregate demand for labor but the wage will continue to equal the MRS.

Then, in a spot labor market, a natural metric of wage rigidity is the elasticity of the observed wage rate with respect to the MRS. Since the perfectly competitive wage will equal the MRS under a wide variety of different assumptions about labor demand, the elasticity of observed wages with respect to the MRS provides a measure of how far observed wages are rigid, compared to competitive wages, and this measure remains equally valid and useful under a wide variety of different assumptions about labor demand. Of course, to measure wage rigidity in this way, it is necessary to have a time series for the MRS. The literature on the cyclicality of the labor wedge Chari, Kehoe and McGrattan (2007) has shown that it is straightforward to calculate a value of the MRS from aggregate data under standard assumptions about preferences.

Our approach differs from simply measuring wage rigidity in terms of the elasticity of wages with respect to the MRS because we allow for search frictions in labor markets. With search frictions, there is no longer any reason to expect that a flexible wage would equal the MRS. Instead, we derive a similarly general expression for the Nash wage which holds under a wide variety of different assumptions about firm and match heterogeneity and about the determinants of labor demand. Just as different assumptions about labor demand affect employment and wages but do not affect the basic equality between wages and the MRS in the competitive case, so different assumptions about labor demand also affect employment and wages but do not affect the Nash wage equation in the search theoretic case. In effect, the Nash wage equation we will derive is a search theoretic analogue to the supply curve for labor in a competitive market. That is, the Nash wage equation defines a locus of points that the wage rate should satisfy, conditional on labor market stocks and flows, and this locus is unaffected by the determinants of labor demand, just as the supply curve is unaffected by the determinants of labor demand in the competitive case.

As such, we define the Nash wage elasticity as the elasticity of observed wages with respect to the Nash wage derived from our Nash wage equation. We now derive the Nash wage equation formally.

2.2 The Nash wage without heterogeneity

In this section, we derive an equation for the Nash wage in a broad framework which nests a substantial number of different SAM models but does not allow for firm or match heterogeneity. We extend the results to a substantially more general setting in the next section.

We first outline the assumptions of our framework with no firm or match heterogeneity. Time is discrete. The economy consists of measure 1 of households and some measure of firms. Households live in large families, made up of employed and unemployed agents. Each large family shares consumption among its members. Unemployed agents match at the start of each period with vacancies v_t posted by firms in period t, according to the constant returns to scale continuously differentiable matching function $M_t = \overline{m}_t M(u_{t-1}, v_t)$, where $\overline{m}_t$ is a possible shock to the efficiency of the matching function, u_{t-1} is the number of unemployed at the end of the period $t-1$ and start of period t, and v_t is the number of vacancies posted in period t. The unemployed therefore find jobs at the job finding rate $f_t = \frac{M_t}{u_{t-1}}$. There is no on-the-job search.

At the start of period t, fraction s_t of employed agents separate from jobs. We allow that s_t may evolve stochastically over time in response to shocks. The measure of households who are unemployed, u_t, evolves over time according to the following law of motion:

$$u_t = (1 - f_t)u_{t-1} + s_t(1 - u_{t-1}) \tag{1}$$

Preferences: The members of each large family act to maximize the expected value of :

$$U = \sum_{t=0}^{\infty}(1-\rho)^t u(c_t),$$

where c_t is the consumption of the family and $u(\cdot)$ is strictly increasing and concave. employed agents earn wage rate w_t in period t. Unemployed agents engage in home production. Each unemployed agent produces z_t units of consumption each period, where z_t changes over time according to some stochastic process.

It is not realistic to interpret z_t as literally representing home production. We instead view z_t as a blackbox for the opportunity cost of employment, which, in a more general model, would include the utility value of the time that an unemployed person does not need to spend working, adjustments to reflect that the unemployed face different tax rates to wage earners, the various cash and in-kind benefits an unemployed person is entitled to, the possibility that these benefits may expire after a certain period of unemployment, and the utility cost of applying for these benefits. In a richer model that incorporates all these features, Chodorow-Reich and Karabarbounis (2016) show that it is possible to derive time series for z_t from aggregate and survey data under various assumptions about preferences. They point out that, as far as SAM models are concerned, what matters for aggregate wage and employment dynamics is the behavior of z_t, rather than the various components of z_t. As such, for simplicity, we do not model the components of z_t, and instead simply treat z_t as home production. In our empirical analysis we calculate the Nash wage using estimated series of z_t from Chodorow-Reich and Karabarbounis (2016), so that our conclusions depend on behavior of z_t that they argue fits the data.

Let $\mathcal{W}_t$ and $\mathcal{U}_t$ denote, respectively, the marginal present value of to the household of having an extra employed and unemployed agent. These evolve according to the following Bellman equations:

$$\begin{aligned}
\mathcal{W}_t = \quad & w_t + \mathbb{E}_t\left[(1-\rho)\frac{u'_{c_{t+1}}}{u'_{c_t}}[(1-s_{t+1})\mathcal{W}_{t+1} + s_{t+1}\mathcal{U}_{t+1}]\right], \\
\mathcal{U}_t = \quad & z_t + \mathbb{E}_t\left[(1-\rho)\frac{u'_{c_{t+1}}}{u'_{c_t}}[(1-f_{t+1})\mathcal{U}_{t+1} + f_{t+1}\mathcal{W}_{t+1}]\right],
\end{aligned}$$

where $\frac{u'_{c_{t+1}}}{u'_{c_t}}$ is the household's stochastic discount factor.

Firms: Firms post vacancies, which each cost κ_1 per period. Fraction q_t of vacancies are assumed to match with workers each period. If a vacancy matches with a worker, the firm hires the worker at additional hiring cost κ_0.

The total number of new matches M_t each period must satisfy:

$$M_t = q_t v_t = f_t u_{t-1}.$$

It follows that $q_t = \frac{u_t f_t}{v_t}$.

We assume, for notational convenience, that vacancies match with unemployed agents before production takes place, and so newly hired workers are productive in the period that they are hired.

Employed agents provide a gross flow value to the firm of r_t in period t, which we allow to evolve over time according to some stochastic process. We are completely agnostic about the determinants of r_t. The term r_t can be interpreted as the marginal revenue product of a worker – so it might depend on the markup as well as on the labor productivity and on the number of hours that a worker works.[7] More generally, if workers provide other useful services to a firm apart from producing output, such as research and development or training of other workers, then these may also enter r_t. Since we are agnostic about the determinants of r_t or about how it varies over time, our framework can nest any friction in goods or financial markets that maps into aggregate quantities via its effect on productivity or markups (and, therefore, on r_t). This might include, for instance, the effect of sticky prices in goods markets as in Ravenna and Walsh (2008), or working capital constraints, as in Christiano, Eichenbaum and Trabandt (2015).

Let $\mathcal{J}_t$ and $\mathcal{V}_t$ denote, respectively, the marginal present value of an extra worker and an extra vacancy to a firm. These satisfy the following Bellman equations:

$$\mathcal{J}_t = r_t - w_t + \mathbb{E}_t\left[\frac{u'_{c_{t+1}}}{u'_{c_t}}[(1 - s_{t+1})\mathcal{J}_{t+1} + s_{t+1}\mathcal{V}_{t+1}]\right], \tag{2}$$

$$\mathcal{V}_t = -\kappa_1 + (1 - q_t)\mathbb{E}_t\left[\frac{u'_{c_{t+1}}}{u'_{c_t}}\mathcal{V}_{t+1}\right] + q_t(\mathcal{J}_t - \mathcal{V}_t - \kappa_0). \tag{3}$$

Here, $\mathcal{J}_t$ appears on the right hand side of the Bellman equation for the vacancy in period t, because vacancies that are filled in period t already become productive that period.

Firms are able to create vacancies for free, so in equilibrium vacancy posting satisfies the

[7] It might seem that allowing for time-varying hours should require the disutility of working a particular number of hours to enter the value function of an employed worker. However, we may instead follow Chodorow-Reich and Karabarbounis (2016) and normalize so that the disutility of working the current mandated number of hours features as part of the value of being unemployed, and so this is incorporated into z_t.

free entry condition $\mathcal{V}_t = 0$. Substituting this into (3) and rearranging, we obtain:

$$\tilde{\mathcal{J}}_t = \kappa_0 + \frac{\kappa_1}{q_t} = \kappa_0 + \frac{\kappa_1 v_t}{u_{t-1} f_t} = h_t, \tag{4}$$

where h_t denotes the expected hiring cost.

Worker Share of Match Surplus: We now derive an equation that defines the share of match surplus that accrues to workers in in our framework. Below, we use this to derive a formula for the Nash wage. We define the worker share of match surplus, β_t, as the worker match surplus, divided by the total surplus. That is:

$$\beta_t = \frac{\mathcal{W}_t - \mathcal{U}_t}{\mathcal{J}_t - \mathcal{V}_t + \mathcal{W}_t - \mathcal{U}_t}.$$

Using that $\mathcal{V}_t = 0$ and $\mathcal{J}_t = h_t = \kappa_0 + \frac{\kappa_1 v_t}{u_{t-1} f_t}$, this can be written as:

$$\mathcal{W}_t - \mathcal{U}_t = \beta_t \left[\mathcal{W}_t - \mathcal{U}_t + \kappa_0 + \frac{\kappa_1 v_t}{u_{t-1} f_t} \right],$$

which rearranges to,

$$\mathcal{W}_t - \mathcal{U}_t = \frac{\beta_t}{1 - \beta_t} \left[\kappa_0 + \frac{\kappa_1 v_t}{u_{t-1} f_t} \right]. \tag{5}$$

Subtracting the Bellman equation for an unemployed agent from the Bellman equation for an employed agent, and substituting in (5) to eliminate $\mathcal{W}_t$ and $\mathcal{U}_t$ terms, we obtain the following dynamic equation for β_t:

$$\begin{aligned} \frac{\beta_t}{1 - \beta_t} \left(\kappa_0 + \frac{\kappa_1 v_t}{u_{t-1} f_t} \right) &= w_t - z_t \\ &+ \mathbb{E}_t \left[(1 - \rho) \frac{u'_{c_{t+1}}}{u'_{c_t}} (1 - s_{t+1} - f_{t+1}) \frac{\beta_{t+1}}{1 - \beta_{t+1}} \left(\kappa_0 + \frac{\kappa_1 v_{t+1}}{u_t f_{t+1}} \right) \right]. \end{aligned} \tag{6}$$

Evidently, the share of match surplus that goes to workers depends on the wage w_t, as is intuitive. Note that we have made no assumptions about how wages are actually set – the dynamic equation (6) characterizes the implied share of match surplus that is going to workers, for *any* well-behaved stochastic process governing w_t.

We assume that the economy fluctuates around a steady state. In the steady state, equation (6) implies that:

$$\frac{\beta}{1 - \beta} = \frac{(w - z)}{[1 - (1 - f - s)(1 - \rho)] h}, \tag{7}$$

where, abusing notation, we simply omit the time t subscript to denote the steady state value of a variable. Here we used that $h = \kappa_0 + \frac{\kappa_1 v}{uf}$.

The Nash Wage: We define the Nash wage, w_t^N, as the value that the wage w_t would have to take each period in order for the worker surplus share β_t to remain constant over time at its steady state value β, where β_t is calculated according to equation (6). Then, it follows that w_t^N satisfies:

$$\begin{aligned} \frac{\beta}{1-\beta}\left(\kappa_0 + \frac{\kappa_1 v_t}{u_{t-1} f_t}\right) =& w_t^N - z_t \\ &+ \mathbb{E}_t\left[(1-\rho)\frac{u'_{c_{t+1}}}{u'_{c_t}}(1 - s_{t+1} - f_{t+1})\frac{\beta}{1-\beta}\left(\kappa_0 + \frac{\kappa_1 v_{t+1}}{u_t f_{t+1}}\right)\right], \end{aligned} \tag{8}$$

where $\frac{\beta}{1-\beta}$ is given by equation (7). In our empirical analysis, we assume that the steady state values of variables are equal to their average in the sample period. Thus, w_t^N is the value that w_t would need to take each period in order for $\frac{\beta_t}{1-\beta_t}$ to remain forever equal to its average value over the sample period.

We refer to w_t^N defined in this way as the Nash wage, since, under the Nash sharing rule, the worker share of match surplus β_t is given by the bargaining strength of workers. The standard assumption in the SAM literature is that this is constant over time so that $\beta_t = \beta$, in which case equations (6) and (8) imply that $w_t = w_t^N$.

Two further remarks are in order regarding the relationship between w_t^N and the concept of Nash bargaining in SAM models. First, it is more precise to say that w_t^N is defined according to the Nash sharing rule rather than the Nash bargaining solution. In the basic DMP model, the two coincide: Nash bargaining implies that the worker gets a constant fraction β of the match surplus. However, in some models with frictions, such as Schoefer (2021), the Nash bargaining solution does not imply that the worker's share of match surplus remains constant over time, even though the bargaining weight of workers remains constant. The wage-setting arrangements in such models are therefore not consistent with our notion of the Nash wage, although we conjecture that the the Nash bargaining solution will nevertheless deliver something close to a constant share of the match surplus going to workers in many such models in practice, in which case our Nash wage will continue to provide a good approximation to the outcome of Nash bargaining.

Second, since our Nash wage implies a constant share of surplus going to workers, it rules out the possibility of shocks to worker bargaining power, as considered by, for instance, Shimer (2005). If, in reality, there are shocks to worker bargaining power, this could lead to systematic errors in the time series we derive for the Nash wage and bias our estimates of the Nash wage elasticity. We discuss in Section 3 how we use other structural shocks as instruments for the Nash wage in our estimation strategy to avoid these biases.

2.3 Nash Wage Equation in a More General Environment

The framework used in the previous section to derive the Nash wage equation (8) was relatively general on some dimensions. We made almost no assumptions about the stochastic process underlying the opportunity cost of labor z_t, wages w_t, the marginal revenue product of labor r_t, the separation rate s_t or possible shocks to the matching function. Thus, the framework in the previous nests many different models with different assumptions about how these variables are determined.

Nevertheless, the framework in the previous section was special in that, for instance, it did not allow for firm or match heterogeneity. We now generalize our framework to allow for rich heterogeneity in firms and matches, including heterogeneity in firms' discount factors (for instance, due to financial frictions), as well as allowing for endogenous separations, job-to-job transitions and time-varying labor force participation. Our approach is to derive an equation for the Nash wage while making as few assumptions as possible, in order to create a framework which nests as many different SAM models as possible. We show that, even in this much more general case, the equation describing the Nash wage looks very similar to the equation derived in the previous section.

We now outline the assumptions of our more general framework. We maintain the same assumptions as in Section 2.2 except where noted.

Labor Market Flows: As before, there are measure 1 of households who live in large families. We allow for the possibility that some members of a family are economically inactive, i.e. not in the labor force. Let i_t denote the measure of agents who are economically inactive in period t.

All unemployed agents are identical. We let f_t denote the fraction of unemployed agents who find a job in period t by successfully matching with vacancies. We do not require that all matches are accepted – since we allow for the possibility that poor quality matches are rejected. Thus f_t denotes the probability that an unemployed agent finds a match in period t that she accepts.

It is assumed that agents may shift between being unemployed or economically inactive, but economically inactive agents cannot go straight into employment without first becoming an unemployed agent who looks for a job in some period t, and potentially finding a job in period $t+1$.[8]

Then, the law of motion of the measure of unemployed agents at the end of period t, u_t, is:

$$u_t = (1 - f_t)u_{t-1} + s_t(1 - i_{t-1} - u_{t-1}) - (i_t - i_{t-1}),$$

[8] It is well known that, in the data, economically inactive individuals do find jobs without previously being registered as unemployed. We interpret such individuals as people who were, in truth, looking for work and therefore unemployed, but were mismeasured as economically inactive.

where s_t denotes the average separation rate of all employed agents. For simplicity, we do not model agents' choices of whether to be unemployed or economically inactive, we merely assume that i_t evolves over time according to some endogenous stochastic process. Explicitly modelling this motivation would not affect our conclusions provided the flow value of unemployment, z_t, is taken as given.

Household Bellman Equations: We allow that workers in different matches and/or at different firms may earn different wages, and that the wages of a worker in a match may evolve idiosyncratically and endogenously over time due to, for instance, match-specific human capital accumulation, or long-term wage contracting as in Rudanko (2009). We also allow the separation rate into unemployment to vary over time and across matches and firms. This could be due to e.g. endogenous separations, where low productivity matches have a higher probability of separating. Let $w_t^{i,k}$ and $s_t^{i,k}$ denote the wage and separation rate in match k at firm i at time t.

We also allow for possible job-to-job transitions, which occur at the start of each period, simultaneously with separations into unemployment. Let $\lambda_t^{i,k}$ denote the probability of a worker in match k at firm i transferring to a new job at the start of period t. This may vary over time and across matches, since workers may be more likely to look for other jobs if their match quality is low.[9]

The Bellman equations for an unemployed agent, and a for an employed agent in the match k, are as follows:

$$\mathcal{U}_t = z_t + \mathbb{E}_t\left[(1-\rho)\frac{u'_{c_{t+1}}}{u'_{c_t}}[(1-f_{t+1})\mathcal{U}_{t+1} + f_{t+1}\tilde{\mathcal{W}}_{t+1,t+1}]\right], \tag{9}$$

$$\mathcal{W}_t^{i,k} = w_t^{i,k} + \mathbb{E}_t\left[(1-\rho)\frac{u'_{c_{t+1}}}{u'_{c_t}}[(1-s_{t+1}^{i,k}-\lambda_{t+1}^{i,k})\mathcal{W}_{t+1}^{i,k} + \lambda_{t+1}^{i,k}\tilde{\mathcal{W}}_{t+1}^{i,k,T} + s_{t+1}^{i,k}\mathcal{U}_{t+1}]\right], \tag{10}$$

where $\mathcal{W}_t^{i,k}$ denotes the value of a worker in match k at firm i at time t and $\tilde{W}_{t+1}^{i,k,T}$ denotes the expected value that the worker in match k at firm i at the start of $t+1$ expects to have, if she transitions directly to a new job in period $t+1$. $\tilde{\mathcal{W}}_{t,\tau}$ denotes the average value among workers at time t, who were most recently unemployed at the start of period τ, and found a job during period τ. By the usual abuse of the law of large numbers, we assume therefore that an unemployed agent who finds a job in period $t+1$ has expected value $\tilde{W}_{t+1,t+1}$ in that period.

Firms: Firms, i, and matches, k, are heterogeneous in terms of the marginal flow value to the firm of the match, which we denote by $r_t^{i,k}$. This may be due to heterogeneous productiv-

[9] Of course, our setting also nests models with no job-to-job transitions, which amounts tofixing $\lambda_t^{i,k} = 0$ for all i,k,t.

ity or markups across firms. If firms have concave production functions or downward-sloping demand curves then $r_t^{i,k}$ will also depend on the number of workers employed by a firm.

Firms may hire out of unemployment, or may hire already employed agents, which precipitates a job-to-job transition. For mathematical tractability, we assume that the unemployed and the already employed match with firms in different submarkets, with potentially more than one submarket for matching with the already employed, as occurs in models of directed search with job-to-job transitions. Then, when a firm posts a vacancy, it decides whether to target the unemployed or the already employed.

As before, we assume a vacancy posting cost of κ_1 and a fixed hiring cost of κ_0 if the vacancy successfully yields a new hire. Let $q_t^{i,u}$ denote the probability that a vacancy posted by firm i that targets the unemployed successfully turns into a match (and an employment relationship) in period t. We allow that $q_t^{i,u}$ varies across firms because, in a setting with firm and match heterogeneity, it is possible that the matches at lower productivity firms are less likely to be accepted, and so less likely to turn into employment relationships.[10]

Then, the Bellman equations for a firm with a match $\mathcal{J}_t^{i,k}$ and a vacancy at firm i that targets the unemployed $\mathcal{V}_t^{i,u}$ are as follows:

$$\mathcal{J}_t^{i,k} = r_t^{i,k} - w_t^{i,k} + \mathbb{E}_t\Big[m_{t+1}^i(1 - s_{t+1}^{i,k})\mathcal{J}_{t+1}^{i,k}\Big], \tag{11}$$

$$\mathcal{V}_t^{i,u} = -\kappa_1 + q_t^{i,u}(\tilde{\mathcal{J}}_{t,t}^i - \mathcal{V}_t^{i,u} - \kappa_0). \tag{12}$$

Here $\tilde{\mathcal{J}}_{t,\tau}^i$ denotes the expected value of a match at firm i at the start of t if the worker was hired out of unemployment in period τ. m_{t+1}^i is the firm i's stochastic discount factor, which we allow to potentially depend on the firm i – this might occur if, for instance, some firms value their cashflow today more relative to the future due to short-term financing constraints.

The Bellman equations above already incorporate that a firm can freely create and dispose of new vacancies. This means that it must be the case that $\mathcal{V}_t^{i,u} \leq 0$, with equality if the firm is maintaining at least one vacancy in period t.

Then, equation (12) implies that, if firm i hires in period t, then,[11]

$$\tilde{\mathcal{J}}_{t,t}^i = \kappa_0 + \frac{\kappa_1}{q_t^{i,u}}. \tag{13}$$

[10] For instance, if a worker and firm observe the idiosyncratic match productivity of a match before deciding whether to go ahead with the match, then it may be that matches at a low productivity firm will only be accepted if the idiosyncratic match-specific component of productivity is particularly good, which may be a low probability event.

[11] Note that, in many models, it is possible for equation (13) to hold in equilibrium for multiple firms with different values of $q_t^{i,u}$ and $\tilde{\mathcal{J}}_{t,t}^i$. In particular, in a model with large firms facing downward-sloping demand curves or concave production functions, firms with a higher $q_t^{i,u}$ will, all else equal, hire more and will see the marginal value of $\tilde{\mathcal{J}}_{t,t}^i$ fall for these firms until (13) holds with equality.

Worker Share of Match Surplus: We now derive an expression for the worker's share of match surplus in this more general framework. We define β_t as the average share of match surplus at time t that is earned by workers who are newly hired out of unemployment in that period (where the average is across new matches of such workers in period t). As will be seen below, it is this measure of worker surplus share for which there exists a mathematical formulation in terms of labor market flows that is almost identical to equation (6) above. Thus, we define β_t as:

$$\beta_t = \frac{\tilde{\mathcal{W}}_{t,t} - \mathcal{U}_t}{\tilde{\mathcal{W}}_{t,t} - \mathcal{U}_t + \frac{\int_i q_t^{i,u} v_t^{i,u} \tilde{\mathcal{J}}_{t,t}^i \mathrm{d}i}{\int_i q_t^{i,u} v_t^{i,u} \mathrm{d}i}}.$$

Here, the term $\tilde{\mathcal{W}}_{t,t} - \mathcal{U}_t$ represents the average match surplus of workers newly hired out of unemployment at time t, as defined above. The term $\tilde{\mathcal{J}}_{t,t}$ represents the expected surplus each firm i gets from hiring such workers, and the integral reflects that this should be averaged across firms i in proportion to their share of total hiring out of unemployment, with the hiring of firm i out of unemployment given, in expectation, by $q_t^{i,u} v_t^{i,u}$.[12]

Rearranging this, and using that (13) holds for all firms that set $v_t^{i,u} > 0$, we obtain:

$$\tilde{\mathcal{W}}_{t,t} - \mathcal{U}_t = \left(\frac{\beta_t}{1-\beta_t}\right) \frac{\int_i q_t^{i,u} v_t^{i,u} \tilde{\mathcal{J}}_{t,t}^i \mathrm{d}i}{\int_i q_t^{i,u} v_t^{i,u} \mathrm{d}i} = \left(\frac{\beta_t}{1-\beta_t}\right) \left(\kappa_0 + \frac{\kappa_1 v_t^u}{\int_i q_t^{i,u} v_t^{i,u}}\right),$$

where $v_t^u = \int_i v_t^{i,u} \mathrm{d}i$ denotes the total number of vacancies targeted at the unemployed.

The total number of unemployed agents that find jobs must equal the total number of such vacancies that match, so that $\int_i q_t^{i,u} v_t^{i,u} \mathrm{d}i = u_{t-1} f_t$. Then, we can rewrite the equation above as:

$$\tilde{\mathcal{W}}_{t,t} - \mathcal{U}_t = \left(\frac{\beta_t}{1-\beta_t}\right) \left(\kappa_0 + \frac{\kappa_1 v_t^u}{u_{t-1} f_t}\right). \tag{14}$$

In order to use this to derive a dynamic equation for β_t similar to (6), it is necessary to characterize $\tilde{\mathcal{W}}_{t,t}$. We do this by gradually integrating the Bellman equation for $\mathcal{W}_t^{i,k}$ across workers and employment histories. This requires some additional notation. Let $\mathscr{P}(j, m, t | i, k, \tau)$ denote a probability measure representing the probability that a worker who left unemployment at time τ, obtaining the match k in firm i will subsequently find themselves in period t in match m in firm j, without having spent a spell of unemployment in between (where these probability measures are based on the information available at the start of period τ). Let $f_\tau^{i,k}$ denote a probability measure representing the probability that an unemployed agent at the start of period τ successfully forms the match k with firm i in period τ (where these probability measures are based on the information available at the

[12] Here we are making the usual abuse of the law of large numbers by taking the expectations of the values of firms and hiring costs when integrating across firms.

start of period τ). Let

$$\overline{\mathcal{W}}^{i,k}_{t,\tau} = \frac{\int_{j,m} \mathcal{W}^{j,m}_t \mathrm{d}\mathscr{P}(j,m,t|i,k,\tau)}{\int_{j,m} \mathrm{d}\mathscr{P}(j,m,t|i,k,\tau)}, \qquad \overline{w}^{i,k}_{t,\tau} = \frac{\int_{j,m} w^{j,m}_t \mathrm{d}\mathscr{P}(j,m,t|i,k,\tau)}{\int_{j,m} \mathrm{d}\mathscr{P}(j,m,t|i,k,\tau)},$$

$$\overline{s}^{i,k}_{t,\tau} = \frac{\int_{j,m} s^{j,m}_t \mathrm{d}\mathscr{P}(j,m,t|i,k,\tau)}{\int_{j,m} \mathrm{d}\mathscr{P}(j,m,t|i,k,\tau)}, \qquad \tilde{s}_{t,\tau} = \frac{\int_{i,k} \overline{s}^{i,k}_{t,\tau}(\overline{\mathcal{W}}^{i,k}_{t,\tau} - \mathcal{U}_t)\mathrm{d}f^{i,k}_\tau}{\int_{i,k}(\overline{\mathcal{W}}^{i,k}_{t,\tau} - \mathcal{U}_t)\mathrm{d}f^{i,k}_\tau}.$$

That is, $\overline{\mathcal{W}}^{i,k}_{t,\tau}$, $\overline{w}^{i,k}_{t,\tau}$ and $\overline{s}^{i,k}_{t,\tau}$ denote the average values of $\mathcal{W}$, w and s that a worker expects to obtain at time t, if the worker is hired in match k by firm i at time τ, and remains continuously employed (without a spell of unemployment) between time τ and time t. $\tilde{s}_{t+1,\tau}$ is the average separation rate at time $t+1$ of workers who were hired out of unemployment in period τ (and have not since become unemployed) where the average is weighted across matches in proportion to the surplus of those matches.

Then, it follows from equation (10) that $\overline{\mathcal{W}}^{i,k}t,\tau$ evolves according to:[13]

$$\overline{\mathcal{W}}^{i,k}_{t,\tau} = \overline{w}^{i,k}_{t,\tau} + \mathbb{E}_t\left[(1-\rho)\frac{u'_{c_{t+1}}}{u'_{c_t}}[(1-\overline{s}^{i,k}_{t+1,\tau})\overline{\mathcal{W}}^{i,k}_{t+1,\tau} + \overline{s}^{i,k}_{t+1,\tau}\mathcal{U}_{t+1}]\right] \tag{15}$$

Integrating across all new workers hired out of unemployment at time τ, we obtain, after some rearrangement:

$$\tilde{\mathcal{W}}_{t,\tau} = \tilde{w}_{t,\tau} + \mathbb{E}_t\left[(1-\rho)\frac{u'_{c_{t+1}}}{u'_{c_t}}[(1-\tilde{s}_{t+1,\tau})\tilde{\mathcal{W}}_{t+1,\tau} + \tilde{s}_{t+1,\tau}\mathcal{U}_{t+1}]\right].$$

Repeatedly recursively substituting (14) and (9) into this equation to eliminate $\mathcal{W}$ and $\mathcal{U}$ terms, we obtain the dynamic equation that describes β_t:

$$\begin{aligned}\frac{\beta_t}{1-\beta_t}\left(\kappa_0 + \frac{\kappa_1 v^u_t}{u_{t-1}f_t}\right) &= w^{UC}_t - \Phi_t - z_t \\ &+ \mathbb{E}_t\left[(1-\rho)\frac{u'_{c_{t+1}}}{u'_{c_t}}(1-\tilde{s}_{t+1,t} - f_{t+1})\frac{\beta_{t+1}}{1-\beta_{t+1}}\left(\kappa_0 + \frac{\kappa_1 v^u_{t+1}}{u_t f_{t+1}}\right)\right],\end{aligned} \tag{16}$$

[13] To prove formally that (10) implies (15), first note that (10) implies (15) when $t=\tau$. This follows from integrating (10) across all matches that a worker could move to as part of an on-the-job transition. Then, by a symmetrical argument, note that if (10) implies (15) for some $t=n$ and τ, then (10) implies (15) for $t=n+1$ and τ. The result then follows by induction.

where,

$$w_t^{UC} = \tilde{w}_{t,t} + \mathbb{E}_t \sum_{j=1}^{\infty} (1-\rho)^j \frac{u'_{c_{t+j}}}{u'_{c_t}} \Bigg[\tilde{w}_{t+j,t} \left(\prod_{k=1}^{j} (1 - \tilde{s}_{t+k,t}) \right) - \left(\frac{1-\tilde{s}_{t+1,t}}{1-\tilde{s}_{t+1+j,t+1}} \right) \tilde{w}_{t+j,t+1} \left(\prod_{k=1}^{j} (1 - \tilde{s}_{t+1+k,t+1}) \right) \Bigg], \tag{17}$$

$$\Phi_t = \mathbb{E}_t \sum_{j=2}^{\infty} (1-\rho)^j \left(\frac{u'_{c_{t+j}}}{u'_{c_t}} \right) \mathcal{U}_{t+j} \Bigg[- \left(\frac{\tilde{s}_{t+j,t}}{1-\tilde{s}_{t+j,t}} \right) \left(\prod_{k=1}^{j} (1 - \tilde{s}_{t+k,t}) \right) + \left(\frac{\tilde{s}_{t+j,t+1}}{1-\tilde{s}_{t+j,t+1}} \right) \left(\frac{1-\tilde{s}_{t+1,t}}{1-\tilde{s}_{t+1+j,t+1}} \right) \left(\prod_{k=1}^{j} (1 - \tilde{s}_{t+1+k,t+1}) \right) \Bigg], \tag{18}$$

$$\mathcal{U}_t = z_t + \mathbb{E}_t \sum_{j=1}^{\infty} (1-\rho)^j \frac{u'_{c_{t+j}}}{u'_{c_t}} \left[z_{t+j} + f_{t+j} \left(\frac{\beta_{t+j}}{1-\beta_{t+j}} \right) h_{t+j} \right]. \tag{19}$$

Equation (16) is identical to equation (6), which described β_t in the model with no firm or match heterogeneity, except for the following differences:

1. The vacancy rate v_t in equation (6) is replaced by v_t^U: the number of vacancies targeted at the unemployed.

2. The separation rate s_t in equation (6) is replaced by $\tilde{s}_{t+1,t}$: the average next period separation rate of workers newly hired out of unemployment, where the average is weighted by the match surplus.

3. The wage rate w_t is replaced in equation (16) with the wage component of the *user cost of labor*, w_t^{UC}, based on workers hired directly out of unemployment. The user cost of labor is a concept first developed by Kudlyak (2014), who observed that in models where wages in a match continue depend on conditions under which the worker was first hired, the macroeconomically relevant measure of the cost of labor is not the current wage rate, but instead depends on the wage of newly hired workers and also the future wage changes that these newly hired workers expect in future.
   Our expression for w_t^{UC} is the same as the expression for the wage component of the user cost of labor in Kudlyak (2014) except for two key differences. First, we allow for a time varying stochastic discount factor and a separation rate that varies across matches and over time, which complicates the user cost equation. Second, our derivation of w_t^{UC} makes clear that the correct measure of the user cost of labor for our purposes depends on the expected present and future wages of workers *hired directly*

out of unemployment. For instance, the key first term in our user cost equation, $\tilde{w}_{t,t}$, is the average wage of workers newly hired out of unemployment. In contrast, Kudlyak measures the user cost using the wages of all newly hired workers – many of whom are workers transitioning from one job to another. As argued by Gertler, Huckfeldt and Trigari (2020), the behavior of the wages of workers transitioning from one job to another can give a very misleading impression of the cost of labor for firms, in cases where workers are more likely to move to higher quality matches in booms. At the same time, contrary to what Gertler, Huckfeldt and Trigari (2020) and Bellou and Kaymak (2021) have suggested, it is possible to accurately estimate the relevant notion of user cost in the models encompassed by our framework without needing to measure, control or make assumptions about match quality, since there are no measures of match quality in the equation (17).

4. There is an additional term Φ_t, which is non-zero if (and only if) the probability of a worker losing their job and entering unemployment depends on the number of periods that the worker has been employed.[14] The term Φ_t enters equation (16) because if, for instance, holding a job for longer reduces the likely future separation rate, then a worker who is hired at t and retains their job into $t+1$ is less likely to be unemployed at, e.g. time $t+10$ than a worker who is hired at $t+1$. The consequence is that the value of being unemployed at time $t+10$ therefore affects the value of accepting a job today, and so future unemployment values $\mathcal{U}_{t+j}$ enter into the expression for Φ_t.

The Nash Wage: In the more general case, we define the Nash wage w_t^N as the time series that the user cost of labor w_t^{UC} would take each period according to equation (16), if it were the case that the share of match surplus going to each worker newly hired from unemployment β_t, remained forever constant at some steady state value β (which we take, in our empirical analysis, to be the average observed value of β_t in the sample period). Fixing $\beta_t = \beta$ in (16), this implies the Nash wage equation:

$$\begin{aligned} w_t^N &= \Phi_t^N + z_t + \frac{\beta}{1-\beta}\left(\kappa_0 + \frac{\kappa_1 v_t^u}{u_{t-1}f_t}\right) \\ &\quad - \mathbb{E}_t\left[(1-\rho)\frac{u'_{c_{t+1}}}{u'_{c_t}}(1-\tilde{s}_{t+1,t}-f_{t+1})\frac{\beta}{1-\beta}\left(\kappa_0 + \frac{\kappa_1 v_{t+1}^u}{u_t f_{t+1}}\right)\right], \end{aligned} \tag{20}$$

[14]This will be typically true in, for instance, models with endogenous separations – where a longer period in which a worker has been employed may e.g. indicate a likely higher match quality and therefore a lower chance of future separation.

where β is given by the steady state value

$$\frac{\beta}{1-\beta} = \frac{(w - z - \Phi^N)}{[1 - (1 - f - s)(1 - \rho)]h},, \tag{21}$$

and where Φ_t^N and Φ^N are calculated by substituting $\beta_t = \beta$ into equation (18) and (19).

2.4 Cases Nested by Our Modelling Framework

Our framework in the previous section deliberately imposes minimal structure in order to encapsulate a wide variety of models and mechanisms discussed in the literature. Since we can derive the equation for the Nash wage without imposing more structure, it follows that our equation for the Nash wage holds across a very large range of different models. The consequence of this is that our estimates for the Nash wage elasticity in Section 3 will be valid across a large class of models.

Here we briefly outline some of the many cases nested by our framework. Of course, an important special case of our framework is a discrete time version of the canonical DMP model in Shimer (2005). Our framework captures this special case if all matches are assumed to be homogeneous, r_t^k is the same across matches and equal to aggregate productivity, separation rates are constant over time, and stochastic discount factors are the same across agents and over time.

Equally, our model also covers significant departures from the framework of Shimer (2005). First, the model allows for a wide variety of frictions outside the labor market. Any friction that affects the labor market via the flow value of labor r_t^k is covered. This may occur, for instance, if there are distortions to product markets, such as product market price rigidities. Working capital constraints as in Christiano, Eichenbaum and Trabandt (2015) are also covered. Likewise, we allow for firms' discount rates to vary over time and differ from household discount rates, which covers cases where financial frictions make firms behave as if they are relatively impatient, as in Schoefer (2021).

Second, the model allows for rich heterogeneity across firms and matches and over time. Since r_t^k can vary across matches over time, our framework allows for the possibilities that workers improve at their jobs over time, that matches persistently vary in quality, or if there are match-specific productivity shocks. Equally, allowing r_t^k and firms' discount rates to vary across firms and over time allows the model to capture cases where the effects of goods and financial market frictions differ across firms. Since we allow that there can be variation across firms and time in the probability that vacancies match with workers and match specific time varying separation rates, our model also covers cases where the probability of matching and separating depend endogenously on e.g. match specific productivity.

Third, our framework allows for time varying labor force participation and job-to-job transitions.

Fourth, our framework allows for essentially any wage-setting protocol, and by allowing the wage to depend upon both the current period t and the period at which the match began, we allow for history dependence in wages, as in e.g. Rudanko (2009). While our Nash wage equation by definition captures the wage set according to the Nash sharing rule, we nowhere assume that actual wages are set in this way, since there is no assumption that they equal Nash wages.

Fifth, our framework allows for the possibility of a wide range of macroeconomic shocks. Any shock that operates via r_t^k is covered, such as productivity shocks, markup shocks or shocks that increase the costs of working capital. Since we did not specify a matching function, we also allow for the possibility of variation in matching efficiency over time. We also allow for the possibility of shocks to the separation rate or to firms' discount rates, where the latter could be a consequence of financial shocks.

It is worth emphasizing that we are not making the absurd assumption that none of these many features matter for labor markets. Our framework absolutely allows them to matter for both equilibrium unemployment and wage determination. However, our common Nash wage equation reveals that, under the Nash sharing rule, the wage has to be set according to the same equation across these many cases, just as workers have to be on their supply curve equation in a competitive labor market, regardless of what is assumed about labor demand.

2.5 The Nash Wage Elasticity

In the Section 3, we take the equations (16) and (20) for the Nash wage w_t^N and the worker bargaining share β_t in the model allowing for heterogeneity, and use these equations to compute time series for these two variables using data on wages labor market flows, and series for the opportunity cost of employment z_t from Chodorow-Reich and Karabarbounis (2016). We use the resulting time series for the Nash wage to estimate the Nash wage elasticity, as we now discuss.

Computing series for w_t^N and β_t: To compute series for w_t^N and β_t, we make we make several assumptions that simplify equations (16) and (20), due to data limitations.

First, we impose $\Phi_t = \Phi_t^N = 0$ for all t. It is not possible to evaluate Φ_t or Φ_t^N without either data or assumptions about how the separation rate varies across jobs and over time due to job tenure effects and varying match surplus across matches. Neither the sign nor the cyclicality of Φ_t are straightforward to determine, since a decreasing separation rate with job tenure implies both $\left(\frac{\tilde{s}_{t+j,t}}{1-\tilde{s}_{t+j,t}}\right) < \left(\frac{\tilde{s}_{t+j,t+1}}{1-\tilde{s}_{t+j,t+1}}\right)$ and $\left(\frac{1-\tilde{s}_{t+1,t}}{1-\tilde{s}_{t+1+j,t+1}}\right) < 1$. In most models in the literature, the probability of a worker separating from a job and entering unemployment is either completely or approximately unrelated to the length of time the unemployed individual has been in a job. In such cases, Φ_t is either exactly or approximately equal to zero. For this reason, we judge that $\Phi_t = 0$ may be a plausible approximation. Future work could investigate how far allowing for time variation in Φ_t affects estimates of the Nash wage elasticity.

Second, we set $\tilde{s}_{t+1,t} = s_{t+1}$. Recall that the former is a weighted average separation rate, while the latter is simply the economy-wide average separation rate. $\tilde{s}_{t+1,t}$ will tend to be greater than s_{t+1} insofar as workers who have been in a job less time are more likely to separate, but will tend to be less than s_t insofar as matches with a higher surplus are less likely to separate. In practice, the $\tilde{s}_{t+1,t}$ terms are of very small quantitative significance in equations (16) and (20), and so we find that our results are essentially unaffected by different assumptions about $\tilde{s}_{t+1,t}$, provided $\tilde{s}_{t+1,t}$ is of the same order of magnitude as s_{t+1}.

Third, we assume that v_t^u is proportional to v_t. Specifically, we fix $v_t^u = v_t$, where setting the constant of proportionality to equal 1 is a normalization, because doubling κ_1 and halving v_t^u leaves equations (16) and (20) unchanged. We make this assumption because we do not know of data available for our long time period regarding what fraction of vacancies are targeted primarily at the unemployed. Setting $v_t^u = v_t$ makes our Nash wage equation consistent with the case of no job-to-job transitions, which is a common, if counterfactual, assumption in models in the literature. Furthermore, making v_t^u proportional to v_t may roughly approximate reality because the job finding rate of the unemployed and job-to-job transitions seem to have a roughly similar cyclicality in US data (Mukoyama, 2014).

For the empirical analysis, it is convenient to work with log-linearized forms of the equations equations (16) and (20). Log-linearizing (20) around the steady state using (21), setting $\Phi_t = 0$, $\tilde{s}_{t,t+1} = s_{t+1}$ and $v_t^u = v_t$ and using hat variables to denote log deviations from the steady state, we obtain

$$\begin{aligned}\frac{w^{UC}\hat{w}_t^N - z\hat{z}_t}{w^{UC} - z} &= \left(\frac{\kappa_1 v}{huf}\right)(\hat{v}_t - \hat{u}_t - \hat{f}_t) \\ &\quad + \left(f\hat{f}_t + s\hat{s}_t\right)\frac{(1-\rho)}{1-(1-f-s)(1-\rho)} + \mathbb{E}_t[\hat{A}_{t+1} - \hat{A}_t], \qquad (22)\end{aligned}$$

where

$$\hat{A}_t = \frac{(1-\rho)(1-f-s)}{1-(1-f-s)(1-\rho)}\left[\frac{f\hat{f}_t + s\hat{s}_t}{1-f-s} - \left(\frac{\kappa_1 v}{huf}\right)(\hat{v}_t - \hat{u}_t - \hat{f}_t) + \sigma\hat{c}_t\right]. \qquad (23)$$

Log-linearizing equation (16) for β_t, imposing $\Phi_t = 0$ and recursively substituting in (22), we obtain:

$$\frac{\hat{\beta}_t}{1-\beta} = \left(\frac{w^{UC}}{w^{UC}-z}\right) \mathbb{E}_t \sum_{j=0}^{\infty} \left[1-(1-f-s)(1-\rho)\right](1-\rho)^j (1-f-s)^j (\hat{w}^{UC}_{t+j} - \hat{w}^N_{t+j}). \tag{24}$$

This equation confirms that the deviation of the worker's share of the match surplus from its steady state value is proportional to a discounted sum of expected future deviations of the user cost of labor from Nash wages.

We use equations (22) and (24) to infer the level of the deviation of the worker surplus share, $\hat{\beta}_t$, and the Nash wage, $\hat{w}^N_t$, from empirical data. Given values of these, we propose a measure of aggregate wage rigidity which we term the Nash Wage Elasticity (NWE). The NWE represents the percentage change in the actual wage rate, w_t, when the Nash wage, w^N_t, changes by 1%. An NWE of 1 would imply that Nash bargaining provides an accurate model of wage fluctuations. On the other hand, if the NWE is positive but close to 0, this would imply a wage rate which is relatively insensitive to the macroeconomic factors that influence the Nash wage.

Specifically, we assume a relationship of the form

$$\hat{w}^{UC}_t = \gamma \hat{w}^N_t + \varepsilon_t \tag{25}$$

where γ is the NWE and ε_t is a disturbance term. We estimate this equation by OLS and using various instruments for the Nash wage to address concerns of possible measurement error.

The next section discusses the data series and details of the specifications used to estimate the Nash wage. The Nash wage elasticity is closely related to the procyclicality of wages. As we discuss in the next section, our derived value of the Nash wage turns out to be highly procyclical. Consequently, higher values of γ suggest more procyclical wages. Generally, we find γ far below 1, and, accordingly, our measures of the user cost of labor is less procyclical than the Nash wage. As such, we find that the Nash wage tends to be lower than the user cost of labor in recessions. Furthermore, since we showed above that $\hat{\beta}_t$ depends on the difference between the Nash wage and the actual user cost of labor, we find that $\hat{\beta}_t$ is strongly countercyclical.

3 Data and Calibration

We compute time series for the model-implied Nash Wage $\hat{w}^N_t$ and worker surplus share ($\hat{\beta}_t$) in US data using the equations (22) and (24) using various measures of the user cost of labor

(w_t^{UC}) various measures of the opportunity cost of employment (z_t), calibrated parameters time series for the job finding rate, separation rate, unemployment and consumption derived from US data. The national statistical sources used for these series are reported in Appendix A). We use the headline series of the level of unemployed workers (u^l), number employed workers (e) and number of unemployed workers for less than 5 weeks (u^s) monthly released by the U.S. Bureau of Labor Statistics (BLS) to derive f and s as in Shimer (2005):

$$f_t = 1 - \frac{u_t^l - u_{t-1}^s}{u_t^l} \tag{26}$$

$$s_t = \frac{u_{t+1}^s}{e_t(1 - \frac{1}{2}f_t)} \tag{27}$$

The job vacancy rate (v) is taken from Petrosky-Nadeau and Zhang (2020). This series has the advantage of being significantly longer than the BLS' Job Openings and Labor Turnover Survey (JOLTS) and is constructed from a range of other sources, for which we refer to the cited paper.

For the cost of labor, we consider multiple empirical measures. In the previous section, it was shown that the relevant measure of the cost of labor that should ideally be used to calculate the Nash wage elasticity is the user cost of labor based on newly hired workers out of unemployment. The user cost of labor incorporates not only the wage earned by a newly hired worker today, but also the possibility that being hired today rather than tomorrow affects a worker's likely wages in future periods, thereby making it more or less expensive for a firm to hire a worker today versus tomorrow. Unfortunately, no empirical measure of this has been constructed and it would be difficult to construct a reliable such measure given the available data. In particular, to calculate the user cost accurately, a relatively long individual panel is needed to incorporate the possibility that being employed today affects a worker's wages some distance into the future, as implied by models of implicit contracts such as Rudanko (2009). One such panel is the National Longitudinal Survey of Youth (NLSY), however the NLSY does not ask whether an individual has been hired out of unemployment, and so the wages of new hires out of unemployment cannot be distinguished from those of job switchers. Kudlyak (2014) and Basu and House (2016) have constructed measures of the user cost of labor from the NLSY, based on lumping together all new hires, whether they are job switchers or new hires out of unemployment. This is problematic since Gertler, Huckfeldt and Trigari (2020) have found that the wages of job switchers and wages of new hires out of unemployment have quite different cyclical properties, with the latter being substantially more cyclical. If this is the case, the user cost series of Kudlyak (2014) and Basu and House (2016) may have quite a substantial procyclical bias, which would substantially bias upwards estimates of the NWE based on this series (since the Nash wage is very procyclical, as we discuss below).

For this reason, we consider five different empirical proxies for the user cost of labor: the BLS average weekly earnings wage series, two series from Basu and House (2016) and two series from Haefke, Sonntag and Van Rens (2013). We consider all these series because of the substantial debate in the literature regarding the true cyclicality of the cost of labor (Gertler, Huckfeldt and Trigari, 2020; Bellou and Kaymak, 2021). As we discuss, we believe that some of the series are likely to lead to downward biased estimates of the NWE and other series are likely to lead to upward biased estimates of the NWE. Therefore, by considering the range of measures, we hope to provide a range of estimates of the NWE, with the true value likely to fall somewhere within this estimated range.

We call the first series the CES wage as it is based on the monthly Current Employment Survey (CES) and captures the U.S. employed population average hourly earnings. It is the most widely used measure of an actual wage as it gauges the effective salary across the whole pool of continuously employed workers at a given time. There are two reasons why this headline series may have a countercyclical bias, thus failing to be a representative 'user cost' series: a) since the CES wage is aggregate, this measure suffers from likely countercyclical composition bias if the workers hired in booms have on average lower quality characteristics than the workers hired in recessions and b) an aggregate wage will have a countercyclical bias if the wages of newly hired workers are more influend by the current state of the labor market than are those of job stayers (Basu and House, 2016; Kudlyak, 2014). Given this likely countercyclical bias, we anticipate that the CES series will tend to understate the Nash wage elasticity.

As a second proxy for the user cost of labor, we use a measure of the new hire wage computed from NLSY data by Basu and House (2016), which specifically corrects for composition bias by controlling for individual fixed effects. As discussed above, this new hire wage includes both job changers and those of newly hired workers out of unemployment, whose wages might have quite different cyclical properties. Another concern with the NLSY series is that the NLSY consists of only a single cohort, which might not be representative of the wider population.

The third series we use is the 'user cost of labor' calculated by Basu and House (2016) from NLSY data (this is similar to the series calculated by Kudlyak (2014)). We refer to this series as the NLSY user cost, to distinguish it from the true user cost of labor in our theoretical framework. This series also adjusts for individual fixed effects to address composition bias. As discussed above, this series gets close conceptually to the relevant concept of the user cost of labor, but may overstate the Nash wage elasticity, possibly by a significant amount, since it lumps together job switchers with newly hired workers out of unemployment.

The remaining two wage series are calculated by Haefke, Sonntag and Van Rens (2013) on the basis of Current Population Survey (CPS) hourly earnings. The first such wage series, which we call Haefke New Hire, is the wage of newly hired workers out of unemployment, since the CPS has information on recent past unemployment status and so can distinguish this group from job switchers. The second series, which we call Haefke All, is a wage of all workers. Both series adjust for composition bias using controls for education, demographic characteristics and expierience. The composition bias adjustment means that the 'Haefke All' series is less at risk of countercyclical bias than the CES series discussed above. Nevertheless, since neither of these series is based on panel data, neither of them can fully capture the possible dynamic effects of being hired today on future wages, which should be taken into account as part of the user cost of labor. Thus, neither of these measures fully capture the user cost. Since these two indices have some missing values due to the unavailability of more granular information in the third and forth quarters of 1985 and 1995, we resort to linear interpolation to work with a continuous series.

Next, we multiply our wage indices, which are hourly, by average weekly hours (according to the CES) worked to state our cost of labor series in terms of a per-worker cost. This transformation is necessary to make the derivative wage indicators consistent with the SAM model stated above, where wage is intended as the pay for being employed rather than a hourly rate.

We obtain measures of the opportunity cost of employment, z_t, from Chodorow-Reich and Karabarbounis (2016). As discussed above on page 11, these measures represent the combined advantages of being unemployed relative to the mean marginal product of labor in terms of benefits in cash and in kind, taxes, and more free time. The reader is referred to Chodorow-Reich and Karabarbounis (2016) for details of how these series for z_t are constructed. Using different assumptions on preferences, Chodorow-Reich and Karabarbounis derive four different time series for z_t, computed on the basis of a) separable utility in hours and consumption (SEP); b) Constant Frisch Elasticity (CFE) and two different Cobb-Douglas parametrizations (CD1 and CD2). For completeness, we derive our results using all four series for z_t. As we show in Table 1 below, these different series imply very different levels for the average value of z_t over time, but Chodorow-Reich and Karabarbounis robustly find z_t to be highly procyclical, contra the assumption of constant z commonly considered in the SAM literature. The intuition is that the marginal utility of consumption relative to leisure is higher in downturns and dominates the countercyclicality of unemployment benefits.

Chodorow-Reich and Karabarbounis's different assumptions about preferences imply different assumptions about σ. To maximize consistency with their approach, we use the value of σ associated with each z_t series in Chodorow-Reich and Karabarbounis (2016)

3.1 Steady State Variable Values and Calibrated Parameters

Having obtained time series for s_t, f_t, u_t, z_t and so on, our log-linearization approach requires that we calculate the steady state values of these variables as well as their deviations from the steady state. For each variable, we assume that the log HP-filtered value from the data isolates the deviation of the variable from its steady state value. For almost all variables, we consider the longer term (whole sample) average as the steady state, value. These values are shown in Table 1.

Inferring the steady state value of the user cost of labor is less straightforward, since many of our series for this control for individual fixed effects or characteristics, so it is not clear what these series imply for the average user cost of labor over time. Instead, we calibrate the steady state value of the user cost of labor based on the hiring first order condition for firms, in order to maximize consistency with the literature.

This approach requires first that we calibrate the hiring costs κ_0 and κ_1, representing the fixed hiring cost and vacancy posting cost. As a baseline, we set the fixed hiring cost to zero (the traditional assumption in the literature) and set the vacancy posting cost to 0.44. This second parameter deserves more discussion, since there is no consensus in the literature on the total value of hiring costs (relative to the steady state marginal product of labor) Both Hagedorn and Manovskii (2008) and Michaillat and Saez (2021) calibrate the costs based on microdata, both assuming fully variable costs ($\kappa_0 = 0$). We try different combinations of fixed/variable hiring costs consistent with a steady state hiring cost of 0.579, halfway between the value used by Hagedorn and Manovskii (2008) and Michaillat and Saez (2021). Our steady state costs are stated in the following equation:

$$h = \kappa_0 + \kappa_1 \left(\frac{v}{uf} \right) \simeq 0.58 \tag{28}$$

At the same time, since there is also no consensus on the likely split between fixed and variable hiring costs, we explore multiple calibrations of κ_0 and κ_1 below, keeping the steady state hiring cost at 0.579, but varying the fraction of fixed hiring costs in the total from 0 to 90%. This implies values of κ_0 ranging from 0 to 0.52, and values of κ_1 ranging from 0.044 to 0.44.

For the user cost of labor, we assume that steady state w^{UC} is the value of the wage we would expect to see in the steady state if all workers were homogeneous and paid the same wage and there are no goods market frictions in the steady state. Hence, we calculate it in accordance to (12):

$$\mathcal{J} = r - w + (1 - \rho)(1 - s)\mathcal{J} \tag{29}$$

Its first order condition is $\mathcal{J} = h$. Normalizing r = 1, we obtain that in the steady state:

$$w = 1 - (1 - (1 - \rho)(1 - s))h \simeq 0.98 \tag{30}$$

We note that 0.98 is consistent with the SAM literature (e.g. Pissarides (2009) has the steady state wage at 0.98).

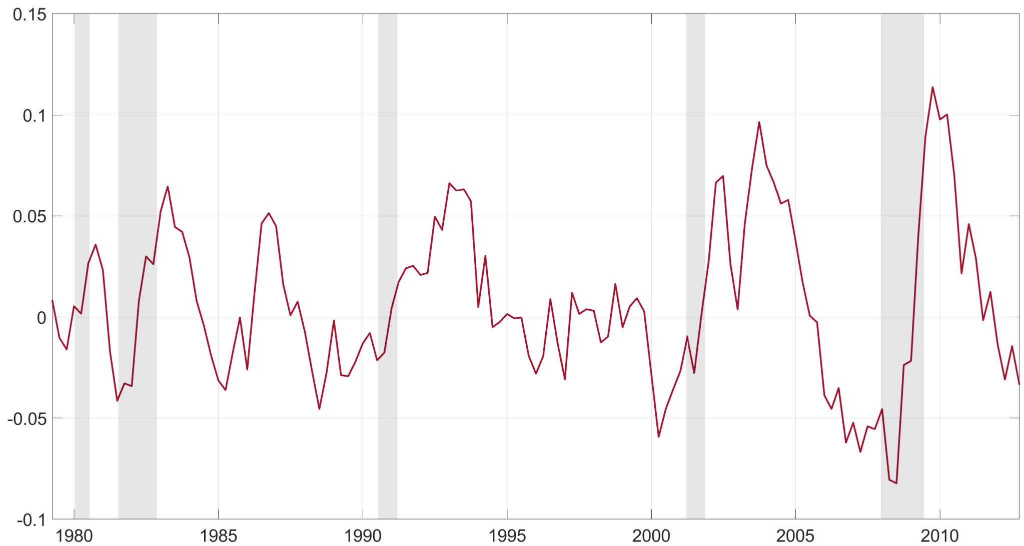
We summarise the calibration in the Table 1 below, which shows the range for κ_0 and κ_1 consistent with the steady state h of 0.58 and the σ parameter considered by Chodorow-Reich and Karabarbounis (2016) for the calculation of the four z series. In the analyses below, we do not show results for other values of σ, ρ or the total steady cost of hiring (i.e. we only vary the ratio $\frac{\kappa_0}{\kappa_1}$). This is because we find that plausible alternatives to these parameter assumptions make truly negligible difference to our empirical results – affecting NWE estimates by less than 1%.

Table 1: Steady States and Calibration

Variables in Steady State		**Description**
u	0.064	Unemployment Rate
f	0.37	Finding Rate
w	0.98	Wage
z	0.47, 0.76, 0.96	Opportunity cost of employment
s	0.03	Separation Rate
v	0.03	Vacancy rate
β	0.68	Bargaining Share
h	0.58	Hiring Costs
Calibrated Parameters		
k_0	0, 0.29, 0.52	Fixed Hiring Costs
k_1	0.44, 0.22, 0.044	Proportional Hiring Costs
σ	1, 1.52, 1.25, 1.19	Risk Aversion Coefficient
ρ	0.012	Discount Rate

4 Results

Figure 1: Worker Surplus Share (β_t) based on the NLSY user cost of labor

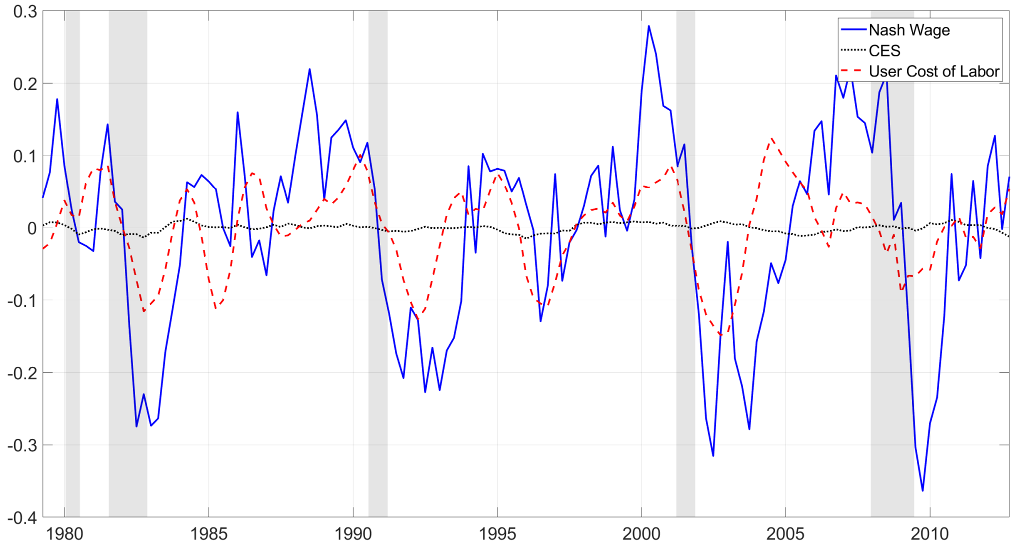


Before estimating the NWE, we graph values of the worker surplus share and Nash wage computed according to the log-linearized expressions for these in Section 2.5. In Figure 1 we graph the worker surplus share. The baseline series is computed using the input series described in the previous section. The left-hand side only requires arithmetic operations whereas on the right-hand side the terms in the expectations operator have been forecast with a reduced-form VAR with y_t = [1 year interest rate; Real GDP; GDP Deflator; f_t; s_t; w_t; z_t; v_t; u_t; c_t] up to horizon $j = 50$, although the geometrical decay of the summation term makes it effectively nil after the 20^{th} period.

In these equations, the expected forward value $\hat{w}^N_{t+j}$ appears in expectations and since we stated in Eq. (22) the Nash wage as a combination of known parameters j periods ahead, given the VAR forecasts, we recursively iterate the VAR to calculate the expectations $\mathbb{E}_t \hat{w}^N_{t+j}$ which we use in equation (24) to calculate the bargaining share and Nash wage.

In Figure 2 we chart the empirical Nash Wage, calculated as in (22). Comparing Figures 1 and 2, it is evident that the measured worker share of match surplus is strongly countercyclical and the Nash wage is strongly procyclical. The countercyclical worker surplus share suggests that, in recessions, workers are earning more than they would if their share of the surplus was constant (as in the Nash sharing rule). This is consistent with the evidence for wage rigidity we document below.

Figure 2: Nash Wage $\hat{w}_t^N$



4.1 Estimating the Nash Wage Elasticity

In this section, we estimate the NWE. To do so, we estimate 60 linear regressions for which the NWE is the slope coefficient of a regression where the Nash wage is the independent variable and the dependent variable is a measure of the cost of labor. The Nash Wage Elasticity is represented by the coefficient γ in the following ordinary least squares (OLS) regression:

$$\hat{w}_t^{UC} = \gamma \hat{w}_t^N + \varepsilon_t \tag{31}$$

We estimate 60 different such regressions by varying the cost of labor measure $\hat{w}_t^{UC}$ by alternating between the five proxies for the user cost of labor discussed above, and by using 12 different combinations of the four series for the opportunity cost of labor z with three different combinations of fixed and variable hiring costs. In our equation for the Nash wage, what matters is the size of variable hiring costs relative to fixed hiring costs. To capture the likely range of such variation, we consider specifications with only variable costs ($\kappa_0 = 0, \kappa_1 = 0.43$), as is standard in the literature, specifications where the fixed hiring cost is roughly half of total hiring costs ($\kappa_0 = 0.29$, $\kappa_1 = 0.22$) and specifications where the fixed hiring cost is 90% of total hiring costs ($\kappa_0 = 0.52, \kappa_1 = 0.04$).

In Table 2 we find estimates of the the NWE ranging from 0.01 to 1.56. However, the vast majority of estimates are below 0.1 (and usually statistically indistinguishable from zero), and only a few estimates are above 0.6. This small number of estimates above 0.6 all use the NLSY user cost of labor (our most procyclical labor cost series) *and* use either almost entirely fixed hiring costs, or use the CD2 series of z. This series of z has an average value of 0.96 – close to the calibration of Hagedorn and Manovskii (2008), which is viewed by most of the subsequent literature as implausibly high (Chodorow-Reich and Karabarbounis, 2016; Christiano, Eichenbaum and Trabandt, 2021). Given the extreme assumptions needed to find an NWE above 0.6, we interpret our estimates as clearly supporting an NWE below 0.6, and favoring an NWE of 0.1 or below.

Table 2: Results of Regression (31). We calculate the NWE for 5 wage indexes and 12 distinct calibrations of z and hiring costs. Newey–West Standard Errors in brackets.

Z Series	Steady state z	κ_0	κ_1	CES	NLSY New Hire	NLSY User Cost	Haefke All	Haefke New Hire
SEP	0.47	0.00	0.44	0.01	0.02	0.24	0.01	0.02
				(0.01)	(0.04)	(0.04)	(0.01)	(0.02)
SEP	0.47	0.29	0.22	0.01	0.02	0.39	0.03	0.03
				(0.01)	(0.07)	(0.07)	(0.01)	(0.03)
SEP	0.47	0.52	0.04	0.02	0.02	0.65	0.06	0.04
				(0.02)	(0.12)	(0.15)	(0.02)	(0.07)
CFE	0.47	0.00	0.44	0.01	0.01	0.24	0.01	0.02
				(0.01)	(0.04)	(0.04)	(0.01)	(0.02)
CFE	0.47	0.29	0.22	0.01	0.02	0.39	0.03	0.03
				(0.01)	(0.07)	(0.07)	(0.01)	(0.03)
CFE	0.47	0.52	0.04	0.02	0.02	0.67	0.06	0.05
				(0.02)	(0.13)	(0.15)	(0.02)	(0.07)
CD1	0.76	0.00	0.44	0.02	0.04	0.53	0.03	0.04
				(0.01)	(0.09)	(0.10)	(0.01)	(0.05)
CD1	0.76	0.29	0.22	0.03	0.06	0.81	0.06	0.06
				(0.02)	(0.15)	(0.16)	(0.02)	(0.08)
CD1	0.76	0.52	0.04	0.06	0.07	1.16	0.13	0.09
				(0.04)	(0.24)	(0.26)	(0.04)	(0.15)
CD2	0.96	0.00	0.44	0.19	0.38	1.56	0.26	0.13
				(0.06)	(0.40)	(0.54)	(0.06)	(0.30)
CD2	0.96	0.29	0.22	0.21	0.40	1.39	0.28	0.12
				(0.06)	(0.40)	(0.57)	(0.07)	(0.32)
CD2	0.96	0.52	0.04	0.22	0.40	1.18	0.28	0.10
				(0.06)	(0.39)	(0.59)	(0.07)	(0.33)

Next, we estimate the NWE using the (hp-filtered) unemployment rate as an instrument for the Nash wage. To do so, we estimate an IV regression with the same second stage equation as Eq. (31), but where the hp-filtered unemployment rate instruments for the Nash wage in the first stage.

$$w_t = \theta w^n + \xi_t \tag{32}$$

This approach has two advantages. First, it reduces concerns of measurement error in the Nash Wage. Second, it corresponds to dividing the elasticity of actual wages with respect to the unemployment rate by the elasticity of Nash wages with respect to the unemployment rate.[15] Since the reduced-form literature on wage cyclicality commonly computes the elasticity of wages with respect to the unemployment rate, this approach has the virtue of easy comparison to that literature.

We report the results in Table 3. Throughout, IV estimates are relatively similar to OLS estimates. This is because the Nash Wage is strongly correlated with the unemployment rate (correlation $\simeq$ 0.8) in most specifications. Again, in most of cases NWE is close or indistinguishable from zero. Intuitively, as we discuss in Section 4.3, this is because the elasticity of the Nash wage with respect to the unemployment rate is much higher than the elasticity of the actual cost of labor with respect to the unenmployment rate, for most measures of the cost of labor. The NWE based on NLSY User Cost is markedly positive and in some few instances also greater than 1, but with the caveat of being high under a relative non-standard calibration – using the CD1 and CD2 specification entailing a high value of z, and/or a high value of fixed hiring costs κ_0.

[15] That is, we choose this specification as analog of the ratio θ_1/θ_2 in the following OLS regressions:

$$w_t = \theta_1 y_t + \xi_{t,1}$$

$$w_t^n = \theta_2 y_t + \xi_{t,2}$$

This definition of an IV estimate is also useful for our approach to estimating the NWE conditional on monetary shocks below.

Table 3: Results of Regression (32). We calculate the NWE for 5 wage indexes and 12 distinct calibrations of z and hiring costs. Newey–West Standard Errors in parentheses.

Z Series	Steady state z	κ_0	κ_1	CES	NLSY New Hire	NLSY User Cost	Haefke All	Haefke New Hire
SEP	0.47	0.00	0.44	0.01	- 0.01	0.27	0.02	0.01
				(0.01)	(0.05)	(0.07)	(0.01)	(0.02)
SEP	0.47	0.29	0.22	0.01	- 0.01	0.41	0.03	0.02
				(0.01)	(0.08)	(0.10)	(0.01)	(0.04)
SEP	0.47	0.52	0.04	0.01	- 0.02	0.68	0.05	0.03
				(0.02)	(0.13)	(0.16)	(0.02)	(0.07)
CFE	0.47	0.00	0.44	0.01	- 0.01	0.28	0.02	0.01
				(0.01)	(0.05)	(0.07)	(0.01)	(0.02)
CFE	0.47	0.29	0.22	0.01	- 0.01	0.42	0.03	0.02
				(0.01)	(0.08)	(0.10)	(0.01)	(0.04)
CFE	0.47	0.52	0.04	0.02	- 0.02	0.70	0.06	0.03
				(0.02)	(0.14)	(0.17)	(0.02)	(0.07)
CD1	0.76	0.00	0.44	0.01	- 0.02	0.62	0.04	0.02
				(0.02)	(0.12)	(0.15)	(0.02)	(0.05)
CD1	0.76	0.29	0.22	0.02	- 0.03	0.91	0.07	0.03
				(0.03)	(0.18)	(0.22)	(0.03)	(0.08)
CD1	0.76	0.52	0.04	0.03	- 0.04	1.46	0.12	0.06
				(0.05)	(0.28)	(0.35)	(0.04)	(0.14)
CD2	0.96	0.00	0.44	0.09	- 0.12	4.11	0.29	0.15
				(0.13)	(0.79)	(0.98)	(0.11)	(0.35)
CD2	0.96	0.29	0.22	0.11	- 0.14	4.85	0.35	0.18
				(0.15)	(0.94)	(1.15)	(0.13)	(0.42)
CD2	0.96	0.52	0.04	0.12	- 0.16	5.66	0.42	0.22
				(0.18)	(1.09)	(1.35)	(0.16)	(0.50)

4.2 NWE Conditional on Monetary Shocks

To estimate the NWE conditional on monetary shocks we borrow from the dynamic fiscal multiplier literature. The multiplier is defined as the cumulative change in GDP relative to government spending on an exogenous impulse. It is often approximated as the ratio of the integral of GDP and government spending impulse response functions (IRFs) at an arbitrary horizon h. Ramey (2016). Nekarda and Ramey (2021) have extended this framework to analyse the conditional response of markup to monetary policy, government spending, productivity and investment specific technology shocks.

An handy way to retrieve IRFs and their ratio is by setting up a vector auto-regression (VAR), a methodology that is simple to implement and provides an intuitive identification when structural restrictions are made explicit. Ramey (2016, 2018); Barnichon, Debortoli and Matthes (2021) use a more direct and assumptions-free way to calculate the dynamic fiscal multiplier by comparing the impulse response functions of a h-step ahead local projection (LP). This methodology is appealing because no identifying assumptions are needed and LPs can be computed in a single equation rather than stating a full system. Control variables are usually included to avoid serial correlation and improve the regression fit instead of the cross lags of variables.

In this regard, the literature offers two alternative testing routes. They yield identical results, but it is useful to recall them in order to compare such methodologies with the Regressions stated for the unconditional NWE and provide further intuition. The first method is based on the calculation of LP impulse response functions (IRFs) h periods ahead as:

$$w_{t+j} = a_h + \gamma_j C_t + \psi_h x_t + u_{t+h} \tag{33}$$

Where the dependent variable is the variable subject to an identified shock x_{t+j} and C_t is a matrix of control variables. ψ_h is the response of y_t on impulse. In this setting the multiplier can be calculated as the ratio of cumulative IRFs without having to specify a full blown structural model.

The second method consolidates the IRF ratio in a single equation, providing a direct point-estimate of the multiplier by means of a direct instrumental local projection (IV-LP). As in footnote 15, this IV-LP consolidates the LP in Eq. (33) in a single equation by using the identified shock as an excluded instrument for the Nash wage.

$$\sum_{j=0}^{h} w_{t+j} = m_h \sum_{j=0}^{h} w^n_{t+j} + \gamma_j C_t + u_{t+j} \tag{34}$$

Adapting this equation to our case, $\sum_{j=0}^{h} w_{t+j}$ is the cumulative measure of wage, $\sum_{j=0}^{h} w^n_{t+j}$ is the Nash Wage and C_t is a matrix of control variables. The identified shock the is used as an external instrument for w^n. The coefficient m_h is then the estimate NWE multiplier at time horizon h. The IV-LP approach has the advantage to be conducive to the calculation of standard error [16] and weak instrument first stage statistics.

We use the IV-LP to retrieve the NWE measures conditioning on externally identified shocks akin to Nekarda and Ramey (2021). We consider an exogenous monetary shock. Hence we instrument the Nash Wage using the Romer and Romer narrative series for mon-

[16]which have to be heteroscedasticity and autocorrelation robust (HAC), since the IV-LP error term is MA

etary policy Romer and Romer (2004) as updated in Wieland and Yang (2020).

Our specification is exactly Eq. (34) where the matrix of controls contains 1 lag of the shock, log GDP, Nash Wage and wage proxy. The multiplier is calculated at 4 quarters after the monetary policy shock. We summarise the results in Table 4.

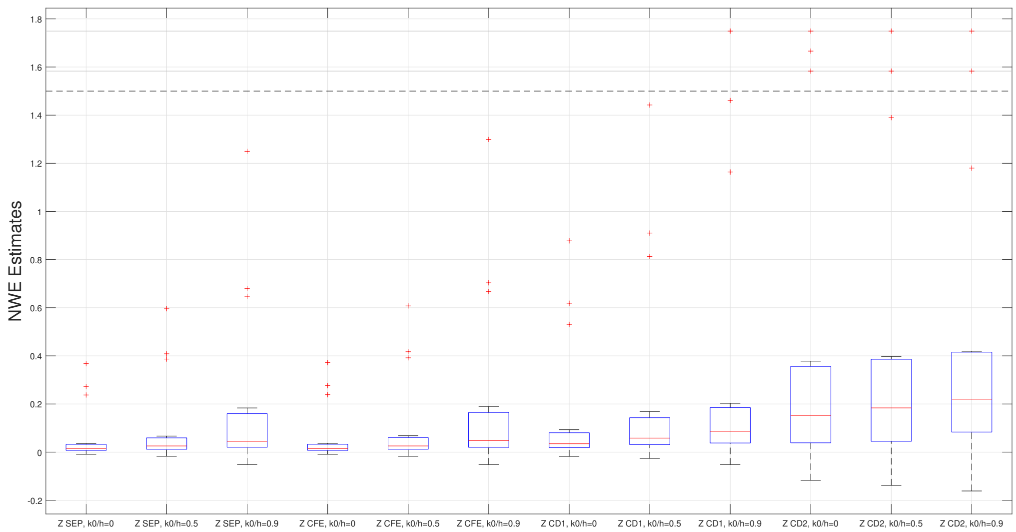
Table 4: Results of Regression (34). We calculate the NWE for 5 wage indexes and 12 distinct calibrations of z and hiring costs. Newey–West Standard Errors in parentheses.

Z Series	Steady state z	κ_0	κ_1	CES	NLSY New Hire	NLSY User Cost	Haefke All	Haefke New Hire
SEP	0.47	0.00	0.44	0.01	0.04	0.37	0.02	- 0.01
				(0.01)	(0.06)	(0.18)	(0.01)	(0.04)
SEP	0.47	0.29	0.22	0.02	0.07	0.60	0.04	- 0.02
				(0.02)	(0.09)	(0.30)	(0.02)	(0.07)
SEP	0.47	0.52	0.04	0.03	0.18	1.25	0.09	- 0.05
				(0.04)	(0.21)	(0.63)	(0.06)	(0.16)
CFE	0.47	0.00	0.44	0.01	0.04	0.37	0.02	- 0.01
				(0.13)	(0.14)	(0.13)	(0.01)	(0.04)
CFE	0.47	0.29	0.22	0.02	0.07	0.61	0.04	- 0.02
				(0.13)	(0.15)	(0.13)	(0.02)	(0.07)
CFE	0.47	0.52	0.04	0.03	0.19	1.30	0.09	- 0.05
				(0.13)	(0.17)	(0.15)	(0.06)	(0.16)
CD1	0.76	0.00	0.44	0.02	0.09	0.88	0.02	- 0.01
				(0.13)	(0.15)	(0.14)	(0.01)	(0.04)
CD1	0.76	0.29	0.22	0.03	0.17	1.44	0.04	- 0.02
				(0.13)	(0.17)	(0.15)	(0.02)	(0.07)
CD1	0.76	0.52	0.04	0.01	0.20	2.85	0.09	- 0.05
				(0.11)	(0.19)	(0.17)	(0.06)	(0.16)
CD2	0.96	0.00	0.44	0.02	0.09	8.27	0.02	- 0.01
				(0.11)	(0.19)	(0.23)	(0.01)	(0.04)
CD2	0.96	0.29	0.22	0.04	0.06	11.06	0.04	- 0.02
				(0.11)	(0.20)	(0.24)	(0.02)	(0.07)
CD2	0.96	0.52	0.04	0.08	0.05	15.26	0.09	- 0.05
				(0.11)	(0.20)	(0.25)	(0.06)	(0.16)

The NWE conditioning on the monetary policy shock is similar to the OLS and the IV specifications. The NWE is generally very small or nil one year after a monetary policy shock. This suggests that the actual wage is not sensitive to moves in the NWE also contingent on shocks.

In Figure 3 we summarise all the 180 estimates for the NWE derived so far in a box plot. The 12 boxes represent the 12 alternative calibrations, using the four different series for z and three different calibrations of hiring costs. The outliers are all estimates involving the NLSY user cost series. Estimates with a higher steady state z and CD2 are more dispersed and present more outliers. The rest of estimates are very concentrated around 0. Taken together, this evidence is strongly suggestive that the NWE is positive but much closer to 0 than to 1.

Figure 3: Box Plot of NWE across all regressions.



4.3 What is driving our results?

Across many different measures of wages and different empirical specifications, we have estimated values of the NWE that are positive, but substantially below 1. Perhaps surprisingly, this is true in many specifications even when we use the highly procyclical NLSY user cost of labor from Basu and House (2016). Key to understanding our results is that our measured series for the Nash wage are highly procyclical, more procyclical even than the NLSY user cost of labor.[17] Since the actual cost of labor is significantly less procyclical than the Nash wage, it follows that the NWE is substantially below 1.

In this section, we discuss why we find that the Nash wage is so procyclical. We first provide some informal intuition, before making the discussion more precise.

Informally, the key to our results is that firm match surpluses appear to be procyclical, whereas worker match surpluses appear highly countercyclical, across different measures of the cost of labor. The reason that the firm match surplus appears to be procyclical is that the firm's hiring decision implies that the firm match surplus must equal the hiring cost, and hiring costs are procyclical since vacancies take longer to be filled when unemployment is low. On the other hand, the worker match surplus is strongly countercyclical, as the value of

[17]The procyclicality of the Nash wage and actual wage were shown in Table 3 above.

unemployment is much lower in recessions due to the longer time required to find a job. With a procyclical firm surplus and a strongly countercyclical worker surplus, it follows that the worker's share of the total surplus is strongly countercyclical, and so the Nash wage must be substantially more procyclical than the actual wage. Then, the NWE must be substantially below 1. In principle, if wages were procyclical enough then this would be consistent with a procyclical worker match surplus and an NWE of 1, but it turns out that this would require wages to be a lot more procyclical than any of our empirical measures of labor cost.

To develop this argument formally, assume, for simplicity, that all variables follow approximate random walks, so that for each variable $\mathbb{E}_t[x_{t+1}] \simeq x_t$. Assume also that the separation rate is roughly constant, so that $\hat{s}_t = 0$. Then, equation (22) gives the following approximate formula for the Nash wage.

$$\frac{w^{UC}\hat{w}_t^N - z\hat{z}_t}{w^{UC} - z} - f\hat{f}_t\left(\frac{(1-\rho)}{1-(1-f-s)(1-\rho)}\right) = \left(\frac{\kappa_1 v}{huf}\right)(\hat{v}_t - \hat{u}_t - \hat{f}_t)$$

Furthermore, log linearizing equation (1), setting $u_{t+1} = u_t$ and $\hat{s}_t = 0$ yields:

$$(f+s)\hat{u}_t = -f\hat{f}_t.$$

Substituting this into the equation above, we obtain:

$$\underbrace{\frac{w^{UC}\hat{w}_t^N - z\hat{z}_t}{w^{UC} - z} + (f+s)\hat{u}_t\left(\frac{(1-\rho)}{1-(1-f-s)(1-\rho)}\right)}_{\text{Deviation of Worker Match Surplus}} = \underbrace{\left(\frac{\kappa_1 v}{huf}\right)\left(\hat{v}_t - \frac{s}{f+s}\hat{u}_t\right)}_{\text{Deviation of Firm Surplus}}.$$

The right hand side of this equation is the deviation from steady state of the hiring cost, which equals the deviation from steady state of the firm's match surplus. Since vacancies are procyclical and unemployment is countercyclical, the firm's surplus is procyclical.

The equation states that this has to equal the deviation from steady state of the worker surplus. The reason that the left hand side of the equation has to equal the right hand side is that the Nash wage $\hat{w}_t^N$ is defined as the wage that keeps the worker share of surplus constant, requiring that the log deviation of the firm and worker surplus are equal.

The worker surplus term on the left hand side has two components. The first term is the deviation of the worker surplus is the change in the wage minus opportunity cost of working. The second term, proportional to $\hat{u}_t$, represents that the worker's match surplus relative to unemployment depends on how long a worker would expect to be unemployed if they were to quit the job. Since the duration of unemployment increases as the unemployment rate increases, this term relates positively to $\hat{u}_t$.

If the wage were too acyclical, so that $\hat{w}_t^N = 0$, then the left hand side of this equation would be countercyclical, since our data series for $\hat{z}_t$ is procyclical (Chodorow-Reich and Karabarbounis, 2016), and the unemployment rate is countercyclical. Since the right hand side is procyclical, $\hat{w}_t^N$ has to be procyclical for the left hand side to equal the right hand side. That is, the firm's match surplus is procyclical, and everything apart from wages makes the worker surplus countercyclical. As such, the wage rate would have to be quite procyclical in order for the worker's surplus to be as procyclical as the firm's, which is required by Nash bargaining.

How procyclical then does $\hat{w}_t^N$ have to be? To answer this question, rewrite the left hand side of the equation above as:

$$\left[\frac{w^{UC}\left(\frac{\hat{w}_t^N}{\hat{u}_t}\right) - z\left(\frac{\hat{z}_t}{\hat{u}_t}\right)}{w^{UC} - z} + \left(\frac{(1-\rho)(f+s)}{1-(1-f-s)(1-\rho)}\right)\right]\hat{u}_t.$$

For this to be procyclical, it must be negatively related to $\hat{u}_t$. That is, we need that:

$$\frac{w^{UC}\frac{\hat{w}_t^N}{\hat{u}_t} - z\frac{\hat{z}_t}{\hat{u}_t}}{w^{UC} - z} + \left(\frac{(1-\rho)(f+s)}{1-(1-f-s)(1-\rho)}\right) < 0$$

Using that $\frac{\hat{z}_t}{\hat{u}_t} < 0$, this can be rearranged to:

$$\left|\frac{\hat{w}_t^N}{\hat{u}_t}\right| > \left(1 - \frac{z}{w^{UC}}\right)\left(\frac{(f+s)(1-\rho)}{\rho + (f+s)(1-\rho)}\right) + \frac{z}{w^{UC}}\left|\frac{\hat{z}_t}{\hat{u}_t}\right|$$

The right hand side term in round brackets will be very close to 1 in an empirically plausible calibration. Since the last term is also positive, this requires in practice that:

$$\left|\frac{\hat{w}_t^N}{\hat{u}_t}\right| > 1 - \frac{z}{w^{UC}}.$$

This condition implies that the Nash wage must be extremely procyclical. In particular, for our baseline series of z, the right hand side is equal to 0.53. The left hand side is the percentage change in Nash wages when the number unemployed increases by 1%. If this exceeds 0.53, and the average unemployment rate is 6.3%, then a 1 percentage point increase in the unemployment rate (i.e. from 6.3% to 7.3%) must reduce the Nash wage by more than $0.53/0.063 = 8.4$ percent. Note that this is merely the minimum level of cyclicality of the Nash wage required for the worker match surplus to be at all procyclical. If the firm surplus is highly procyclical (which will be the case if κ_0 is substantially below 1) then Nash bargaining requires for the worker match surplus also to be highly procyclical, implying that a 1% decrease in the unemployment rate must increase the Nash wage by substantially more than 8.4 percent. As such, the Nash wage is highly procyclical. The implication, then, is that the measured wage would have to be roughly this procyclical for us to find an NWE close to 1. Since none of our series for the cost of labor are anything like this procyclical (see Table 3), we find an NWE considerably below 1.

5 Business Cycle Implications of Wage Rigidity

In this section, we assess the business cycle implications of our estimated level of aggregate wage rigidity. For simplicity, we restrict attention to the environment with homogeneous firms and matches and no on-the-job search outline in Section 2.2. Furthermore, as in much of the theoretical literature, we restrict attention to an economy which experiences only one shock, to the marginal revenue product of labor r_t, which follows an exogenous stochastic process. Shocks to r_t could be interpreted as, for instance, productivity shocks or markup shocks, or aggregate demand shocks in a model with sticky prices in goods markets. In addition, we assume for simplicity that the matching function and separation rate are time invariant so that $M_t = M(u_{t-1}, v_t)$ and $s_t = s$. We investigate the consequences of different assumptions about wage setting in this environment.

First, we show analytically that, if all variables follow approximate random walks, then the NWE is approximately a sufficient statistic for the contribution of wage rigidity to the cyclical volatility of unemployment in such a model. This is because we show that there is a tight mathematical relationship between the NWE and the Fundamental Surplus, which Ljungqvist and Sargent (2017) have shown is a useful predictor of the cyclical volatility of unemployment in many search models.

Next, we show via simulations that the NWE does indeed closely predict the volatility of unemployment in a simple SAM model with shocks to r_t, just as the link with the Fundamental Surplus would lead to us to expect. To do this, we calibrate a very simple log-linearized business cycle model based on our search and matching framework. When the NWE is as low as most of our empirical estimates, we show that wage rigidity amplifies unemployment fluctuations in the model more than sevenfold compared to the case of Nash bargaining, and that such a model can easily account for around half of the empirical volatility of unemployment over the business cycle if the only shocks to r_t are productivity shocks.

Lastly, we investigate how far our results are consistent with various other models of wage setting in the literature, including models with constrained efficient wages, such as many directed search models, and rigid wage models based on Hall (2005), Gertler and Trigari (2009) and Christiano, Eichenbaum and Trabandt (2016).

5.1 The NWE and the Fundamental Surplus

We now revisit, in our framework, recent results of Ljungqvist and Sargent (2017), who show in the context of a number of SAM models, including some with sticky wages, that the elasticity of the unemployment rate with respect to productivity shocks depends closely on a term that they call the 'Fundamental Surplus'. We extend their results to the class of models studied in this section.[18] Furthermore, we show that the fundamental surplus depends closely on the Nash wage elasticity, and that the Nash wage elasticity is therefore a strong predictor of the effect of wage rigidity on the volatility of unemployment. The relationship between the Fundamental Surplus and the Nash wage elasticity is so close that it is very difficult for a model in the class we study to deliver a high volatility of unemployment unless it either has a low Nash wage elasticity or has a high value of $\frac{z}{w}$.

We assume, as in Section 4.3 that all variables follow approximate random walks and set, for each variable x, that $\hat{x}_t \simeq \hat{x}_{t-1} \simeq E_t[\hat{x}_{t+1}]$ – i.e. we consider the long run effects of an almost permanent shock. This is essentially the same as Ljundqvist and Sargent's approach of studying the comparative statics of model steady states with respect to parameters, and is a valid approximation insofar as shocks are highly persistent.

To derive the fundamental surplus formula, log-linearize the matching function, to infer that (under the random walk assumption):

$$\hat{f}_t \simeq (1-\phi)(\hat{v}_t - \hat{u}_t) \tag{35}$$

where ϕ is the elasticity of the matching function with respect to unemployment, in the neighborhood of the steady state. Substitute (4) into (2) to eliminate $\mathcal{J}$ terms and log-linearize. Then, combine with (22) and (35) and use that $\hat{w}_t = \epsilon_N \hat{w}_t^N$, and $\hat{z}_t = \epsilon_z \hat{w}_t$, where ϵ^N is the Nash wage elasticity and ϵ_z is the procyclicality of z_t relative to wages. After rearrangement, we obtain:

$$-\frac{\hat{u}_t}{\hat{r}_t} \simeq \phi(1-u)\tilde{\Upsilon} \underbrace{\left(\frac{w + \delta_F h}{\alpha_0 \delta_F h + (w-z)\tilde{\epsilon}\left[1 - \frac{\delta_F}{\delta}(1-\alpha_0)\right]} \right)}_{\text{Inverse Fundamental Surplus Ratio}} \hat{r}, \tag{36}$$

[18] This class was specified immediately above.

where

$$\tilde{\epsilon} = \frac{\epsilon_N}{1 - \epsilon_z \epsilon_N \frac{z}{w}},$$
$$\delta = 1 - (1 - f - s)(1 - \rho),$$
$$\delta_F = 1 - (1 - s)(1 - \rho),$$
$$\alpha_0 = 1 - \frac{\kappa_1}{h}.$$

and

$$\tilde{\Upsilon}^{-1} = \left(\frac{\delta\tilde{\epsilon} - \delta_F\tilde{\epsilon}}{\left(\frac{1-\beta}{\beta}\right)\alpha_0\delta_F + \delta\tilde{\epsilon} - (1-\alpha_0)\delta_F\tilde{\epsilon}} \right)\phi + \left(\frac{\left(\frac{1-\beta}{\beta}\right)\alpha_0\delta_F + \alpha_0\delta\tilde{\epsilon}}{\left(\frac{1-\beta}{\beta}\right)\alpha_0\delta_F + \delta\tilde{\epsilon} - (1-\alpha_0)\delta_F\tilde{\epsilon}} \right)(1-\phi).$$

The left hand side of (36) is the size of response of unemployment to a shock to $\hat{r}_t$. Thus, squaring this equation gives the cyclical volatility of unemployment relative to $\hat{r}_t$. Ljungqvist and Sargent (2017) assume $\epsilon_z = 0$, and consider cases where $\epsilon_N = 1$ (Nash bargaining) and $\epsilon_N = 0$ (the completely sticky wage of Hall (2005) discussed below). After some rearrangement, it can be shown that the values of $\tilde{\Upsilon}$ and of the Inverse Fundamental Surplus Ratio are exactly the same in these special cases as found by Ljundqvist and Sargent (using different notation). The 'Fundamental Surplus' refers to the reciprocal of the Inverse Fundamental Surplus Ratio.

It is immediate that wage behavior only enters the right hand side of equation (36) via the Nash wage elasticity. As such, insofar as the random walk approximation is accurate, the Nash wage elasticity is an accurate summary statistic for the effect of wage rigidity on the cyclical volatility of unemployment.

We now discuss how this formula shows that the Fundamental Surplus term, and, in particular, the Nash wage elasticity and the ratio $\frac{z}{w}$ are the key drivers of the cyclical volatility of unemployment. Ljundqvist and Sargent show that $\tilde{\Upsilon}$ is bounded below by 1 and above by $\min[\phi; 1-\phi]^{-1}$, a result that can be straightforwardly seen to also hold in our setting by inspecting the expression for $\tilde{\Upsilon}^{-1}$. The standard view in the literature is that the data supports $\phi \simeq 0.5$ (Petrongolo and Pissarides, 2001) in which case $\tilde{\Upsilon} \in [1, 2]$. Then, the only way to get a high volatility of unemployment relative to $\hat{r}_t$ (which is the easiest way to get the model to produce large fluctuation in unemployment) is to make the Inverse Fundamental Surplus Ratio large and the Fundamental Surplus small. Given the very small size of the terms in δ and δ_F in the equation for the Fundamental Surplus, it is virtually impossible to make the Fundamental Surplus small unless the term $\frac{(w-z)\tilde{\epsilon}}{w}$ is small – in other words, either the Nash wage elasticity (which is the main term in $\tilde{\epsilon}$) is small, or z is close to

w – workers are roughly indifferent between being unemployed and employed. This echoes the conclusion of Christiano, Eichenbaum and Trabandt (2021) that wage rigidity is essential to allow SAM models without very high z to deliver large fluctuations in unemployment.[19]

5.2 Numerical Simulations

We build a very simpled calibrated SAM model based on the model framework laid out in Section 5. Our simulations reveal that the NWE closely predicts the volatility of unemployment in the model, relative to the volatility of r_t, which is the driving shock.

We parametrize the model laid out in Section 5 in the simplest possible way while allowing the NWE to vary. In the next section we show results of simulations of this model that show that changes in the NWE predict the cyclical volatility of unemployment well, just as implied by the formal analysis of the Fundamental Surplus above.

We assume that all matches are homogeneous and that the marginal revenue product of labor r_t follows an AR(1) process with quarterly autocorrelation equal to 0.97, roughly the autocorrelation of labor market tightness in our sample period. We are agnostic about whether changes in the marginal revenue product of labor are due to changes in markups (e.g. as a consequence of aggregate demand shocks with nominal rigidities in goods markets) or because of changes in productivity. The separation rate s_t is time invariant and set equal to the steady state separation rate in our empirical analysis.

Workers and firms match according to a Cobb Douglas matching function:

$$M_t = \overline{M} v_t^{1-\phi} u_t^{\phi},$$

where $\overline{M}$ is a constant and $\phi = 0.5$.

To examine the aggregate effects of our estimated level of wage rigidity, we assume that the wage satisfies:

$$\hat{w}_t = \gamma \hat{w}_t^N, \tag{37}$$

where γ is the Nash wage elasticity, and the Nash wage is determined by the log-linearized equation (22) that was derived in Section 2. We consider values of the Nash wage elasticity γ in the range from 0 to 1.

For simplicity, we assume linear utility, so that $\sigma = 0$. This entails that $\hat{c}_t$ drops out of the model equations, which allows us to avoid making assumptions about goods markets and the determination of aggregate consumption.

[19]Ljundqvist and Sargent also argue that, for instance, large fixed hiring costs κ_0 will bring down the fundamental surplus. The formulation here, where κ_0 does not directly appear in the fundamental surplus formula, makes clear that large fixed hiring costs can significantly shrink the fundamental surplus (only) insofar as they increase the equilibrium steady state ratio $\frac{z}{w}$.

Finally, it is necessary to specify the cyclicality of the flow value of unemployment. For this we consider two cases, one case where z_t is acyclical, and one where it is proportional to w_t, in which case:

$$\hat{z}_t = \hat{w}_t \tag{38}$$

All other variables are calibrated in line with the steady state values we used in Section 3 above.

Table 5 shows the standard deviation of unemployment relative to the marginal revenue product of labor for different levels of the NWE, and for the case of acyclical and procyclical z. All moments are hp-filtered, in accordance with our empirical analysis. To adjudicate the accuracy of the fundamental surplus formula, (36), the volatility implied by that formula is also shown. We see that, except for very low values of the NWE, the funamental surplus formula provides a relatively good guideline of the likely effect of wage rigidity on the volatility of unemployment.

The relative volatility of unemployment increases substantially as the Nash wage elasticity falls. With a Nash wage elasticity of 0.1, somewhat higher than most side of our estimates, the relative volatility of unemployment is more than 7 times as high as in the case of a Nash wage elasticity of one. Thus, our empirical findings suggest that wage rigidity may be increasing the cyclical volatility of unemployment more than sevenfold compared to what would be occurring under flexible wages.[20] In our data, the relative cyclical volatility of unemployment is roughly 11 times that of productivity. Thus, our estimates suggest that if r_t represented shocks to productivity alone (i.e. we ignored e.g. aggregate demand shocks) then the empirical level of wage rigidity can account for around half of the cyclical volatility of unemployment. Thus, wage rigidity goes a long way to explaining the 'Shimer puzzle' that unemployment is far more volatile relative to productivity than implied in a simple model with Nash bargaining: it can explain around half of the Shimer puzzle just with productivity shocks alone.

5.3 Implications of the NWE for Non-Nash Wage Models

We now study how far our estimates of the NWE are informative for various models of non-Nash bargaining. We investigate the implications of our NWE estimates for four non-Nash approaches to modeling wages from the recent literature, first models in which the labor market is constrained efficient, as in many models of directed search (Wright et al., 2021),

[20] A caveat with this analysis is that, in a richer model, changes in the flexibility of wages could have additional repercussions for labor demand and therefore the volatility of r_t. For instance, if a higher level of wage flexibility led to a less countercyclical capital-labor ratio, this might make r_t less volatile. Alternatively, if changes in wages affect the aggregate demand for goods, this could also affect r_t if goods markets feature nominal rigidities.

Table 5: NWE and Simulated Relative Unemployment Volatility

NWE	Acyclical z		z Proportional to w	
	Relative Volatility of Unemployment	Volatility Implied by FS formula	Relative Volatility of Unemployment	Volatility Implied by FS formula
0	21.1	46.1	21.1	46.1
0.05	8.6	13.5	8.4	13.2
0.1	5.4	7.9	5.2	7.6
0.2	3.1	4.3	2.8	3.9
0.3	2.2	3.0	1.9	2.6
0.4	1.7	2.3	1.4	1.9
0.5	1.3	1.8	1.0	1.4
0.6	1.1	1.5	0.8	1.1
0.7	1.0	1.3	0.7	0.9
0.8	0.9	1.2	0.5	0.7
0.9	0.8	1.0	0.4	0.6
1	0.7	0.9	0.4	0.5

then three models designed to generate rigid wages: the approaches of Hall (2005), Gertler and Trigari (2009) and Christiano, Eichenbaum and Trabandt (2016). We show that the wage setting assumptions in these papers can, with small changes, be incorporated into the framework of Section 2, at least in the case of homogeneous firms and matches.

We show that the constrained efficient wage setting model delivers a wage that is weakly more procyclical than the Nash wage provided the matching function displays as much complementarity between unemployment and vacancies as the main matching functions considered in the literature. Therefore, if wages were set in a way consistent with constrained efficiency, we would expect to estimate a value of the NWE greater than 1. As such, our low estimates of the NWE indicate, first, that many directed search models are likely to struggle to explain the pattern of wages we see in the data and, second, that wages in the data appear to be more rigid than is consistent with constrained efficiency.

We show that each of the three approaches to rigid wages implies a wage setting equation where the aggregate wage is a function of the Nash wage, and (possibly) hiring costs and the flow value of unemployment.[21] We study the cyclical implications of these three approaches to rigid wages by incorporating the wage setting equation of each into the business cycle model studied in Section 5.2. Our simulation allows us to infer the values of the Nash wage elasticity implied by these rigid wage models, as well as the resulting cyclical volatility of unemployment.

The simplest approach to rigid wages of the three is the approach of Hall (2005). In this

[21] Consistently with the framework of Section 2 the wage setting equation implied by these three approaches to rigid wages does not depend on many other features of the economic environment such as frictions in goods markets.

model, firms are homogeneous and each firm's wage is assumed to be fixed provided that the fixed wage is consistent with positive match surplus for both worker and firm. If the steady state wage rate is consistent with positive match surplus for both worker and firm, then this will continue to be true in the neighborhood of the steady state, and so the wage will remain fixed in the neighborhood of the steady state. In that case, it follows that $\hat{w}_t = 0$, and so the Nash wage elasticity implied by the Hall (2005) model is exactly zero. This is not far from some of our estimates of the NWE in Section 4. Since this model is a special case of the model in Section 5.2, the row of Table 5 corresponding to an NWE of 0 shows the results implied by the wage setting assumption of Hall (2005).

We now study the cyclical properties of the other three non-Nash models mentioned: constrained efficient wages, staggered wage bargaining and alternating offer bargaining.

5.3.1 Constrained Efficient Wages and Directed Search

We suppose that the wage is set in such a way that, in the absence of goods market or financial market frictions, the equilibrium level of unemployment is constrained efficient. This allows us to infer the wage behavior implied by the many directed search models which entail constrained efficiency in the absence of frictions in other markets.[22]

We suppose that the matching function has an elasticity of substitution between unemployment and vacancies that is weakly less than 1, so that the elasticity of matches with respect to unemployment, ϕ_t, is weakly decreasing in the number of unemployed. This assumption nests the cases normally considered in the literature, including the common Cobb-Douglas matching function which has an elasticity of substitution of 1, as well as, for instance, urnball matching functions.

A constrained efficient allocation would set vacancies according to the following first order condition of a benevolent social planner:

$$
\begin{aligned}
p_t - z_t - \frac{\kappa_1}{1-\phi_t}\left(\frac{v_t}{u_{t-1}f_t}\right) + \mathbb{E}_t\Bigg[\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)\frac{\kappa_1}{1-\phi_t}\left(\frac{v_{t+1}}{u_t f_{t+1}}\right)(1-s_{t+1}-\phi_t f_{t+1}) \\
- \kappa_0 + \kappa_0\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)(1-s_{t+1})\Bigg] = 0,
\end{aligned}
$$

where p_t denotes the marginal product of labor. The intuition for the first line of this first order condition is as follows. Suppose the planner creates enough extra vacancies at time t to create an extra position at t, and reduces vacancies at $t+1$ to leave employment at $t+1$ unchanged. The benefit of this at time t is that there is p_t extra output, but one fewer worker is unemployed so the flow value of unemployment z_t is lost. Furthermore, the planner

[22] See Wright et al. (2021) for a discussion of the relationship between directed search and constrained efficiency.

has to create $\frac{1}{1-\phi_t}\left(\frac{v_t}{u_{t-1}f_t}\right)$ vacancies at time t, because each vacancy has a filling rate of $\frac{v_t}{u_{t-1}f_t}$, and the elasticity of matches with respect to vacancies is $1-\phi_t$. On the other hand, fraction $1-s_{t+1}$ of the extra hired workers are still employed at $t+1$ so the planner can create correspondingly fewer vacancies then, but also has to create extra vacancies at $t+1$ in proportion to $\phi_t f_{t+1}$, since there will be one fewer unemployed at the start of $t+1$ per extra worker hired at t, and so, all else equal, this will lead to $\phi_t f_{t+1}$ fewer matches at $t+1$ because the elasticity of matches with respect to unemployment is ϕ_t. The intuition for the second line is simply that hiring an extra worker at t costs κ_0 but requires $1-s_{t+1}$ fewer hires at $t+1$ since $1-s_{t+1}$ of extra employees will still be employed then.

Now, suppose there are no goods or financial market frictions, so that the marginal revenue product of labor satisfies $r_t = p_t$. The constrained efficient wage is then one such that, given this wage, firms' optimal hiring decisions will achieve the same allocation as the planner. Substituting (4) and (2) into the planner's first order condition, to eliminate p_t, reveals that this implies that the constrained efficient wage, w_t^E should satisfy:

$$\begin{aligned}\frac{\phi_t}{1-\phi_t}\left(\kappa_0+\frac{\kappa_1 v_t}{u_{t-1}f_t}\right) =& w_t^E - z_t \\ &+\mathbb{E}_t\left[(1-\rho)\frac{u'_{c_{t+1}}}{u'_{c_t}}(1-s_{t+1}-f_{t+1})\frac{\phi_t}{1-\phi_t}\left(\kappa_0+\frac{\kappa_1 v_{t+1}}{u_t f_{t+1}}\right)\right],\end{aligned}$$

Comparing this with equation (8) makes clear that the constrained efficient wage is the same as the Nash wage, except setting the worker bargaining power β equal to ϕ_t (i.e. the well known Hosios condition), and ignoring the fixed cost of hiring κ_0. Now, first consider the Cobb-Douglas matching function, which holds $\phi_t = \phi$ fixed. Then, since our results in Section 3 made clear that adding a fixed cost of hiring will make the Nash wage less cyclical (thereby raising estimates of the Nash wage elasticity) it follows that the constrained efficient wage will be more procyclical than the Nash wage if $\kappa_0 > 0$. Now, alternatively, with a matching function that has an elasticity of substitution strictly less than 1, ϕ_t will tend to be procyclical, since it is decreasing in the number of unemployed and increasing in vacancies. This adds an additional procyclical element to the efficient wage, which is increasing in ϕ_t. This further accentuates the tendency for the constrained efficient wage to be more procyclical than the Nash wage.

Thus, we may conclude that if wages were set in a way consistent with constrained efficiency, as in many directed search models, wages would be more procyclical than Nash wages and we would expect the Nash wage elasticity to be greater than 1. As a consequence of this, it follows that the low Nash wage elasticity we find in the data not only indicates that directed search models may have difficulty in matching the empirical behavior of wages over the business cycle, but also indicates that the movement of wages over the business cycle is likely to be more rigid than is consistent with constrained efficiency.

5.3.2 Staggered Wage Bargaining:

The first model of rigid wages we consider is a staggered wage bargaining model based on Gertler and Trigari (2009), henceforth GT. In this model, each firm pays all its workers the same wage. At the start of each period, each firm draws an idiosyncratic iid shock which determines whether it renegotiates its wages with its workers or not. Fraction λ of firms retain the same wage as they had in the previous period, while fraction $1-\lambda$ of firms negotiate a new wage with all their workers according to Nash bargaining,

To isolate the effect of wage rigidity on unemployment fluctuations, we amend GT's staggered wage bargaining model so that it is consistent with the modeling framework outlined in Section 2, with as few additional assumptions as possible. This allows us to compare the wage implied by staggered wage bargaining with the Nash wage derived in Section 2.

To this end, we make one change to the wage bargaining framework in GT. In GT, the firm, when negotiating wages with its existing workers, takes into account that this wage will affect the wages of new workers it hires. The effect of this assumption in GT is to lead firms to bargain as if their discount rate is somewhat lower, and the effective firm discount rate is time varying and depends on the firm's expectations of its future hiring, and also of how its future hiring will be affected by the wage rate it negotiates. This adds considerable complexity to the bargaining problem, and also entails that the GT wage bargaining solution depends on convex costs of hiring, which are a feature of GT but are inconsistent with the framework of Section 2. To avoid this complexity and to maintain consistency with the framework of Section 2, we do not assume convex costs of hiring. Furthermore, we assume that when a firm renegotiates its wages with workers, the outcome of this negotiation depends only on the match surplus the firm earns from its current workers, and the match surplus of these workers, and does not depend on the effect of wages on future hiring.

As such, we assume that, when a firm renegotiates wages with workers, the renegotiated wage for each match k satisfies the Nash bargaining solution:

$$\mathcal{W}_t^k - \mathcal{U}_t = \beta[(\mathcal{W}_t^k - \mathcal{U}_t) + (\mathcal{J}_t^k - \mathcal{V}_t^i)] = \beta[(\mathcal{W}_t^k - \mathcal{U}_t) + \mathcal{J}_t^k],$$

where β is the worker bargaining share and the Bellman values $\mathcal{J}_t^k$, $\mathcal{V}_t^i$, $\mathcal{W}_t^k$ and $\mathcal{U}_t$ are as defined in Section 2 and so evolve according to the same Bellman equations as in Section 2.

Since all matches in the same firm are the same, we let $\mathcal{J}_t^i(w)$ denote the match surplus of the firm i if it pays the wage w. Likewise, $\mathcal{W}_t^i(w)$ is the match surplus of the worker in firm i if they are paid wage w.

Let $\overline{\mathcal{J}}_t^i$ be the expected value of a firm i at the start of the period t, before it discovers whether or not it will renegotiate its wages that period. That is:

$$\overline{\mathcal{J}}_t^i = \lambda \mathcal{J}_t^i(w_{t-1}^i) + (1-\lambda)\mathcal{J}_t^i(w_t^{*i})$$

where w_{t-1}^i is the wage paid by the firm in the previous period, and w_t^{*i} is the wage that would be negotiated if the firm renegotiates its wages.

Define $\overline{W}_t^i$ similarly.

The Bellman equations in Section 2 imply that:

$$\begin{aligned}
\mathcal{J}_t^i(w_t^*) &= \overline{\mathcal{J}}_t^i - (w_t^{*i} - \overline{w}_t^i) + \lambda(1-s_{t+1})E_t[m_{t+1}(\mathcal{J}_{t+1}^i(w_t^{*i}) - \overline{\mathcal{J}}_{t+1}^i)] \\
\mathcal{W}_t^i(w_t^*) &= \overline{\mathcal{W}}_t^i + (w_t^{*i} - \overline{w}_t^i) + \lambda(1-s_{t+1})E_t[m_{t+1}(\mathcal{W}_{t+1}^i(w_t^{*i}) - \overline{\mathcal{W}}_{t+1}^i)],
\end{aligned}$$

where the expected wage of at the start of the period (before it is known whether renegotiation will happen) is:

$$\overline{w}_t^i = \lambda w_t^{*i} + (1-\lambda)w_{t-1}^i.$$

When wages are renegotiated, the bargaining solution satisfies:

$$\mathcal{W}_t^i(w_t^{*i}) - \mathcal{U}_t = \beta[(\mathcal{W}_t^i(w_t^{*i}) - \mathcal{U}_t) + \mathcal{J}_t^i(w_t^{*i})],$$

Combining this with the previous two Bellman equations and rearranging, we obtain:

$$\left(\frac{\beta}{1-\beta}\right)\left(\overline{\mathcal{W}}_t^i\right) = \overline{\mathcal{J}}_t^i - \frac{w_t^{*i} - \overline{w}_t^i}{1-\beta} + \lambda(1-s_{t+1})E_t\left[m_{t+1}\left(\left(\frac{\beta}{1-\beta}\right)\left(\overline{\mathcal{W}}_{t+1}^i - \overline{\mathcal{J}}_{t+1}^i\right)\right)\right]$$

Averaging across all firms (and so dropping i superscripts), log-linearizing around the steady state and rearranging, we obtain:

$$-\frac{\hat{\beta}_t}{1-\beta} = \left(\frac{w}{\beta h}\right)\left(\frac{1}{1-\lambda(1-s)(1-\rho)} \cdot \frac{\lambda}{1-\lambda}\right)(\hat{w}_t - \hat{w}_{t-1})$$

Substituting in equations (21) and (16), we obtain:

$$\hat{w}_t - \hat{w}_{t-1} = (1-\delta)\mathbb{E}_t[\hat{w}_{t+1} - \hat{w}_t] + \frac{\delta}{\psi}(\hat{w}_t^N - \hat{w}_t) \tag{39}$$

where

$$\begin{aligned}
\delta &= 1-(1-f-s)(1-\rho) \\
\psi &= \frac{w-z}{\beta h}\left(\frac{1}{1-\lambda(1-s)(1-\rho)}\right)\left(\frac{\lambda}{1-\lambda}\right).
\end{aligned}$$

Equation (39) is the wage setting equation for the staggered wage bargaining model. This resembles New Keynesian Phillips curve equations, in that the rate of growth of wages depends on the deviation of wages from the negotiated (Nash) level, and also depends on the expected rate of growth of wages next period.

We now study the business cycle properties of the staggered wage bargaining model. In particular, we keep the business cycle model assumptions unchanged from Section 5.2, except that we replace the wage equation (37) assumed there, and replace it with the wage equation (39). For simplicity, we limit attention to the $\hat{z}_t = 0$ case. In Table 6 below, we consider various values of λ. Since λ is the probability that a firm is unable to renegotiate its wages in a period, this parameter determines the level of wage rigidity in the staggered bargaining model.

For each value of λ, the second column of the Table 6 shows the estimated NWE obtained from simulating the model for 10,000 periods and performing an OLS regression of the wage on the Nash wage in the simulated data, in accordance with the first approach we used to estimate the Nash wage in Section 3. The remaining columns of Table 6 mirror those Table 5: they show the relative volatility of unemployment implied by the staggered bargaining model, and then they show the relative volatility predicted by the Fundamental Surplus formula, given the estimated NWE in the second column.

GT originally calibrated λ at 0.88. As Table 6 shows, this is consistent with an NWE of 0.03, which is rather lower than the majority of our empirical estimates. On the other hand, if the model is recalibrated with $\lambda = 0.66$, the implied NWE is slightly higher than the majority of our estimates. This suggests that the staggered bargaining model is consistent with the level of wage rigidity we estimate, provided that λ is calibrated at a somewhat lower level than assumed by GT.

The last two columns of Table 6 also reveal that the NWE implied by the staggered bargaining model provides a useful guideline to the cyclical volatility of unemployment in that model, using the Fundamental Surplus formula. Nevertheless, the staggered bargaining model delivers a rather lower level of unemployment volatility than one would expect given the Nash wage elasticity. This is because the staggered bargaining model delivers a low NWE only in the short run (recall that we estimated the NWE delivered by the model using hp-filtered simulated data). In the long run, the staggered bargaining model implies that the wage should be fully flexible. Consequently, firms may e.g. hire more in recessions than the low NWE of the staggered bargaining model would suggest, because they anticipate that while wages are sticky now, they will fall in future.

Table 6: NWE and Relative Unemployment Volatility under Staggered Bargaining

λ	Estimated NWE	Relative Volatility of Unemployment	Volatility Implied by FS formula
0.01	0.99	0.70	0.95
0.02	0.98	0.70	0.96
0.11	0.83	0.72	1.12
0.22	0.61	0.78	1.50
0.33	0.41	0.89	2.20
0.44	0.26	1.10	3.44
0.55	0.14	1.46	5.76
0.66	0.07	2.14	10.25
0.77	0.03	3.47	19.06
0.88	0.01	6.58	33.75
0.99	0.00	18.07	45.68

5.4 Alternating Offer Bargaining

Following Hall and Milgrom (2008) and Christiano, Eichenbaum and Trabandt (2016) we consider a wage setting protocol in which wages are determined by an alternating offer bargaining game. The details of the bargaining game follow Christiano, Eichenbaum and Trabandt (2016) (henceforth CET) exactly. We suppose that each period is divided into $M = 60$ sub-periods. At the start of the first sub-period, the firm makes an initial wage offer to the worker, which the worker can accept or reject. If the wage offer is rejected, play proceeds to the next sub-period. In odd sub-periods, if the firm and worker have not yet reached agreement, then the firm gets to make a wage offer to the worker, which the worker can accept or reject. Every offer the firm makes costs the firm γ in processing costs. In even sub-periods, if the firm and worker have not yet reached agreement, then the worker makes an offer to the firm, which the firm can accept or reject. If neither have reached agreement by the end of the last sub-period, the match terminates. Additionally, each time an offer is rejected, bargaining breaks down and the match is terminated with probability ς. In each sub-period in which the two sides have not reached agreement, the worker does not produce the flow value r_t^k and does not get paid, but receives the flow value of unemployment z_t.

CET show that the solution of the bargaining game is that the worker accepts the firm's offer in the first sub-period and the wage satisfies:

$$\mathcal{J}_t = \mu_1(\mathcal{W}_t - \mathcal{U}_t) - \mu_2\gamma_t + \mu_3(r_t - z_t)$$

where $\mu_i = \frac{\alpha_{i+1}}{\alpha_1}$ and

$$\alpha_1 = 1 - \varsigma + (1-\varsigma)^M$$

$$\alpha_2 = 1 - (1-\varsigma)^M$$

$$\alpha_3 = \alpha_2 \left(\frac{1-\varsigma}{\varsigma}\right) - \alpha_1$$

$$\alpha_4 = \left(\frac{1-\varsigma}{2-\varsigma}\right)\frac{\alpha_2}{M} + 1 - \alpha_2$$

where ς is the probability that bargaining breaks down each day.[23].

Into this, we substitute the firm's Bellman equation to eliminate r_t, substitute that $\mathcal{W}_t - \mathcal{U}_t = \frac{\beta_t}{1-\beta_t} \cdot \mathcal{J}_t$ (where β_t is the worker's share of match surplus) to eliminate $\mathcal{W}_t$ and $\mathcal{U}_t$, and substitute the firm's optimal hiring decision $\mathcal{J}_t = h_t$ to eliminate $\mathcal{J}_t$. We obtain the following form of the alternating offer bargaining solution:

$$h_t = \mu_1 h_t \cdot \frac{\beta_t}{1-\beta_t} - \mu_2\gamma + \mu_3(w_t + h_t - \mathbb{E}_t[m_{t+1}(1-s_{t+1})h_{t+1}] - z_t)$$

Log-linearizing this around the steady state and using equations (16) and (21), we obtain:

$$\hat{w}_t = \left(\frac{\mu_1}{\mu_1+\mu_3}\right)(\hat{w}_t^N - \hat{w}_t^A) + \hat{w}_t^A - \left(\frac{\mu_3}{\mu_1+\mu_3}\right)(1-\delta)\mathbb{E}_t\hat{w}_{t+1}^A + \left(\frac{\mu_3}{\mu_1+\mu_3}\right)(1-\delta)\mathbb{E}_t\hat{w}_{t+1} \tag{40}$$

where, $\hat{w}_t^N$ is the Nash wage (deviation from the steady state) and $\hat{w}_t^A$ is the deviation of an alternative wage, given by:

$$\hat{w}_t^A = \left(\frac{h}{\mu_3 w}\right)\left[1 - \mu_3 - \frac{\beta\mu_1}{1-\beta}\right]\hat{h}_t + \frac{(1-\rho)(1-s)h}{w}\mathbb{E}_t\left[(\sigma c_t - \sigma c_{t+1}) + \hat{h}_{t+1} - \frac{s\hat{s}_{t+1}}{1-s}\right] + \frac{z}{w}\hat{z}_t. \tag{41}$$

As with the staggered wage bargaining model above, we study the cyclical properties of the business cycle model in Section 5.2, replacing the wage equation there (equation (37)) with the wage setting equations (40) and (41). Again, we fix $\hat{z}_t = 0$. The key parameter that determines the rigidity of wages under alternating offer bargaining is the probability ς that bargaining breaks down. As we did for λ with the staggered bargaining model, we vary the level of this parameter, estimate the resulting NWE on model simulated data and compare the cyclical volatility of unemployment with what would be implied by the model's implied NWE and the Fundamental Surplus formula. The results are in Table 7 below. We see that, across parameter values, the alternating offer model delivers an NWE around 0.7, significantly higher than almost all our estimates. The volatility of unemployment predicted

[23] Here, we have written CET's result in terms of our own notation

by the model is close to what one would expect from its NWE, based on the fundamental surplus formula.

Table 7: NWE and Relative Unemployment Volatility under Alternative Offer Bargaining

ς	Estimated NWE	Relative Volatility of Unemployment	Volatility Implied by FS formula
0.000	0.69	0.84	1.34
0.002	0.70	0.87	1.32
0.003	0.71	0.89	1.30
0.005	0.72	0.90	1.29
0.006	0.72	0.91	1.28
0.008	0.73	0.92	1.27
0.009	0.73	0.93	1.26
0.011	0.74	0.93	1.26
0.012	0.74	0.94	1.25
0.014	0.74	0.94	1.25
0.015	0.75	0.95	1.25
0.017	0.75	0.95	1.24

It is surprising that the alternating offer bargaining model generates an NWE so close to 1, given that a key purpose of the model was to generate wage rigidity. Inspection of equation (40) and (41) indicates that a major reason for the cyclicality of the wage under the alternating offer bargain is the cyclicality of hiring costs. CET likewise note that the alternating offer bargain fits their macroeconomic data far better with fixed rather than variable hiring costs. For this reason, we also study the alternating offer bargaining model with primarily fixed hiring costs. Specifically, we reduce κ_1 from 0.43 to 1, and recalibrate κ_0 to maintain the average total hiring cost. Results of the simulations of this model are presented in Table 8 below. In this case, the results are very sensitive to ς, but with values of ς close to zero, the model achieves an NWE close to 0.3 and correspondingly larger unemployment fluctuations. Thus, our results support CET's assertion that fixed hiring costs help the model fit the data. This is still higher than many of our estimates. As such, our estimates generally support a level of wage rigidity that is as great or greater than implied by the alternating offer bargaining model.

Table 8: NWE and Relative Unemployment Volatility under Alternative Offer Bargaining with Mostly Fixed Hiring Costs

ς	Estimated NWE	Relative Volatility of Unemployment	Volatility Implied by FS formula
0.000	0.33	3.33	3.99
0.002	0.42	2.71	3.19
0.003	0.50	2.36	2.75
0.005	0.56	2.14	2.48
0.006	0.61	1.99	2.30
0.008	0.64	1.87	2.16
0.009	0.68	1.79	2.06
0.011	0.71	1.72	1.98
0.012	0.73	1.67	1.92
0.014	0.75	1.63	1.87
0.015	0.77	1.60	1.83
0.017	0.78	1.57	1.80

6 Conclusion

In this paper, we develop a new measure of aggregate real wage rigidity, the Nash wage elasticity. The NWE is simply the elasticity of the measured marginal cost of labor with respect to the Nash wage, where the bargaining share is set to equal the actual wage in a steady state. A completely rigid wage implies an NWE of 0, and if wages were set by Nash bargaining then the NWE should in theory equal 1.

We build a broad modelling framework that encompasses a wide variety of cases studied in the literature, and show that the framework delivers equations for the worker share of match surplus and the Nash wage that can be calculated from empirical data. When taking these equations to US data from 1979-2012, we find that the worker share of match surplus is strongly countercyclical and that the Nash wage is substantially more procyclical than the observed cost of labor. These findings hold for a range of different measures of the cost of labor that range from practically acyclical to strongly procyclical.

Across 180 regressions, which variously use different wage measures and use either simple OLS or different instruments for the Nash Wage, we obtain estimates of the Nash wage elasticity that are mainly between 0 and 0.1. We only obtain Nash wage elasticity estimates above 0.65 for the most procylical series of the cost of labor (the user cost from the NLSY) and, even with this series, only for specifications that assume a relatively high value of fixed hiring costs and/or the opportunity cost of employment.

We investigate the business cycle implications of the our small estimated values for the NWE. We find that a small NWE makes an enormous difference to fluctuations in unemployment. We show that there is a tight link between the NWE and the Fundamental Surplus of Ljungqvist and Sargent (2017), with smaller values of the NWE greatly shrinking the Fundamental Surplus and increasing the volatility of unemployment in SAM models with shocks to the marginal revenue product of labor. In a simple SAM model with such shocks, an NWE of 0.1 yields fluctuations in unemployment that are more than seven times as large as occur when the NWE is 1. In this sense, the vast majority of cyclical movements in unemployment can be attributed to the effects of wage rigidity.

Finally, we compare our estimated NWE with the implications of various non-Nash models of wage setting. This includes constrained efficient wage setting, as in many directed search models, and three models of sticky wages. We find that our estimated NWE implies much more rigid wages than is consistent with the constrained efficient wage model. Our NWE estimates do suggest wages may be less rigid than assumed by Hall (2005) and Gertler and Trigari (2009), but more rigid than assumed by Christiano, Eichenbaum and Trabandt (2016).

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Appendix

A Data Sources and Data

Name	Description	Source	ID	Notes
u	Unemployment Rate	Fred	UNRATE	
u^l	Unemployment Level	Fred	UNEMPLOY	
u^s	Short tern unemployment	Fred	UEMPLT5	unemployed for less than 5 weeks
e^l	Employment Level	Fred	CE16OV	
c	Personal Consumption	Fred	A794RX0Q048SBEA	
	Labour Productivity	BLS	NFBUS	
w	NLSY New Hire Wage	Basu and House (2016)		
w^{CES}	Average Hourly Wage	Fred	AHETPI	
w^{UL}	NLSY User Cost of labor	Kudlyak (2014)		reported in Basu and House (2016)
z	Elasticity	Chodorow-Reich and Karabarbounis (2016)		
f	Finding Rate	Calculated		Eq. (26)
s	Separation rate	Calculated		Eq. (27)
v	Vacancy rate	Petrosky-Nadeau and Wasmer (2013)		
Forecasting VAR				
1-Year T-Rate	Market Yield on U.S. Treasury Securities at 1-Year Constant Maturity	Fred	GS1	
Real GDP	Real Gross Domestic Product	Fred	GDPC1	
GDP Deflator	GDP Implicit Price Deflator	Fred	USAGDPDEFQISMEI	
Shock Measures				
MP	Romer and Romer Narrative Series	Romer and Romer (2004)		updated in Wieland and Yang (2020)