

Capital Deaccumulation and the Large Persistent Effects of Financial Crises

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January 10, 2018

Abstract

In a panel of OECD and emerging economies, I find that recessions are associated with larger initial drops in investment and more persistent drops in output if they occur simultaneously with banking crises. Furthermore, the banking crises that are followed by more persistent output slumps are associated with particularly large initial drops in investment. I show that these patterns can arise in a model where a financial shock temporarily increases the costs of external finance for investing entrepreneurs. This leads to a drop in investment and a persistent slump in output. Critical to the model is the distinction between different types of capital with different depreciation rates. Intangible capital and equipment have high depreciation rates, leading these stocks to drop substantially when investment falls after a financial shock. If wages display some rigidity, this induces a slump in output and employment that persists for roughly a decade, through the contribution of equipment and intangibles to production and labor demand. I find that this mechanism can account for almost a third of the persistent drop in output and employment in the US Great Recession (2007-2014). In the model, TFP and government spending shocks lead to relatively smaller declines in investment and less persistent drops in output, so the model is also consistent with the more transitory output drops seen after non-financial recessions, where such shocks may have been more important.

*I am indebted especially to my PhD advisor Mark Bilal, and to George Alessandria, Yan Bai, Yongsung Chang and Romans Pancs for their generous help and guidance throughout this project. For very helpful suggestions and comments, I thank Parantap Basu, Chitrakha Basu, Corina Boar, Huberto Ennis, Andreas Hornstein, Nobuhiro Kiyotaki, Dan Lu, Gabriel Mihalache, Pierre Sartre, Felipe Schwartzman, Tatsuro Senga, Nicholas Trachter, William Taylor, and seminar participants at the Federal Reserve Bank of Richmond, the 2015 Money, Macro and Finance Research Group Annual Meeting, and the Fall 2015 Midwest Macro Meeting. Any remaining errors are my own.

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1 Introduction

This paper proposes a model to account for the long and protracted slumps in output that often follow financial crises. The length of the slump that followed the financial crisis of 2008 in many countries has been seen as puzzling in the literature, with many real and monetary models with financial shocks predicting much sharper recoveries (see, for instance, Kydland and Zarazaga (2013); Galí, Smets and Wouters (2012)).¹ However, the long slump after 2008 does not appear to have been particularly exceptional. In a panel of 40 OECD and emerging economies from 1970 onwards, I find that output typically shows no sign of returning to trend 6 years after a recession featuring a banking crisis. This finding is consistent with a large empirical literature.² By contrast, the majority of banking crises in the dataset last fewer than 4 years, suggesting that the slump in output persists substantially beyond the period of financial stress. Research on the behavior of credit spreads also supports this interpretation. For instance, Krishnamurthy and Muir (2015) find that increases in the spread predict output losses during banking crises, but output does not immediately return to trend once spreads have returned to normal levels.

I hypothesize that drops in investment during financial crises may be a major force contributing to long subsequent slumps. I find in my panel that recessions marked by banking crises not only feature longer-lived output drops than other recessions, but are also characterized by larger drops in investment at the onset of the recession.³ I also find that banking crises that are followed by more persistent output drops are associated with larger drops in the investment-output ratio around the time of the crisis. Neither of these two findings can be explained by the fact that recessions which are initially more severe are also more persistent. This is because recessions marked by financial crises do not feature much larger initial output drops than other recessions, even in cases where the output drop is ultimately very persistent.

I show that these patterns can arise in a model where financial shocks have persistent effects on output through their effects on investment and the capital stock. In the model I present, a financial shock leads to a particularly sharp drop in investment, which causes a persistent decrease in the stock of equipment and intangible capital and, therefore, output. The quantitative importance I find for this mechanism contrasts with the existing literature, which has found movements in the aggregate capital stock, as conventionally measured, to be of less importance in economic fluctuations. This is because the depreciation rate is sufficiently low that

¹Galí, Smets and Wouters (2012) argue that their New Keynesian model can account for the slowness of the recovery from the US Great Recession, but only if the US economy continued to be hit by repeated negative shocks in the years after 2008. No shock individually has a persistent negative effect on output.

²Empirical studies have found financial crises to be followed by very persistent decreases in output relative to the pre-crisis trend in the US, in the world and in the OECD. Such empirical studies include Cerra and Saxena (2008), Furceri and Mourougane (2012), Jalil (2015), Krishnamurthy and Muir (2015) and Abiad et al. (2009). The latter also find financial crises to be followed by persistent decreases in employment.

³That financial crises are followed by particularly large investment slumps has also been documented by Ottonello (2014).

the capital stock does not fluctuate much except at very low frequencies, both in models and in the data.⁴ The key novel feature of my model that overturns this conclusion and generates larger and persistent output slumps is the important role of equipment and intangible capital in production. Unlike standard models, I disaggregate the capital stock into structures, on the one hand, and equipment and intangibles on the other. These are treated as separate inputs into a Cobb-Douglas production function. Equipment and structures represent the two types of business capital traditionally measured in the national accounts. Intangible capital refers to the assets firms have built up through past investment in product design, marketing and customer support, research and development, human capital and organizational development. Many of these assets are still not included in US national accounts. Crucially, I calibrate a relatively large stock of intangible capital, based on the estimates of Corrado and Hulten (2010).⁵ The calibration implies a correspondingly large role for equipment and intangibles in production.⁶

The propagation mechanism in the model results from movements in the stock of equipment and intangible capital, rather than from movements in structures. This is because equipment and intangible capital differ from structures in their depreciation rates. Equipment and intangible capital face estimated depreciation rates of around 20% per year, whereas the depreciation rate of structures is around 4% per year. I model a financial shock as a transitory increase in the riskiness of investment, which reduces the ability of entrepreneurs to borrow externally. This leads to a short-lived rise in the credit spread and a large drop in investment. The decrease in investment causes a substantial decrease in the stock of equipment and intangibles, due to the high depreciation rate of these types of capital. By contrast, the stock of structures does not decrease as much, due to the low depreciation rate of structures. Nevertheless, since equipment and intangibles contribute more to production than do structures, the net effect is a decrease in output which persists until the stock of equipment and intangibles can be replenished. This takes many years. I find that the model produces a decrease in output after a financial shock that is around 50% larger than occurs in an almost identical setting with only one type of capital and a depreciation rate typical of the literature.

As the model would suggest, the US Great Recession after 2008 witnessed a persistent slump in output, a large decrease in the stock of equipment and intangibles relative to trend and a smaller decrease in the stock of structures. However, the crisis of 2008, like many other financial crises, was also followed by a large and persistent decrease in employment. I find that the model can replicate this feature of the data, provided that wages are rigid to an extent that appears consistent with US aggregate wage data. In order to allow for the possibility of wage

⁴See Schwartzman (2012), and Bigio (2012) for more discussion of this.

⁵These estimates are not atypical of the recent intangible capital literature. This is discussed in more depth in footnote 35 on page 28.

⁶For the model to match long-run average investment rates in structures, equipment and intangibles in the steady state, I find that the elasticity of output with respect to equipment and intangibles needs to be 0.38, versus an elasticity of 0.15 for structures.

rigidity, I model the labor market using search frictions, with wages set by an alternating offer bargaining protocol similar to Hall and Milgrom (2008). This bargaining procedure nests the traditional Nash Bargaining solution of the labor search literature as a special case, but can also lead to greater wage rigidity endogenously, depending on the model parameters. I estimate the parameters of this bargaining process using a Bayesian approach, based on NIPA and BLS data on wages, output and employment. I find that the data favors a parametrization that implies more rigid wages than Nash bargaining. At the estimated parameter values, a financial shock leads to a slump in employment, with employment taking six years to recover half-way to the steady state after the shock. By contrast, if standard Nash bargaining is used instead, employment hardly falls after a financial shock and the fall in output is consequently much more muted.

I find that the calibrated model can account for approximately one third of the the persistent decrease in output and employment observed during the US Great Recession. In particular, I analyze how the model economy responds to a series of financial shocks that lead to the the same spikes in the credit spread as in occurred in the US over the 2007-2014 period. In the model economy, these shocks produce a decrease in output and employment around a third as large as the fall in these variables relative to trend during 2007-2010. Moreover, I find that the resultant behavior of aggregate variables in the model shows striking similarities to actual economic developments between 2007 and 2014 in three other key respects. First, output and employment show no tendency to return to their steady state levels over the seven years after the financial crisis. Second, the model economy experiences a significant decrease in the stock of equipment and intangible capital after the financial shocks. Third, it experiences a smaller decrease in the stock of structures. In each of these three respects, the behavior of model variables matches that of detrended variables in the data. In sum, these similarities suggest that the declining stock of equipment and intangibles may be the cause of close to a third of the persistent slump in output and employment during the Great Recession.

I show that the effects of non-financial shocks in the model are consistent with the more transitory decreases in output that appear to occur in non-financial recessions. Specifically, I consider TFP shocks as well as shocks to the level of government employment, as in Michaillat (2014). Due to the existence of financial and search frictions, private employment does not change rapidly in response to changes in government employment in the short run. Consequently, the government employment multiplier is close to 1. Moreover, the responses of output to TFP and government employment shocks is sizable and shows some persistence, but rather less persistence than for financial shocks. In addition, TFP shocks and government employment shocks have less effect on investment than financial shocks. As such, these shocks may account for the more transitory decreases in output and smaller decreases in investment seen in non-financial recessions.

The remainder of the paper is organized as follows. Section 2 discusses how the paper relates to the existing literature. Section 3 considers the empirical behavior of output and investment across a large sample of financial and non-financial recessions. Section 4 presents the model. Section 5 discusses model calibration. Section 6 presents quantitative results and compares movements of variables in the model to the US Great Recession. Section 7 concludes.

2 Relevant Literature

This paper contributes a mechanism through which financial crises have persistent effects on aggregate output through movements in the stock of equipment and intangible capital. As far as I am aware, this is the first paper in the literature to emphasize that the relatively high depreciation rates of these types of capital imply that their stocks can decrease substantially after a financial shock, contributing to persistently low output and employment.

The paper relates most closely to the emerging literature on modeling the persistent economic effects of financial crises, but the mechanism is distinct from those discussed in this literature. Much of the literature on the persistent effects of financial crises draws on endogenous growth models, such as that of Romer (1990). In an endogenous growth setting, a financial crisis can reduce innovation and so have a permanent negative effect on total factor productivity and output. Prominent examples of this approach include Queralto (2013) and Anzoategui et al. (2015).⁷ Critical to the endogenous growth approach is the idea that large scale spillovers arise from technological innovation, leading to persistent negative effects on other firms when a financially constrained firm stops innovating. Unlike in these models, in my setting, the persistent effects of financial crises occur entirely within an exogenous growth context, without any spillover effects of intangible investment. The mechanisms I discuss in this paper are therefore distinct and complementary to this literature.⁸

Other models of the persistent economic effects of financial crises include those of Ottonello (2014), Khan and Thomas (2011) and Schmitt-Grohé and Uribe (2012). As in this paper, Ottonello explores the relationship between what he calls ‘investment slumps’ and slow recoveries after financial crises. However, the channel he focuses on is different, relying on capital becoming unemployed after a financial crisis rather than a deaccumulation of capital after the crisis. Khan and Thomas likewise focus on the misallocation of capital that follows a financial crisis. My approach differs from both these papers in drawing a connection between low investment in

⁷Other examples related to this approach include Bianchi and Kung (2014), Garcia-Macia (2013), Gornemann (2015) and Guerrón-Quintana and Jinnai (2014).

⁸It is also worth noting that investment in equipment and marketing appear to be substantially more volatile in US data than investment in R&D. Since these two forms of investment presumably are not associated with strong spillover effects, this motivates the focus of this paper on the persistent effects of a collapse in investment on output through an alternative channel.

intangibles and the persistence of financial crises.⁹ Finally, Schmitt-Grohé and Uribe (2012) find very persistent effects of financial crises in the context of a liquidity trap. Although this is likely to be a very important mechanism in the Great Recession, it presumably does not apply in the case of financial crises that occurred when interest rates were substantially above the zero-lower bound. This paper, by contrast, considers the effects of financial shocks in a context where money is neutral and so liquidity traps do not play a role.¹⁰

This paper also closely relates to the literature on the measurement of intangible investment and its behavior over the business cycle and during the Great Recession. I rely heavily on the work estimating the stock of intangible capital and intangible investment in the United States by Corrado and Hulten (2010).¹¹ Consistent with the importance accorded to the effect of financial crises on investment in intangible capital in this paper, Campello, Graham and Harvey (2010) find that the tightening of firms' financing constraints in the Great Recession significantly decreased their investment in intangible capital, especially in marketing.¹² Additionally, Hall (2015) and Reifschneider, Wascher and Wilcox (2015) argue that low investment and the decumulation of capital following the financial crisis had a sizeable and persistent negative effect on the economy's aggregate capacity, although these papers do not construct general equilibrium models that give rise to this mechanism.

A small literature has discussed the implications of disaggregating the aggregate capital stock into different sub-categories, such as equipment and structures. However, this literature has not, to my knowledge, simultaneously considered the implications of including intangibles. Furthermore, it has not attempted to account for the persistent effects of financial shocks. Most similar to this paper is Lopez and Olivella (2014), who consider the effect of financial shocks on skilled and unskilled unemployment in a model with two types of capital. They find that allowing for two types of capital amplifies the financial shock when only one type of capital can be pledged as collateral. However, their model does not discuss intangible capital and does not produce persistent decreases in employment following financial shocks. Also relevant is the work of Benhabib, Perli and Sakellaris (2006). They find that disaggregating the capital stock into two types can contribute to the persistence of economic fluctuations, although they do not discuss financial shocks or intangible capital. Chang (1995) observes that the stock of equipment is much

⁹Since it appears difficult to fully account for the severe persistent slumps following financial crises through one channel alone, it is likely that multiple channels are responsible. Thus, the mechanisms discussed by Ottonello (2014) and Khan and Thomas (2011) likely also play an important role explaining in the persistent effects of financial crises, in addition to the channel I discuss.

¹⁰Other models that find persistent effects of financial crises with liquidity traps include Christiano, Eichenbaum and Trabandt (2014) and Eggertsson and Mehrotra (2014).

¹¹Other work attempting to measure intangible capital stocks includes that of Atkeson and Kehoe (2005) and Eisfeldt and Papanikolaou (2014). McGrattan and Prescott (2014) also consider the role played by intangible capital in the Great Recession, and find that the importance of technology shocks in business cycles and the Great Recession is increased when allowance is made for intangible capital.

¹²See Archibugi, Filippetti and Frenz (2013) and the references cited therein for other work discussing the effect of the financial crisis on intangible investment.

more volatile than the stock of structures over the business cycle and considers how distortionary taxes may contribute to this. Slavik and Yazici (2014) analyze optimal dynamic taxation in a model with two types of capital.

This paper also relates to the literature on the interaction between financial frictions and search frictions in labor markets. Prominent examples include Garin (2015) and Petrosky-Nadeau (2014).¹³ Both papers emphasize that the interaction of financial frictions and labor search frictions increases the persistence of movements in employment following economic shocks. Although the mechanism they discuss also operates in this paper to some degree, I find that the role of movements in equipment and intangible capital is quantitatively much more important.

3 Motivating Evidence

A large literature has found that banking crises are associated with particularly persistent declines in output.¹⁴ In this section I present evidence from a sample of developed and emerging economies that the persistent declines in output after financial crises might be due to low levels of investment as a result of the crisis. I consider ‘financial’ and ‘non-financial’ recessions in a sample consisting of OECD countries and countries tracked by JP Morgan’s Emerging Markets Outlook over the period 1970-2014.¹⁵ I define a recession as ‘financial’ if it occurs within a year of a banking crisis, as measured by Reinhart and Rogoff (2009), and ‘non-financial’ otherwise.¹⁶ I use World Bank annual data and drop countries for which GDP or investment data is not available for the whole sample period, leaving a sample of 21 OECD and 19 emerging economies.

Figure 1 shows the average decrease in GDP relative to the 8-year linear pre-recession trend, in the years during and after a financial and non-financial recession. The year of the recession is marked as ‘1’ on the X axis. It is apparent from the figure that both financial and non-financial recessions are associated with decreases in GDP that are surprisingly persistent. There is no evidence of a return of GDP to trend 6 years after the last pre-recession year. Furthermore, Figure 1 is consistent with other findings in the literature that financial recessions are especially associated with persistently poor output performance. Indeed, while the output drop is slightly larger for financial recessions than others in the year of the recession, the gap between the two

¹³Further contributions to this literature include Blanco and Navarro (2014); Chugh (2013); Clymo (2015); Mumtaz and Zanetti (Forthcoming); Quadrini and Sun (2015), and Schoefer (2015).

¹⁴See footnote 2 above for some examples from this literature.

¹⁵Calvo, Izquierdo and Loo-Kung (2006) argue that the Emerging Markets Outlook provides a useful list of emerging economies to study in the context of financial crises, since whether a country appears in the Emerging Markets Outlook can be viewed as a proxy for whether it has well-developed financial markets.

¹⁶Reinhart and Rogoff define a banking crisis as the closure, merger, takeover or large scale government assistance of one or more major financial institutions in a context marked either by bank runs or by a string of such outcomes for similar institutions. The total number of financial recessions in the sample is 33, including 16 in the OECD. The total number of non-financial recessions is 69, including 32 in the OECD.

Figure 1: GDP Decline After Financial vs. Non-Financial Recessions

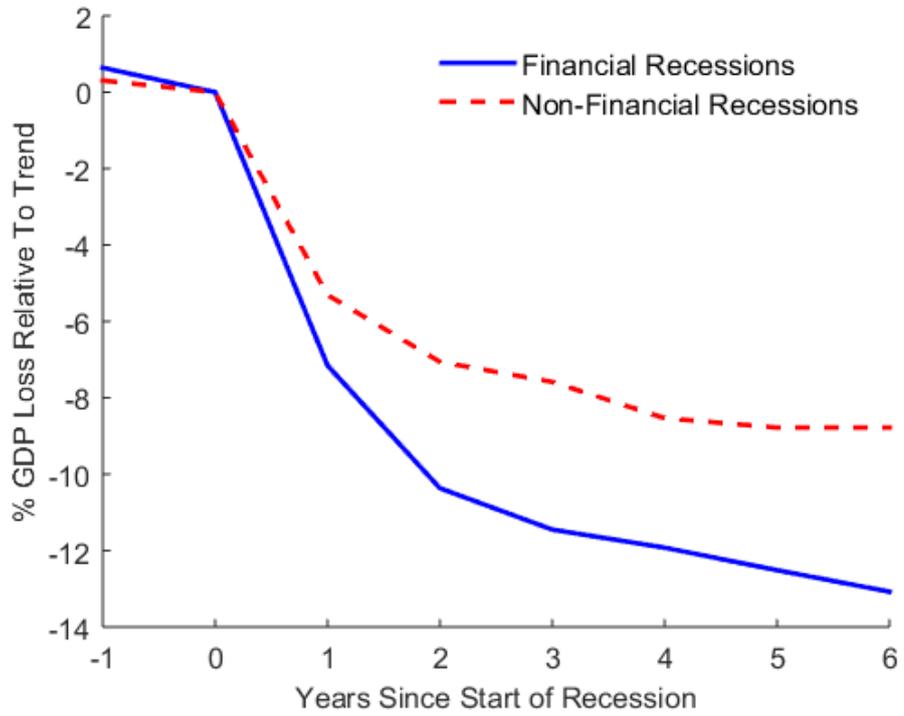
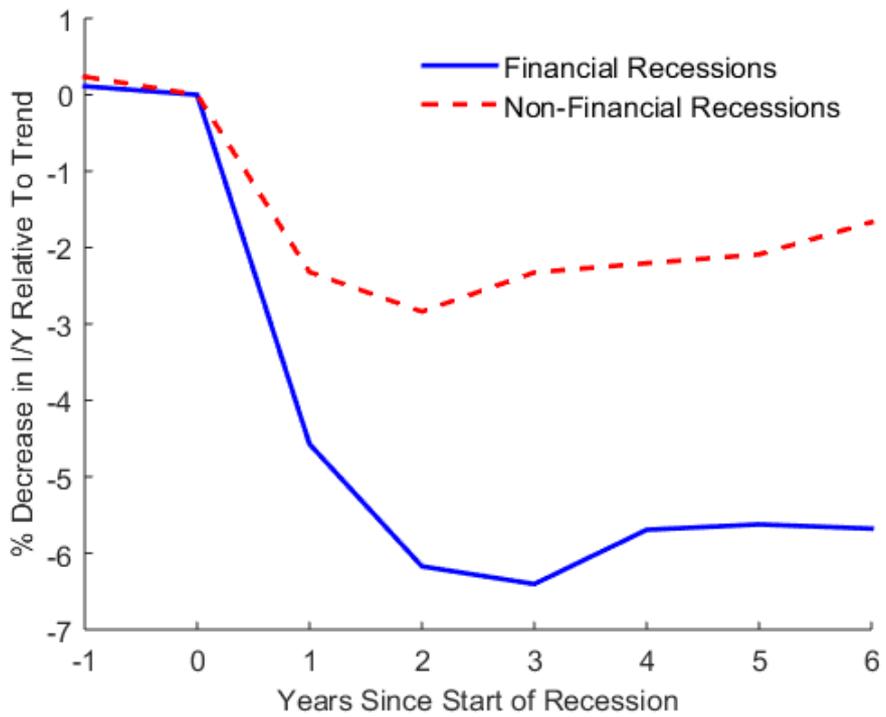


Figure 2: Investment/Output Decline After Financial vs. Non-Financial Recessions



lines increases noticeably over the next 5 years.

However, the difference between financial and non-financial recessions is starker for the behavior of investment. Figure 2 shows the behavior of the investment-output ratio $\frac{I}{Y}$ relative to the pre-recession trend for financial and non-financial recessions. The drop in $\frac{I}{Y}$ in Figure 2 is more than twice as large for financial recessions in the first year of the recession and the following year. By contrast, the drop in GDP in Figure 1 was only moderately larger for financial recessions in these years. Thus, these two figures suggest that financial recessions may be associated with somewhat larger GDP drops than non-financial recessions in the years immediately after the recession. However, they suggest, that financial recessions may see substantially larger drops in $\frac{I}{Y}$ in these years. This pattern of a somewhat larger drop in GDP but a substantially larger drop in $\frac{I}{Y}$ shortly after financial recessions survives if the same figures are plotted in a sample of only OECD countries; a sample of only emerging economies, or in a sample ending in 2006. It also applies if variables are detrended using a 2-30 year or 2-40 year band pass filter instead of a linear trend, or if the measure of financial crisis used is that of Laeven and Valencia (2012) instead of Reinhart and Rogoff.

This begs the question of whether the persistently poor performance of GDP after financial recessions may be related to the drop in investment during the recession. Some evidence suggesting that this may be the case is provided by Figures 3 and 4. These figures show the behavior of GDP and investment in ‘more persistent’ as compared to ‘less persistent’ financial recessions. I define a financial recession as ‘more persistent’ if the output loss relative to trend satisfies the following condition:

$$\min[YLoss_5; YLoss_6] > \max[YLoss_1; YLoss_2; YLoss_3]$$

where $YLoss_i$ denotes the loss in GDP, relative to the linear pre-recession trend, i years after the last pre-recession year. This rough measure of persistence is designed to reduce the risk that recessions are marked as more persistent simply because another recession rapidly follows, or because the recession starts late in the year.¹⁷ In the sample, approximately half of financial recessions are labelled as more persistent by this measure.

It is apparent from Figure 3 that the output loss from more persistent financial recessions is no worse than for less persistent financial recessions in the year of the recession. However, output continues to decline relative to trend for the more persistent financial recessions, whereas the less persistent ones show a gradual recovery. This is hardly surprising – more persistent financial recessions were defined as such because they show larger output losses relative to trend in later years. The pattern in Figure 4, however, does not follow automatically from the way persistence is defined. This figure shows the decrease in the investment-output ratio in the more persistent

¹⁷The patterns are substantively very similar if recessions are instead measured as persistent when $\text{mean}[YLoss_5; YLoss_6] > \text{mean}[YLoss_1; YLoss_2; YLoss_3]$.

Figure 3: GDP Decline After More vs. Less Persistent Financial Recessions

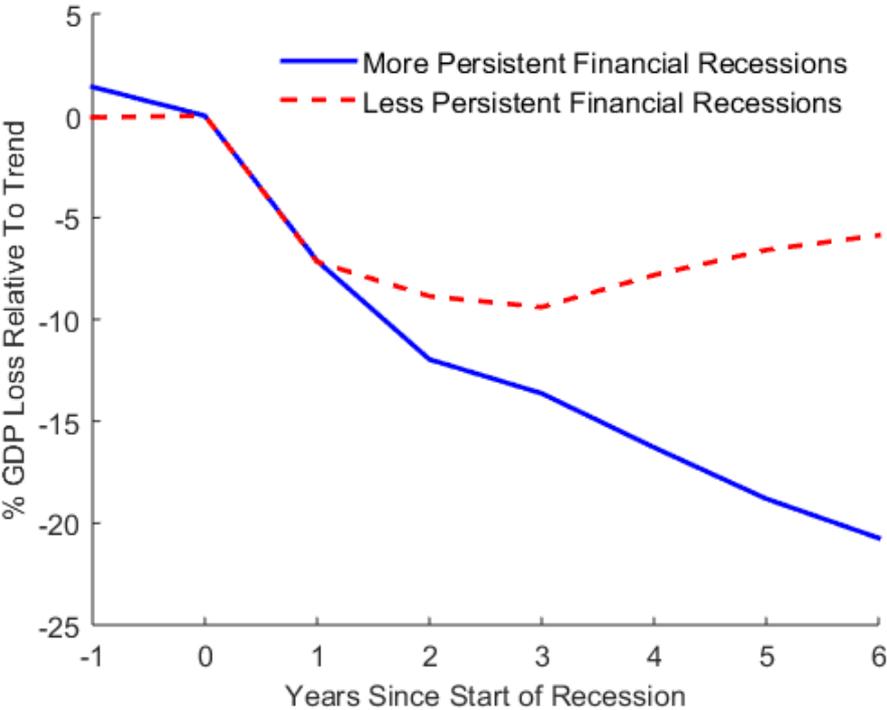
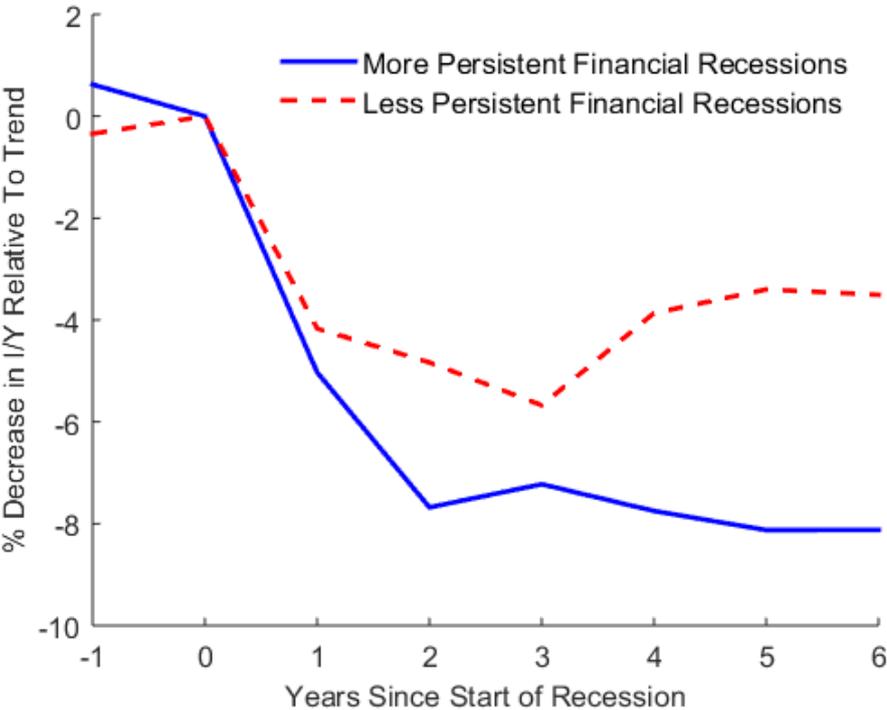


Figure 4: Investment/Output Decline After More vs. Less Persistent Financial Recessions



and less persistent financial recessions. It appears that, even in the first recession year and the subsequent year, more persistent financial recessions see bigger drops in the investment-output ratio than less persistent financial recessions. The difference between the two lines in Figure 4 is especially marked in the year after the first recession year.

This cross national evidence is suggestive that there may be a connection between the behavior of investment during financial recessions and the persistent output drops that follow these recessions.¹⁸ Unfortunately, without data on the types of investment and capital that see big decreases after a financial recession, it is impossible to infer whether there are particularly large decreases in the stocks of equipment and intangible capital in these recessions, a key part of the mechanism for persistent slumps in the model in this paper. Few countries have investment data disaggregated into different types (structures, equipment and intangibles) for a long time span, and many countries have very poor or non-existent measures of intangible capital even today. Fortunately, there is sufficient data to draw conclusions about the dynamics of different types of capital in the US, as structures and equipment have been differentiated in the US national accounts for decades. In Section 6.4, I turn to this data in order to demonstrate the explanatory power of the model developed in this paper, comparing the behavior of types of investment and capital in the US Great Recession with the movements predicted by the model.

4 The Model

The model is built around a DSGE framework with two types of capital: structures and intangibles, which differ only in their depreciation rates. ‘Intangibles’ is intended to proxy for equipment and intangible capital in the data, both of which have relatively high depreciation rates in contrast to structures. There are two kinds of friction in the model: financial frictions and labor market frictions. The financial friction is that production is undertaken by entrepreneurs who are subject to borrowing constraints. Borrowing constraints are modeled as in the canonical framework of Bernanke, Gertler and Gilchrist (1999). The labor market friction is that entrepreneurs wishing to employ workers must pay recruiting firms to post vacancies. Vacancies match with unemployed workers, as in the now standard framework of Mortensen and Pissarides (1994). Entrepreneurs and workers bargain over wages. The next section, 4.1, summarizes the agents in the model and their interactions. In the following sections I discuss the households and perfectly competitive firms first, before considering the financial frictions

¹⁸Such a relationship between the initial drop in investment and the persistence of the fall in output is not predicted by a standard RBC model in which persistent decreases of output are caused by persistent TFP shocks. The reason is that, in such a model, if agents expect TFP and output to be persistently low in the years after the financial crisis, this will lead them to particularly reduce their consumption at the time of the crisis rather than their investment. This contrasts with the data, where persistent drops in output are associated with larger drops in investment.

and wage bargaining.

4.1 Agents

Each household consists of a large family of workers and entrepreneurs who share consumption. In addition to workers and entrepreneurs, there are a large number of competitive banks, competitive capital-goods producing firms and competitive recruiting firms owned by households. Entrepreneurs own structures and intangibles, and employ workers. Entrepreneurs use their workers and capital to produce consumption goods. Capital-producing firms buy consumption goods, structures and intangible capital from entrepreneurs and transform them into new structures and intangibles, which they sell back to entrepreneurs. Recruiting firms carry out recruiting of workers on behalf of entrepreneurs. Banks exist in order to transfer funds from workers to entrepreneurs. Workers can hold risk-free deposits in banks and entrepreneurs can borrow from banks, in order to fund investment and hiring. In addition to workers, entrepreneurs, capital goods producers, recruiters and banks, there is a government, which taxes households and employs workers in order to produce public goods. The inclusion of the government sector allows for consideration of the effect of government employment shocks in the model but has little effect on the dynamic response to other shocks.

4.2 Preferences

All agents act to maximize the expected present discounted utility of their household. The preferences of the representative household are as follows.

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} (1 + \nu N_t + \nu N_{G,t})^\sigma + \frac{Y_{G,t}^{1-\sigma}}{1-\sigma} \right] \quad (1)$$

Here C_t is aggregate consumption, N_t is the measure of workers who are employed in the private sector, $N_{G,t}$ workers are employed in the government sector and $Y_{G,t}$ is total production of public goods by the government sector. It is assumed that the total measure of workers in the economy is equal to 1. Hence, at any time t , there are $1 - N_t - N_{G,t}$ unemployed workers. σ denotes the coefficient of risk aversion and ν determines the disutility of working.¹⁹

¹⁹Total household utility will take this form if the household allocates aggregate consumption C_t between its employed workers and unemployed workers in order to maximize the sum of its workers' utility, where an unemployed worker consuming $c_{u,t}$ gains utility $\frac{c_{u,t}^{1-\sigma}}{1-\sigma} + \frac{Y_{G,t}^{1-\sigma}}{1-\sigma}$ and an employed worker consuming $c_{e,t}$ gains utility equal to $\left(1 - \frac{\nu}{1+\nu}\right)^{-\sigma} \frac{c_{e,t}^{1-\sigma}}{1-\sigma} + \frac{Y_{G,t}^{1-\sigma}}{1-\sigma}$.

4.3 Shocks

I consider three distinct shocks in the model: a shock to the riskiness of investment by entrepreneurs, denoted by ς_t , a government employment shock, affecting $N_{G,t}$ and a shock to total factor productivity, denoted by Z_t . I assume that $\varsigma_t = \bar{\varsigma}e_{\varsigma,t}$, $N_{G,t} = \bar{N}_G e_{g,t}$ and $Z_t = \bar{Z}e_{z,t}$, where $e_{\varsigma,t}$, $e_{g,t}$ and $e_{z,t}$ follow AR(1) processes in logs:

$$\log(e_{\varsigma,t}) = \rho_{\varsigma} \log(e_{\varsigma,t-1}) + \epsilon_{\varsigma,t} \quad (2)$$

$$\log(e_{g,t}) = \rho_g \log(e_{g,t-1}) + \epsilon_{g,t} \quad (3)$$

$$\log(e_{z,t}) = \rho_z \log(e_{z,t-1}) + \epsilon_{z,t} \quad (4)$$

The values of all aggregate shocks are revealed at the start of the period. I assume ς_{t+1} is revealed one period in advance, at time t .

4.4 Capital Goods Producers

The consumption good is the numeraire in this economy. At the end of each period t , capital goods producers buy consumption goods, structures and intangible capital from entrepreneurs and transform these into new structures and intangible capital according to a constant returns technology. They sell these capital goods back to entrepreneurs immediately, who use them for production in the period $t + 1$.

The representative capital goods firm produces structures and intangibles according to the production functions:

$$K_{S,t+1} = (1 - \delta_S)K_{S,t} + I_{S,t} - \kappa_S \left(\frac{I_{S,t}}{I_{S,t-1}}, \frac{I_{S,t}}{K_{S,t}} \right) I_{S,t} \quad (5)$$

$$K_{I,t+1} = (1 - \delta_I)K_{I,t} + I_{I,t} - \kappa_I \left(\frac{I_{I,t}}{I_{I,t-1}}, \frac{I_{I,t}}{K_{I,t}} \right) I_{I,t} \quad (6)$$

where $I_{S,t}$ denotes the consumption goods used by the capital goods firm in the production of new structures and $I_{I,t}$ denotes the consumption goods used by the firm in the production of new intangibles. $K_{S,t}$ and $K_{I,t}$ denote the quantity of existing structures and intangibles, respectively, used by the firm in the production of new structures and intangibles. As will be seen shortly, $K_{S,t}$ and $K_{I,t}$ will be equal, in equilibrium, to the aggregate stock of structures and intangibles in the economy.

Here $\kappa_S(\cdot)$ and $\kappa_I(\cdot)$ are convex adjustment cost functions, given by:

$$\kappa_S \left(\frac{I_{S,t}}{I_{S,t-1}}, \frac{I_{S,t}}{K_{S,t}} \right) = \kappa_1 \left(\frac{I_{S,t}}{I_{S,t-1}} - 1 \right)^2 + \kappa_2 \left(\frac{I_{S,t}}{K_{S,t}} - \frac{I_S^*}{K_S^*} \right)^2 \frac{K_S^*}{I_S^*} \frac{K_{S,t}}{I_{S,t}} \quad (7)$$

$$\kappa_I \left(\frac{I_{I,t}}{I_{I,t-1}}, \frac{I_{I,t}}{K_{I,t}} \right) = \kappa_1 \left(\frac{I_{I,t}}{I_{I,t-1}} - 1 \right)^2 + \kappa_2 \left(\frac{I_{I,t}}{K_{I,t}} - \frac{I_I^*}{K_I^*} \right)^2 \frac{K_I^*}{I_I^*} \frac{K_{I,t}}{I_{I,t}} \quad (8)$$

Here variables with a star superscript denote the value of quantities in the steady state. Thus, there are adjustment costs to both the level of the capital stock and the level of investment.²⁰

Capital goods cannot be stored from one period to the next without the aid of the capital goods producing firms. It may be assumed that structures and intangibles capital units $K_{S,t}$ and $K_{I,t}$ are damaged in period t as a result of being used to produce consumption goods and are in need of repair. In order to be used in period $t + 1$, capital must be converted into new capital units $K_{S,t+1}$ and $K_{I,t+1}$ by a capital goods producer, through the production functions (5) and (6).

Consequently, at the end of period t , entrepreneurs willingly sell their entire stock of structures and intangibles to the capital goods producers, who use them to produce new capital units $K_{S,t+1}$, $K_{I,t+1}$, which they sell back to entrepreneurs, ready for use in the next period. Capital goods producers buy units of structures and intangibles from entrepreneurs at prices $q_{S,t}$ and $q_{I,t}$ and then sell structures and intangibles back to entrepreneurs at prices $\tilde{q}_{S,t}$ and $\tilde{q}_{I,t}$.²¹

Capital goods firms choose the quantity of new structures and intangibles they wish to produce in order to maximize the present discounted value of their profits, which they distribute to the households.

4.5 Government

The public sector produces public goods according to the production function:

$$Y_{G,t} = Z_G N_{G,t} \quad (9)$$

Since $Y_{G,t}$ enters into the household's utility an additively separable way in (1), the value of Z_G does not affect the decisions of any private agent.

²⁰These functional forms are chosen to coincide with capital and investment adjustment cost functional forms considered by Christiano and Davis (2006). In section 6.2, I examine the consequences of removing adjustment costs to the level of capital or to the level of investment for the model.

²¹The assumption that entrepreneurs need to sell capital to capital goods producers who transform it into new capital is technically convenient because it makes adjustment costs to aggregate capital external to the entrepreneur. Very similar assumptions are also made in the related frameworks of Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2014), for the same reason. Since entrepreneurs need the capital goods producers to transform capital from $K_{S,t}$ and $K_{I,t}$ into $K_{S,t+1}$ and $K_{I,t+1}$ in order to produce in the next period, entrepreneurs will be willing to sell their entire stock of capital to capital producers provided $\tilde{q}_{S,t}$ and $\tilde{q}_{I,t}$ are positive. Consequently, it is not in general the case that $q_{S,t} = \tilde{q}_{S,t}$ and $q_{I,t} = \tilde{q}_{I,t}$ in equilibrium.

The government pays public sector employees the wage $w_{G,t}$. It is assumed to set this equal to the average private sector wage w_t . The government funds wage payments to its employees using lump-sum taxes on households. Ricardian equivalence holds in the model, so it is without loss of generality to assume that the government balances its budget every period. That is, the lump sum tax τ_t is equal to total government spending:

$$\tau_t = w_t N_{G,t} \quad (10)$$

4.6 Labor Markets

Recall that workers are either employed or unemployed. Workers who are employed in the private sector are matched with entrepreneurs and separate with probability δ_N at the end of each period. Workers employed by the government also separate from jobs with probability δ_N . Unemployed workers search for jobs randomly and at the end of each period they find a job in the private sector with probability f_t and find a job in the public sector with probability $f_{G,t}$.

The probability of finding a job in the private sector f_t is given by the ratio of aggregate private hiring to the number of unemployed workers, that is:

$$f_t = \frac{N_{t+1} - (1 - \delta_N)N_t}{1 - N_t - N_{G,t}} \quad (11)$$

The probability of finding a job in the public sector is determined in a similar way:

$$f_{G,t} = \frac{N_{G,t+1} - (1 - \delta_N)N_{G,t}}{1 - N_t - N_{G,t}} \quad (12)$$

Entrepreneurs cannot hire workers directly. They must pay a recruiting firm, which posts vacancies, screens and trains workers and then passes these workers on to entrepreneurs. The recruiting firm charges each entrepreneur a price of h_t for use of this service.²² The government does not hire workers directly either: it also pays the recruiting firm the fee h_t to recruit workers on its behalf.

The costs of the representative recruiting firm consist of three parts. First of all, the recruiting firm must post vacancies in order to recruit workers at a fixed cost of h_1 per vacancy. Let v_t denote the number of private sector vacancies and $v_{G,t}$ the number of public sector vacancies posted by the representative recruiting firm in period t . Second, the recruiting firm must pay a convex cost of adjusting the number of vacancies it posts given by $\frac{h_2}{2} \left(\frac{v_t + v_{G,t}}{v_{t-1} + v_{G,t-1}} - 1 \right)^2 (v_t + v_{G,t})$.²³ Third,

²²This assumption can be interpreted as entrepreneurs outsourcing their recruiting and training activities to recruiting firms. That hiring is carried out by recruiting firms is technically convenient because it allows adjustment costs to the level of vacancies that are external to the entrepreneurs.

²³In the calibrated model, these adjustment costs to vacancies are quantitatively tiny. Less than 0.25% of the

once a vacancy has matched with a worker, the recruiting firm must pay an additional cost of h_0 units of output in order for the worker to be hired. These represent training costs.

Let the total number of matches in the labor market in period t be given by the Cobb-Douglas function $M_t = A_M^{1-\psi} (v_t)^{1-\psi} (1 - N_t - N_{G,t})^\psi$. This depends on the number of private and public sector vacancies and the number of unemployed workers, given by $1 - N_t - N_{G,t}$. Then the probability that a vacancy matches with a worker is given by $\left(\frac{A_M(v_t + v_{G,t})}{1 - N_t - N_{G,t}}\right)^{-\psi}$. It is assumed that a law of large numbers holds such that the recruiting firm posts exactly $\left(\frac{A_M(v_t + v_{G,t})}{1 - N_t}\right)^\psi$ vacancies for each worker it recruits.

Therefore, the period t profit made by the representative recruiting firm is equal to:

$$\begin{aligned} & (h_t - h_0)[N_{t+1} - (1 - \delta_N)N_t + N_{G,t+1} - (1 - \delta_N)N_{G,t}] \\ & - h_1(v_t + v_{G,t}) \left[1 + \frac{h_2}{h_1} \left(\frac{v_t + v_{G,t}}{v_{t-1} + v_{G,t-1}} - 1 \right)^2 \right] \end{aligned} \quad (13)$$

The total quantity of private sector vacancies posted is equal to:

$$v_t = [N_{t+1} - (1 - \delta_N)N_t] \left(\frac{A_M(v_t + v_{G,t})}{1 - N_t - N_{G,t}} \right)^\psi \quad (14)$$

and the total quantity of public sector vacancies is given by:

$$v_{G,t} = [N_{G,t+1} - (1 - \delta_N)N_{G,t}] \left(\frac{A_M(v_t + v_{G,t})}{1 - N_t - N_{G,t}} \right)^\psi \quad (15)$$

The representative recruiting firm chooses the number of vacancies it posts so as to maximize the present discounted value of its profits, which it returns to the household.

4.7 Entrepreneurs

An entrepreneur holds three types of physical asset in a period t : structures $k_{S,t}$, intangibles $k_{I,t}$, and workers n_t , which she uses to produce output in the form of consumption goods. Production occurs at the beginning of the period. At the end of each period, the entrepreneur sells some of her output to the household, for consumption purposes, and sells the rest of her output to the capital goods producing firms, who use it to produce new capital, and to recruiting firms, who use it to recruit workers. As explained in section 4.4, the entrepreneur's period t structures and

marginal costs of hiring an extra worker for the recruiting firm in each period are due to the marginal cost of adjusting the total number of vacancies, for realizations of the shocks of 5 or fewer standard deviations away from the mean. Consequently, I find that the model dynamics are hardly affected by variation in the level of these vacancy adjustment costs in the neighborhood of this small value. However, I find that very small adjustment costs to vacancy posting are necessary to prevent indeterminacy from occurring in the model at relevant values of the other parameters.

intangibles are damaged in the process of production and so she cannot use them to produce subsequently. Therefore, she willingly sells her structures and intangibles to capital producing firms at prices $q_{S,t}$ and $q_{I,t}$ after she has used them to produce in period t . The entrepreneur may also sell her workers to other entrepreneurs at price h_t if she so desires. Rather than entrepreneurs literally selling workers to one another, this can be viewed as entrepreneurs selling equity in their private enterprises to one another, thereby selling a fraction of their workers.

After selling her capital and output, the entrepreneur chooses the stock of capital and workers she wants to hold in order to produce in the next period. Denote her choices by $\tilde{k}_{S,t}, \tilde{k}_{I,t}, n_{t+1}$. The entrepreneur buys her capital from capital-producing firms, at prices $\tilde{q}_{S,t}, \tilde{q}_{I,t}$. She buys her workers from recruiting firms or other entrepreneurs at price h_t – recall that h_t was the fee charged by a recruiting firm to an entrepreneur for each worker hired.

After acquiring capital and workers at time t , an entrepreneur draws a shock ξ at the start of period $t+1$ which affects the value of her assets. Specifically, if an entrepreneur buys $\tilde{k}_{S,t}$ and $\tilde{k}_{I,t}$ units of structures and intangibles at the end of period t and draws shock ξ_{t+1} at the start of $t+1$, then the entrepreneur starts period $t+1$ with structures and intangibles given by:

$$\begin{aligned} k_{S,t+1} &= \xi_{t+1} \tilde{k}_{S,t} \\ k_{I,t+1} &= \xi_{t+1} \tilde{k}_{I,t} \end{aligned}$$

As in Bernanke, Gertler and Gilchrist (1999), I assume that ξ has a unit mean lognormal distribution that is independently drawn across time and across entrepreneurs.²⁴ Let ς_{t+1} denote the standard deviation of $\log(\xi)$ at time $t+1$, which varies over time in response to aggregate shocks as discussed in section 4.3 above.

After observing the shock ξ entrepreneurs use their workers and capital to produce output. In addition to affecting the entrepreneur's stock of capital, the shock ξ affects the entrepreneur's total factor productivity for one period. Indeed, an entrepreneur with assets $k_{S,t+1}, k_{I,t+1}$ and n_{t+1} that draws shock ξ_{t+1} produces output y_{t+1} according to:

$$y_{t+1} = Z_{t+1} k_{S,t+1}^{\alpha_S} k_{I,t+1}^{\alpha_I} (\xi_{t+1} n_{t+1})^{1-\alpha_I-\alpha_S} \quad (16)$$

$$\equiv \xi_{t+1} Z_{t+1} \tilde{k}_{S,t}^{\alpha_S} \tilde{k}_{I,t}^{\alpha_I} n_{t+1}^{1-\alpha_I-\alpha_S} \quad (17)$$

where the second line uses that $k_{S,t+1}$ and $k_{I,t+1}$ themselves depend on ξ_{t+1} . Here Z_{t+1} is the aggregate level of total factor productivity, which varies in response to shocks as discussed in section 4.3. Define $\tilde{y}_{t+1} = Z_{t+1} \tilde{k}_{S,t}^{\alpha_S} \tilde{k}_{I,t}^{\alpha_I} n_{t+1}^{1-\alpha_I-\alpha_S}$, so that we may write $y_{t+1} = \xi_{t+1} \tilde{y}_{t+1}$.

Simultaneously with production, the entrepreneur pays wages $w_{t+1}(\xi_{t+1})$, which in general

²⁴The shock ξ can be viewed as capturing the many risks that businesses face. For instance, businesses whose products become obsolete as a result of technological change may find that their capital has fallen substantially in value.

depend on ξ_{t+1} . These wages are set by bargaining, as discussed in section 4.13 below. After period $t + 1$ production, the entrepreneur sells her structures and intangibles to the capital producing firms at prices $q_{S,t}$, $q_{I,t}$. Furthermore, fraction δ_N of her workers exogenously separate from their jobs.

The net effect of this is that the entrepreneur who bought assets $\tilde{k}_{S,t}$, $\tilde{k}_{I,t}$ and n_{t+1} at time t earns a return on these assets R_{t+1}^K equal to:

$$R_{t+1}^K = \left(\frac{\xi_{t+1}\tilde{y}_{t+1} - w_{t+1}(\xi_{t+1})n_{t+1}}{\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}} \right) + \left(\frac{\xi_{t+1}(q_{S,t+1}\tilde{k}_{S,t} + q_{I,t+1}\tilde{k}_{I,t}) + h_{t+1}(1 - \delta_N)n_{t+1}}{\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}} \right)$$

It is convenient to assume now, and verify later on, that the wage paid by the entrepreneur takes the following form in equilibrium:

$$w_{t+1}(\xi_{t+1})n_{t+1} = w_{0,t+1}n_{t+1} + \vartheta(1 - \alpha_I - \alpha_S)\xi_{t+1}\tilde{y}_{t+1} \quad (18)$$

where $w_{0,t+1}$ is an aggregate variable unaffected by ξ_{t+1} or the entrepreneur's decisions, and ϑ is a parameter representing the bargaining weight of labor. Then, the entrepreneur's return can be written as:

$$R_{t+1}^K = R_{0,t+1}^K + \xi_{t+1}R_{1,t+1}^K$$

where

$$R_{0,t+1}^K = \frac{-w_{0,t+1}n_{t+1} + h_{t+1}(1 - \delta_N)n_{t+1}}{\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}} \quad (19)$$

$$R_{1,t+1}^K = \frac{[1 - \vartheta(1 - \alpha_I - \alpha_S)]\tilde{y}_{t+1} + q_{S,t+1}\tilde{k}_{S,t} + q_{I,t+1}\tilde{k}_{I,t}}{\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}} \quad (20)$$

Therefore, the entrepreneur's rate of return is homogeneous of degree zero with respect to the variables $(\tilde{k}_{S,t}, \tilde{k}_{I,t}, n_{t+1})$. As will become evident, the linearity of the entrepreneur's problem ensures that, in equilibrium, all entrepreneurs choose levels of capital and labor proportional to their net worth. Thus, the values of $R_{0,t+1}^K$ and $E_t[R_{1,t+1}^K]$ are invariant across entrepreneurs.

4.8 Financial Markets

Entrepreneurs may finance their purchases of capital and labor at the end of a period t by taking loans from the banks. The banks in turn raise funds by allowing workers to save in risk-free

deposits. Thus, the banks transfer funds from the workers to the entrepreneurs.

The loan obtained by an entrepreneur from a bank is assumed to take the form a standard debt contract. That is, the entrepreneur who borrows b_t at time t , must repay $b_{t+1} = (1 + i_{t+1})b_t$ after producing in period $t + 1$, if she has the resources to repay, and will default on her debt otherwise. The interest rate on the debt i_{t+1} is assumed to depend on the aggregate state at time $t + 1$ as discussed below.²⁵

Entrepreneurs who draw very low ξ_{t+1} may find themselves unable to afford to repay their debt. Let $\bar{\xi}_{t+1}$ denote the cutoff value, such that the entrepreneur will be able to repay her debt if and only if she draws $\xi_{t+1} \geq \bar{\xi}_{t+1}$. Then, in particular it follows that:²⁶

$$(R_{0,t+1}^K + R_{1,t+1}^K \bar{\xi}_{t+1})(\tilde{q}_{S,t} \tilde{k}_{S,t} + \tilde{q}_{I,t} \tilde{k}_{I,t} + h_t n_{t+1}) = b_t(1 + i_{t+1}) \quad (21)$$

Given $\bar{\xi}_{t+1}$ this equation determines i_{t+1} in each state $t + 1$.

If the entrepreneur draws $\xi_{t+1} < \bar{\xi}_{t+1}$, she declares bankruptcy, in which case the bank confiscates all her assets and sells them to other entrepreneurs. However, the bank must also pay a monitoring cost in this case, in order to verify that the entrepreneur truly cannot repay her debt. The monitoring cost is assumed to equal fraction μ of the value of the entrepreneur's assets and output, that is $\mu[R_{0,t+1}^K + \xi_{t+1} R_{1,t+1}^K](\tilde{q}_{S,t} \tilde{k}_{S,t} + \tilde{q}_{I,t} \tilde{k}_{I,t} + h_t n_{t+1})$.²⁷

It is assumed that each bank must pay a risk-free return on all deposits it issues and cannot borrow through any other means. Consequently, each bank must make non-negative profits in every period, in every state of the world. Furthermore, free entry and perfect competition among banks imply that banks cannot make positive expected profits on any loan. Then, an equilibrium will exist in which banks make strictly zero profits on every loan in expectation in every aggregate state.²⁸ That is, the following break-even condition for the bank will hold on

²⁵The details of the debt contract are imposed exogenously here. However, that the contract between the entrepreneur and the bank should take the form of a debt contract can be justified on grounds of costly state verification. See footnote 27.

²⁶More precisely, the equation follows only if $\bar{\xi}_{t+1} \in (0, \infty)$. However, this will always be the case in equilibrium.

²⁷As discussed in Bernanke, Gertler and Gilchrist (1999), the standard debt contract assumed here can be viewed as an incentive compatible solution to a costly state verification problem, in which the entrepreneur privately observes her realization of ξ , and the bank must pay the monitoring cost in order to observe it. The debt contract is optimal given the constraint that the bank make non-negative expected profits on every loan in every aggregate state $t + 1$.

²⁸I will assume here that the equilibrium takes this form. Given the restrictions imposed on banks due the need to issue risk-free deposits and free entry, the only potential deviation from this equilibrium to consider would be a bank that makes positive expected profits on some aggregate states, and negative expected profits on other loans in those states, with zero expected profits on each loan overall. However, such a deviation would increase the proportion of entrepreneurs that default, by making the required repayment from entrepreneurs more stochastic, and so would increase monitoring costs with no benefits, which cannot be profitable.

each loan in each aggregate state $t + 1$:

$$\begin{aligned}
b_t(1 + r_t) &= [1 - F_t(\bar{\xi}_{t+1})](1 + i_{t+1})b_t \\
&\quad + (1 - \mu)(\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}) \int_0^{\bar{\xi}_{t+1}} R_{0,t+1}^K + \xi R_{1,t+1}^K dF_t(\xi)
\end{aligned} \tag{22}$$

Here r_t is the risk free rate paid by the bank to its depositors, which is predetermined at time t and is identical across banks in a competitive equilibrium.

4.9 Evolution of Net Worth

At this point, it may be convenient to review the timing of events experienced by the entrepreneur. At the end of each period t , the entrepreneur sells her existing stock of capital to capital-producing firms and may sell some of her workers to other entrepreneurs. She then buys capital units $\tilde{k}_{S,t}, \tilde{k}_{I,t}$ from capital producing firms and buys workers from other entrepreneurs or recruiting firms. She may choose to finance some of her expenditure on capital and workers by borrowing from a bank. At the start of period $t + 1$, she draws the idiosyncratic shock ξ_{t+1} , affecting the value of her stock of capital and her output. She then produces output and pays wages. After production, she either defaults or does not depending on whether or not she drew $\xi_{t+1} < \bar{\xi}_{t+1}$.

Let x_t denote the value of the entrepreneur's net worth at time t , just before she purchases new capital and hires new workers (by paying a recruiting firm to recruit them). Then her expected period $t + 1$ net worth will be given by:

$$E_t[x_{t+1}] = E_t \left[\int_{\bar{\xi}_{t+1}}^{\infty} (R_{0,t+1}^K + \xi R_{1,t+1}^K)(\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}) - b_t(1 + i_t) dF(\xi, \varsigma_t) \right] \tag{23}$$

$$= (\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}) E_t \left[R_{1,t+1}^K \int_{\bar{\xi}_{t+1}}^{\infty} (\xi - \bar{\xi}_{t+1}) dF(\xi, \varsigma_t) \right] \tag{24}$$

$$= x_t L_t E_t [R_{1,t+1}^K (1 - \Gamma_t(\bar{\xi}_{t+1}))] \tag{25}$$

where L_t and Γ_t are defined by:

$$L_t = \frac{\tilde{q}_{S,t}\tilde{k}_{S,t} + \tilde{q}_{I,t}\tilde{k}_{I,t} + h_t n_{t+1}}{x_t} \tag{26}$$

$$\Gamma_t(\bar{\xi}_{t+1}) = [1 - F_t(\bar{\xi}_{t+1})]\bar{\xi}_{t+1} + G_t(\bar{\xi}_{t+1}) \tag{27}$$

and

$$G_t(\bar{\xi}_{t+1}) = \int_0^{\bar{\xi}_{t+1}} \xi dF_t(\xi) \quad (28)$$

That is, L_t denotes the leverage ratio and $[1 - \Gamma_t(\bar{\xi}_{t+1})]R_{1,t+1}^K$ denotes the fraction of the total return R_{t+1}^K retained by the entrepreneur in expectation. Note that (21) was used in the derivation of (24).

After entrepreneurs have produced, they sell assets to repay their debts, as well as taking on new debts in order to produce once more. Furthermore, after production and default have taken place, fraction χ of entrepreneurs' net worth is exogenously transferred to workers. In addition, fraction $1 - \chi$ of workers' wealth is exogenously transferred to entrepreneurs. This is a modeling device to prevent entrepreneurs from saving enough to outgrow their financial constraints, a common concern in the literature, as discussed by Quadrini (2011). It can be interpreted variously as representing entry and exit, or that firms are not free to vary their payments of dividends.²⁹

Before writing down the entrepreneur's optimization problem, it is convenient to simplify the bank's break even constraint (22). Using (21) to substitute for i_{t+1} in (22) and then using (26), (27) and (28) to substitute for the integral and for b_t , we obtain:

$$[1 - \mu F_t(\bar{\xi}_{t+1})]R_{0,t+1}^K + [\Gamma_t(\bar{\xi}_{t+1}) - \mu G_t(\bar{\xi}_{t+1})]R_{1,t+1}^K = \frac{(L_t - 1)(1 + r_t)}{L_t} \quad (29)$$

4.10 Entrepreneur's Optimization Problem

The entrepreneur wishes to maximize the present value of her household's consumption. This amounts in her case to maximizing the expected present value of the wealth that flows from her to the workers. The entrepreneur's expected net worth after production at time $t + 1$ is given by (25), of which fraction χ flows to the household.

Let $V(x_t, s_t)$ denote the continuation value of an entrepreneur at time t who has net worth x_t , after production and default have taken place. s_t denotes the aggregate state. At this time, the entrepreneur must choose her purchases of structures, intangibles and workers, these choices imply choices of leverage L_t , which, in combination with the bank's break-even condition imply

²⁹One interpretation of this transfer is that the workers' wealth is held by inactive entrepreneurs. Each period fraction $1 - \chi$ of active entrepreneurs remain active, with the rest becoming inactive. In turn, fraction $1 - \chi$ inactive entrepreneurs become active, with the rest remaining inactive. The scales of transfers to and from active entrepreneurs that this implies are particularly large by the standards of the literature. I find that such rapid transfers are necessary in order to prevent entrepreneurs from rapidly outgrowing financing constraints after a negative financial shock, due to the collapse in interest rates induced by the shock. A more elaborate model would provide developed microfoundations for why firms do not outgrow financing constraints so rapidly in practice; this may be related to the fact that the risk-free rate is much less volatile than implied by the model in this paper.

choices of $\bar{\xi}_{t+1}$ for each state s_{t+1} . Then, the Bellman equation for the entrepreneur is:

$$V(x_t, s_t) = \max_{\tilde{k}_{S,t}, \tilde{k}_{I,t}, n_{t+1}, L_t, \{\bar{\xi}_{t+1}(s_{t+1})\}} E_t[\chi m_{t+1} E[x_{t+1}] + (1 - \chi)V(x_{t+1}, s_{t+1})] \quad (30)$$

where the maximization is subject to the conditions that $E[x_{t+1}]$ evolves in accordance with (25), L_t satisfies the definition (26), $R_{0,t}^K$ and $R_{1,t}^K$ satisfy their definitions (19) and (20), and, for each aggregate state s_{t+1} , $\bar{\xi}_{t+1}(s_{t+1})$ satisfies the bank's break even condition (29). Here m_{t+1} denotes the household's stochastic discount factor, given by:

$$m_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)} \quad (31)$$

where $u'(C_t)$ is the derivative of the household's utility function, given in (1), with respect to C_t .

It is easy to verify that the linearity of the entrepreneur's problem ensures that entrepreneurs will choose $\tilde{k}_{S,t}$, $\tilde{k}_{I,t}$ and n_{t+1} proportional to x_t , and therefore that L_t and $\bar{\xi}_{t+1}$ will be invariant across entrepreneurs.³⁰

4.11 Implications for Aggregates

In a slight abuse of notation, let $\tilde{k}_{S,t}(x)$ denote the total stock of structures chosen by an entrepreneur with net worth x at the end of period t . Define $\tilde{k}_{I,t}(x)$, $n_{t+1}(x)$ analogously. Since all entrepreneurs' demands for capital and labor are linear in their individual net worth, the aggregate demands for capital and labor among entrepreneurs are given by:

$$K_{S,t+1} = X_t \tilde{k}_{S,t}(1) \equiv \tilde{k}_{S,t}(X_t) \quad (32)$$

$$K_{I,t+1} = X_t \tilde{k}_{I,t}(1) \equiv \tilde{k}_{I,t}(X_t) \quad (33)$$

$$N_{t+1} = X_t n_{t+1}(1) \equiv n_{t+1}(X_t) \quad (34)$$

Here X_t denotes the aggregate net worth of entrepreneurs. In deriving these expressions, I made use of the fact that the mean of ξ is 1, and that a law of large numbers holds so that the aggregate stock of capital evolves as if all entrepreneurs draw the shock $\xi = 1$.

³⁰To see this, note that it must be the case that $V(x, s_t) \geq xV(1, s_t)$ because an entrepreneur with net worth x can obtain $xV(1, s_t)$ simply by choosing x times as much capital and workers each period as an entrepreneur who has net worth 1 at time t . By an analogous argument, we have that $V(1, s_t) \geq \frac{1}{x}V(x, s_t)$. Then, it must be that an entrepreneur with net worth x can do no better, and no worse, than choosing x times as much capital and workers as an entrepreneur with net worth 1. The strict concavity of the entrepreneur's problem then implies that this must be the unique optimal decision.

Leverage of all entrepreneurs can therefore be obtained by aggregating equation (26):

$$L_t = \frac{\tilde{q}_{S,t}K_{S,t+1} + \tilde{q}_{I,t}K_{S,t} + h_t N_{t+1}}{X_t} \quad (35)$$

Entrepreneurs sell C_t units of output to households, which constitutes aggregate consumption, and sell $I_{S,t} + I_{I,t}$ units to capital goods producers, which constitutes aggregate investment. The total of these must equal the total output produced by entrepreneurs, net of monitoring and hiring costs:

$$C_t + I_{S,t} + I_{I,t} = Z_t K_{S,t}^{\alpha_S} K_{I,t}^{\alpha_I} N_t^{1-\alpha_I-\alpha_S} - h_t(N_{t+1} - (1 - \delta_N)N_t) - \mu(\tilde{q}_{S,t-1}K_{S,t} + \tilde{q}_{I,t-1}K_{I,t} + h_{t-1}N_t)[F_{t-1}(\bar{\xi}_t)R_{0,t}^K + G_{t-1}(\bar{\xi}_t)R_{1,t}^K] \quad (36)$$

Aggregate net worth at time $t + 1$ will equal fraction $1 - \chi$ of the wealth earned by entrepreneurs, plus fraction $1 - \chi$ of the savings of workers. Aggregating equation (25), we have that

$$X_{t+1} = (1 - \chi)X_t L_t R_{1,t+1}^K (1 - \Gamma_t(\bar{\xi}_{t+1})) + (1 - \chi)(1 + r_t)(L_t - 1)X_t \quad (37)$$

where the second right hand side term reflects that the savings of workers are equal to bank lending to entrepreneurs, given by $(L_t - 1)X_t$, and worker savings earn interest rate r_t . Using (19), (20), (35) and the bank break-even condition (29) to substitute for r_t , L_t , $R_{0,t}^K$ and $R_{1,t}^K$, this can be rewritten as:

$$X_{t+1} = (1 - \chi)\{C_{t+1} + I_{S,t+1} + I_{I,t+1} - w_{t+1}N_{t+1} + q_{S,t+1}K_{S,t+1} + q_{I,t+1}K_{I,t+1} + h_{t+1}N_{t+2}\}$$

That is, entrepreneurial net worth is always equal to fraction $1 - \chi$ of profits gross of hiring costs, plus the total value of the stock of capital and next period labor. Here, in an abuse of notation, I use w_t to refer to $E_\xi w_t(\xi)$.

Since the total leverage L_t and default cutoff $\bar{\xi}_{t+1}$ are the same for all entrepreneurs, it follows from (21) and (26) that all entrepreneurs pay the same interest rate on their debt, given by:

$$(R_{0,t+1}^K + R_{1,t+1}^K \bar{\xi}_{t+1}) = \frac{L_t - 1}{L_t} (1 + i_{t+1}) \quad (38)$$

The gap between the expected borrowing interest rate $E_t[i_{t+1}]$ and the rate r_t that workers save at is of particular significance. I shall refer to it as the credit spread in the model. Thus, the spread is defined as:

$$1 + \text{Spread}_t = \frac{E_t[1 + i_{t+1}]}{1 + r_t}$$

4.12 Worker Optimization and Value Functions

Workers can save in risk-free deposits in banks, earning interest rate r_t . The household may use its workers' savings to consume. Therefore, household consumption will satisfy the following Euler equation:

$$u'(C_t) = \beta(1 + r_t)u'(C_{t+1}) \quad (39)$$

The wage earned by a privately employed worker each period will depend on the productivity shock ξ of the entrepreneur he is matched with.³¹ Let $\mathcal{W}_t(\xi_t)$ represent the continuation value of a privately employed worker *at the beginning of period t*, conditional on being matched with an entrepreneur who has drawn shock ξ_t .

Then $\mathcal{W}(\xi_t, s_t)$ satisfies the following Bellman equation:

$$\mathcal{W}_t(\xi_t) = w_t(\xi_t) + (1 - \delta_N)E_t[m_{t+1}\mathcal{W}_{t+1}(\xi_{t+1})] + \delta_N E_t[m_{t+1}\mathcal{U}_{t+1}] \quad (40)$$

That is, the worker's continuation value depends on his wage and the probability of him remaining employed in the next period in the usual way. \mathcal{U}_t is the continuation value of an unemployed worker at the start of t .

The continuation value of a worker employed in the public sector is similarly given by:

$$\mathcal{W}_{G,t} = w_{G,t} + (1 - \delta_N)E_t[m_{t+1}\mathcal{W}_{G,t+1}] + \delta_N E_t[m_{t+1}\mathcal{U}_{t+1}] \quad (41)$$

where $w_{G,t}$ is the public sector wage at time t .

The value of an unemployed worker \mathcal{U}_t satisfies:

$$\mathcal{U}_t = \bar{u}_t + f_t E_t[m_{t+1}\mathcal{W}_{t+1}(\xi_{t+1})] + (1 - f_t - f_{G,t})E_t[m_{t+1}\mathcal{U}_{t+1}] + f_{G,t}E_t[m_{t+1}\mathcal{W}_{G,t+1}] \quad (42)$$

where the flow value of unemployment, \bar{u}_t is given by:

$$\bar{u}_t = \frac{-\left(\frac{\partial u}{\partial N_t}\right)}{\left(\frac{\partial u}{\partial C_t}\right)} = \frac{\nu\sigma C_t}{(\sigma - 1)(1 + \nu N_t + \nu N_{G,t})} \quad (43)$$

The flow value of being unemployed in (43) represents the household's marginal disutility of an extra person working, evaluated in units of consumption. Hence, it equals $\frac{-\left(\frac{\partial u}{\partial N_t}\right)}{\left(\frac{\partial u}{\partial C_t}\right)}$. Using the utility function in equation (1), this is equal to the right hand side of (43). Note that there are

³¹Since all entrepreneurs are identical and make identical decisions in proportion to their net worth, wages across entrepreneurs will only vary due to variation in ξ .

no unemployment benefits.³²

4.13 Wage Bargaining

Wages are set immediately before production takes place, at the beginning of the period, but after aggregate and idiosyncratic shocks have been revealed. Wages are set by bargaining, according to a protocol in which the entrepreneur and worker alternate in making wage offers to one another, as in the closely related frameworks of Hall and Milgrom (2008) and Christiano, Eichenbaum and Trabandt (2013). The details of the bargaining procedure and the derivation of the bargaining solution are slightly complicated and so are left to the appendix. Here, I describe them only briefly.

Bargaining occurs in continuous time within the period – a period is divided into measure 1 of subperiods over which production and bargaining may take place.³³ The entrepreneur gets the opportunity to make an offer immediately at the beginning of the period. If the worker rejects this offer then agents must spend time developing new wage offers to propose. Opportunities for the worker to propose a wage offer to the entrepreneur arrive at a Poisson rate proportional to ϑ throughout the period and opportunities for the entrepreneur to propose a wage offer to the worker arrive at a Poisson rate proportional to $1 - \vartheta$. No production takes place while agents bargain and so bargaining is a costly activity. Bargaining ends and production begins as soon as an offer is accepted. While agents bargain, the bargaining process may completely break down, an event which arrives at Poisson rate ρ . In this case the match between the entrepreneur and worker is terminated, and the worker becomes unemployed. While bargaining, entrepreneurs must pay a flow cost γ to keep producing new wage offers. Workers do not face the disutility of working \bar{u}_t while bargaining. In equilibrium, the entrepreneur's initial wage offer at the beginning of the period is immediately accepted and production begins immediately.

This bargaining procedure leads to an equilibrium wage that satisfies:

$$(1 - e^{-\rho}) \left[\frac{h_t + \pi_t(\xi) - w_t(\xi)}{1 - \vartheta} - \frac{\mathcal{W}_t(\xi) - \mathcal{U}_t}{\vartheta} \right] + e^{-\rho} \left[\frac{\gamma + \pi_t(\xi) - w_t(\xi)}{1 - \vartheta} - \frac{w_t(\xi) - \bar{u}_t}{\vartheta} \right] = 0 \quad (44)$$

where $\pi_t(\xi) = \frac{(1-\alpha_I-\alpha_S)y_t(\xi)}{n_t}$ is the marginal revenue product of the worker. The derivation of

³²The absence of unemployment benefits is consistent with the findings of Chodorow-Reich and Karabarbounis (2013) that effective benefits represent a very small fraction of the wage, once eligibility and cost of takeup are considered.

³³The main contrast between the alternating offer bargaining procedure assumed here and the approach of Christiano, Eichenbaum and Trabandt (2013) is that Christiano et. al. (and Hall and Milgrom (2008)) assume that the worker and employer take turns to make an offer – first the employer makes an offer, then the worker, then the employer and so on. I instead assume that the entrepreneur and worker make offers at a Poisson rate. The advantage of this is that it allows the entrepreneur and worker to have different bargaining strengths, depending on the speed at which they can make wage offers. This yields a solution that nests the standard Nash Bargaining solution used by, e.g. Mortensen and Pissarides (1994), as a special case.

(44) is described in the appendix.

The term $h_t + \pi_t(\xi) - w_t(\xi)$ is the surplus the entrepreneur gains from being matched with a worker, relative to the alternative in which the worker and entrepreneur separate. Similarly $\mathcal{W}_t(\xi) - \mathcal{U}_t$ is the match surplus of the worker relative to separating. In the limit as $\rho \rightarrow \infty$ the second square bracketed term in (44) approaches zero. Therefore, the bargained wage that solves (44) is the one in which the worker's match surplus is proportional to the entrepreneur's, as in the traditional Nash Bargain used by the search and matching literature, including Mortensen and Pissarides (1994). The bargaining weight of the worker is then given by ϑ .³⁴

On the other hand, as $\rho \rightarrow 0$ the alternating-offer wage simplifies to:

$$w_t(\xi) = (1 - \vartheta)\bar{u}_t + \vartheta(\gamma + \pi_t(\xi))$$

In that case, the bargained wage does not depend on the continuation value of the unemployed worker, and so does not depend on the job finding rate f_t . This leads to a wage that is much more insensitive to cyclical fluctuations – that is, wages are endogenously more rigid. It may be verified that the alternating offer bargaining solution 44 implies that the wage satisfies (18) for any value of ρ , provided $w_{0,t}$ is defined accordingly.

4.14 Equilibrium Prices

The capital goods producer chooses how many units of structures, $K_{S,t}$, and how many units of output, $I_{S,t}$, to use in the production of new structures, according to the production function in (5). Likewise the capital goods producer chooses how many units of intangibles, $K_{I,t}$, and output, $I_{I,t}$, to use to produce new intangibles, according to the production function in 6).

Maximization of present discounted profits by the capital goods producer yields the following two first-order conditions for $K_{j,t}$ and $I_{j,t}$, for $j \in \{S, I\}$.

$$1 = \tilde{q}_{j,t} \left[1 - \frac{\kappa_1}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) I_t - \frac{\kappa_1}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \frac{\kappa_2}{K_t} \left(\frac{I_t}{K_t} - \frac{I^*}{K^*} \right) \frac{K^*}{I^*} K_t \right] \quad (45)$$

$$q_t = \tilde{q}_{j,t} \left[1 - \delta_j - \kappa_2 \left(\frac{I_t}{K_t} - \frac{I^*}{K^*} \right) \frac{K^*}{I^*} K_t \left(-\frac{I_t}{K_t^2} \right) - \frac{\kappa_2}{2} \left(\frac{I_t}{K_t} - \frac{I^*}{K^*} \right)^2 \frac{K^*}{I^*} \right] \quad (46)$$

These determine the equilibrium prices $q_{S,t}$, $\tilde{q}_{S,t}$, $q_{I,t}$ and $\tilde{q}_{I,t}$. In the steady state, these conditions imply that $q_{S,t} = 1 - \delta_S$; $q_{I,t} = 1 - \delta_I$; $\tilde{q}_{S,t} = 1$, and $\tilde{q}_{I,t} = 1$.

³⁴In a number of recent contributions to the literature, such as Monacelli, Quadrini and Trigari (2011), the possibility that an entrepreneur may default affects wage bargaining. This mechanism is ruled out here, because whether an entrepreneur defaults is perfectly predictable from the shock ξ , which is known before wage bargaining takes place. Moreover, since the number of workers per entrepreneur is large, each worker acts in the knowledge that his own bargaining behavior will matter too little to affect whether or not the entrepreneur defaults.

The recruiting firm chooses the number of vacancies it posts in order to maximize its present discounted profits. This yields the first order condition:

$$h_t = h_0 + h_1 c_{V,t} \left(\frac{A_M(v_t + v_{G,t})}{1 - N_t - N_{G,t}} \right)^\psi \quad (47)$$

where $c_{V,t}$ is the marginal cost (in terms of present discounted profit) to the recruiting firm per vacancy posted, given by:

$$c_{V,t} = h_1 + h_2 \left(\frac{v_t}{v_{t-1}} - 1 \right) \left(\frac{1}{v_{t-1}} \right) - E_t \left[m_{t+1} c_{V,t+1} h_2 \left(\frac{v_{t+1}}{v_t} - 1 \right) \left(\frac{v_{t+1}}{v_t^2} \right) \right] \quad (48)$$

Equation (47) determines h_t in equilibrium.

4.15 Definition of Equilibrium

At any time t , the aggregate state of the economy can be described by the variables $K_{S,t}, K_{I,t}, N_t, N_{G,t}, Z_t, \varsigma_t$. An equilibrium of the model consists of: a law of motion of these aggregate variables, depending on the realization of the shocks $\epsilon_\varsigma, \epsilon_z, \epsilon_G$; a decision rule for the households determining consumption C_t as a function of the aggregate state; decision rules for the entrepreneur determining capital and labor demand $\tilde{k}_{S,t+1}, \tilde{k}_{I,t+1}, n_{t+1}$, default cutoff $\bar{\omega}_{t+1}$ and leverage L_t as functions of the aggregate state and the state of the entrepreneur (ξ, x) ; a function describing wages w_t paid by an entrepreneur as functions of the aggregate state and the state of the entrepreneur; value functions $V(\cdot)$ and $\mathscr{W}(\cdot)$ describing the entrepreneur and privately employed workers' continuation values as functions of the aggregate state and state of the entrepreneur; value functions $\mathscr{W}_G(\cdot)$ and $\mathscr{U}(\cdot)$ describing the continuation values of the publically employed worker and unemployed worker as functions of the aggregate state, and functions of the aggregate state determining the variables $m_t, I_{S,t}, I_{I,t}, w_{G,t}, r_t, X_t, f_t, f_{G,t}, v_t, v_{G,t}, Y_{G,t}, \tau_t, R_{0,t}^K, R_{1,t}^K, q_{S,t}, q_{I,t}, \tilde{q}_{S,t}, \tilde{q}_{I,t}, h_t, \bar{u}_t$.

The functions describing these variables satisfy the following conditions:

- The stochastic shocks evolve according to equations (2)-(4).
- Capital and labor demands and total consumption and investment are the aggregate of individual entrepreneur decisions, according to (32)-(34) and (36).
- The stocks of structures and intangibles evolve according to the laws of motion (5) and (6).
- The entrepreneur's value function and decisions jointly solve the entrepreneur's optimization problem (30).

- Wages solve the alternating offers bargaining solution (44).
- The worker's value functions $\mathcal{W}(\cdot)$, $\mathcal{W}_G(\cdot)$ and $\mathcal{U}(\cdot)$ solve the Bellman equations (40)-(42).
- The flow value of unemployment u_t satisfies (43).
- $Y_{G,t}$ satisfies the public goods production function (9), public sector wages satisfy $w_{G,t} = w_t$, and τ_t satisfies the government's budget balance (10).
- Consumption satisfies the Euler equation (39) and the stochastic discount factor m_t is given by (31).
- Entrepreneurial net worth is given by (37).
- Job finding rates and vacancies satisfy the accounting identities (11), (12), (14) and (15).
- For each entrepreneur the returns $R_{0,t}^K$ and $R_{1,t}^K$ are given by (19) and (20).
- Capital producing firms choose their levels of capital and output to use to produce new capital, in order to maximize profits and taking prices as given. This implies that prices must be consistent with the first order conditions (45) and (46).
- Recruiting firms choose the number of vacancies to post in order to maximize profits, taking prices as given. This implies that h_t must be consistent with the first order condition (47).

The prices that satisfy these conditions immediately clear capital, financial, goods and labor markets.

5 Model Calibration

Table 1 shows the moments used and the main parameter values chosen for the baseline model. The parameters governing the wage bargaining process are shown in Table 2 and the parameters governing the shock processes are shown in Table 3. I set the time period of the model to be one quarter. Most of the model parameters are calibrated so that the steady state of the model matches various data moments.

Particularly important are the factor shares and depreciation rates for intangibles and structures, α_I , α_S , δ_I , δ_S . I calibrate these so that the steady state of the model produces empirically reasonable values for the stock of and investment in structures, equipment and intangibles, and for the labor share. I take the average investment and stock of business structures and equipment from the NIPA and fixed asset tables, averaged over the period 1995-2007. I infer the level of investment in intangible capital and its stock using Corrado and Hulten's (2010) estimate that, over the 1995-2007 period, intangible investment accounted for 55% of total business

investment and intangible capital accounted for 34% of total business capital.³⁵ I assume that private output is equal to the level of private consumption plus business investment in structures, equipment and intangibles and calculate the implied ratios of investment/private output ratio and capital/private output ratio for equipment, structures and intangibles. Finally, using the NIPA measure of total worker compensation averaged over the 1995-2007 period, I also calculate private labor compensation as a share of private output.

I calibrate the values of β , α_I , α_S , δ_I and δ_S so that the steady state of the model matches the following five moments from these calculations: private total capital/private output; private equipment plus intangible investment/private output; private structures investment/private output, and private labor compensation/private output. The values of the moments used and the corresponding parameter values are shown in Table 1. The implied value of α_I is rather high: 0.38. This arises for three reasons. Firstly, the inclusion of intangible investment increases the value of output above the standard measure. This significantly depresses the fraction of output accruing to labor to 0.47, implying a higher α_S and α_I . Furthermore, wage bargaining implies that some of the returns to an entrepreneur's investment will transfer to her workers, who will be able to bargain for a higher wage if their employer invests more.³⁶ This tends to distort the level of profits below the social return to capital, implying that a higher level of α_I and α_S is required to match the labor share in the data. Thirdly, the share of intangible capital α_I is more than twice as large as the share of structures, α_S , in spite of the fact that the stock of structures and stock of intangibles are relatively similar in the steady state. The reason for this is that the high depreciation rate of intangibles requires that this type of capital receive a high gross rate of return in equilibrium, since otherwise entrepreneurs would not wish to invest in an asset that depreciates quickly. For intangibles to receive a very high gross rate of return relative to structures, their share in production must be very large relative to structures.

I set $\sigma = 2$, as is standard in the literature. In order to calibrate the financial parameters $\bar{\zeta}$, μ and χ , I follow a strategy similar to Bernanke, Gertler and Gilchrist (1999). I match a business failure rate of 3% annually, a credit spread of 1.7% annually³⁷ and an entrepreneurial

³⁵ This can be calculated from Table 1 in Corrado and Hulten (2010). Corrado and Hulten calculate the level of investment in intangibles by summing over measurements of investment in a wide variety of assets including software, research and development, marketing activities and training. They infer the stock of intangibles based on estimates of the depreciation rates of these various assets. It is worth noting that the level of intangible capital calculated by Corrado and Hulten (2010) is actually smaller than many others in the literature. For instance, Eisfeldt and Papanikolaou (2014) estimate that the stock of 'organization capital' alone is greater than the stock of property plant and equipment. Their approach implies that more than 70% of business capital and an even larger fraction of business investment is intangible. See also Hulten and Hao (2008) and McGrattan and Prescott (2014) for methods that imply a somewhat larger fraction of the capital stock that is intangible than the approach used here. Calibrating to larger stocks and investment rates in intangibles would naturally imply a larger response of output to financial shocks in this paper.

³⁶See Acemoglu and Shimer (1999) for a discussion of this issue.

³⁷This is the average value of the credit spread measured by Gilchrist and Zakrajsek (2012) over the period 1973-2012.

Table 1: Calibrated Parameters

Parameter	Value Used	Target Moment	Moment Value	Source
β	0.99	Private Capital/ Annual Output Ratio	1.89	NIPA and Corrado and Hulten (2010)
δ_I	0.049	Private Equipment & Intangible Investment/Output	0.21	
δ_S	0.011	Private Structural Investment/Output	0.036	
α_I	0.38	Private Labor's Share of Business Output	0.47	
α_S	0.15	Ratio of Structural Capital to Total Capital	0.44	
σ	2	Standard	2	
$\bar{\varsigma}$	0.37	Business Failure Rate	0.03	Bernanke, Gertler, Gilchrist (1999)
μ	0.51	Annual Average Credit Spread	0.017	Gilchrist and Zakrajsek (2012)
χ	0.61	Equity-to-Debt Ratio	1.65	Masulis (1983)
κ_1	1.2	Investment Adjustment Costs	1.2	Based on Christiano and Davis (2006) discussion.
κ_2	0.7	Capital Adjustment Costs	0.7	Based on Christiano and Davis (2006) discussion.
\bar{N}_G	0.16	Public Employment/ Total Employment	0.167	Michaillat (2014)
δ_N	0.075	Inflow Rate into Unemployment	0.075	Davis et. al. (2010)/ CPS
h_0	0.37	Training Costs/Quarterly Wage	0.04	Silva and Toledo (2009)
h_1	0.019	Hiring Costs/Quarterly Wage	0.55	
h_2	0.002	Vacancy Adjustment Costs	0.002	Tiny
A_M	1	Normalization		
ψ	0.5	Hiring Cost Elasticity	0.5	Shimer (2005)
ν	0.18	Disutility of Working =40% of Wage		Shimer (2005)
ϑ	0.34	Unemployment Rate	0.06	

net-worth to debt ratio of 1.65.³⁸

I set the parameters of the adjustment cost function based on the literature. Unfortunately, there is not, to my knowledge, any work that estimates the parameters of an adjustment cost function containing adjustment costs to the level of both capital and investment. Christiano and Davis (2006) separately consider adjustment cost functions consisting of just adjustment costs to the level of capital (i.e. $\kappa_1 = 0, \kappa_2 > 0$) and adjustment costs to the level of investment ($\kappa_1 > 0, \kappa_2 = 0$). They find, using a simple neoclassical model, that the volatility of aggregate investment relative to the volatility of the return to capital (based on the stock market) is vaguely in line with the data when either $\kappa_2 = 1$ or $\kappa_1 = 3.5$. Even at these parameter values, they still find excessive volatility of investment relative to the return to capital and suggest that adjustment costs would need to be around twice as large as these numbers in order for the volatility of investment relative to returns to be close to the data. This suggests that in a model with only one kind of adjustment cost, either $\kappa_1 = 3.5$ or $\kappa_2 = 1$ may be a modest figure.³⁹ At the same time, it would be unreasonable to impose both $\kappa_1 = 3.5$ and $\kappa_2 = 1$, since this would produce overall adjustment costs much larger than suggested by Christiano and Davis. If adjustment costs to the level of capital and to the level of investment were viewed as roughly equally important in explaining the low volatility of investment relative to the return to capital, then a natural guess would be to set $\kappa_1 = \frac{3.5}{2}$ and $\kappa_2 = \frac{1}{2}$. However, Roehe (2013) compares the results of an estimated medium-sized DSGE model using adjustment costs to either the level of investment or the level of capital (but not both). He finds that each kind of adjustment costs provides a better fit for particular aspects of the aggregate data, but that the overall fit is better with capital adjustment costs. Based on this argument I infer that adjustment costs to the level of investment are likely relatively less important and adjustment costs to the level of capital relatively more important. Therefore, I set adjustment costs to the level of investment around one third of the Christiano and Davis figure, and adjustment costs to the level of capital around two-thirds of the Christiano and Davis figure. That is, I set $\kappa_1 = 1.2$ and $\kappa_2 = 0.7$.

I set the share of government employment to be 16.7% of total employment in the steady state, as found by Michailat (2014) to be true in CES data.

³⁸Masulis (1983) finds that the equity value/debt varies between 1.3 and 2 for US corporations over the period 1937-1984.

³⁹Models which estimate the elasticity of investment with respect to marginal Q at the firm level frequently find evidence of much larger adjustment costs than this. See, for instance Erickson and Whited (2000) for the case of capital adjustment costs and Eberly, Rebelo and Vincent (2012) for investment adjustment costs. The adjustment cost parameter estimated by Eberly et. al – 1.86 – appears at first glance to be modest compared to the figure of 3.5 used by Christiano and Davis (2006). However, Eberly et. al. use annual rather than quarterly data, suggesting that their estimate should be multiplied by around 4 when moving to the quarterly frequency.

5.1 Calibration of Labor Market Parameters

I calibrate the values of training and vacancy posting costs, h_0 and h_1 respectively, to match estimates by Silva and Toledo (2009) for the fraction of the average wage spent by firms on hiring and training costs in the first quarter of employment. I set h_2 to the extremely small value of 0.002 to minimize the effect of these adjustment costs on model dynamics. With $h_2 < 0.001$, I find that indeterminacy can occur in the model. Setting $h_2 = 0.002$ keeps these adjustment costs as small as possible, but safely above the threshold for indeterminacy. The value $h_2 = 0.002$ implies that adjustment costs to the level of vacancies correspond to less than 0.05% of hiring costs in every period in the calibrated model, for shock realizations of fewer than 5 standard deviations away from the mean.

I set the separation rate δ_N to roughly match the inflow rate into unemployment, which Davis et. al. (2010) find to fluctuate between 2% and 2.5% monthly in the CPS over the 1995-2005 period. The labor bargaining weight ϑ is set to match a steady state unemployment rate of 6%. I set ν so that the flow value of unemployment evaluated in consumption units \bar{u}_t is equal to 40% of the wage in the steady state, as in Shimer (2005).⁴⁰ I set $\psi = 0.5$, following Shimer (2005), and set $A_M = 1$, which is a normalization that has no effect on most aggregate variables.

In addition to these parameters, the behavior of wages in the model depends on the cost to the entrepreneur of proposing wage offers γ and the rate at which wage bargaining breaks down ρ . Since these parameters govern the rigidity of wages in response to aggregate shocks, I estimate them using a Bayesian approach based on the aggregate dynamics of wages in US data.

The alternating offer bargaining solution (44) implies that wages should depend upon current and expected future values of consumption, employment and productivity. Indeed, if the aggregate dynamic process governing these three variables is known, values of the wage w_t , recruiting costs h_t , vacancies v_t in each period, and the worker's continuation values \mathcal{W}_t and \mathcal{U}_t , can all be calculated, given the model parameters, using only the equations governing labour markets and preferences. That is, the values of these variables in each period can be inferred using only the model equations (11), (14), (31), (40), (42), (43), (44), (47), and (48). Therefore, rather than conducting a Bayesian estimation of the whole model, I assume that productivity, consumption and employment follow a reduced-form VAR process with two lags. I estimate this VAR process against the data, and use only the VAR and the eight model equations just mentioned for the purposes of Bayesian estimation of the parameters γ and ρ . The advantage of this approach is that my estimates of these two parameters will not be contaminated if other parts of the model, such as the equations governing financial markets, are mis-specified.

⁴⁰Hall argues in favor of a flow value of unemployment equal to 41% of the wage, based on a constant Frisch elasticity of labor supply of 0.7. Many papers in the search literature, such as Christiano, Eichenbaum and Trabandt (2014), consider a higher flow value of unemployment and justify this as including unemployment benefits. However, Chodorow-Reich and Karabarbounis find that these are negligible on average, once eligibility requirements and cost of take-up are taken into consideration.

I assume that wages are measured with iid error and jointly estimate the parameters of the VAR, the error variance of wages and the parameters ρ and γ . This entails estimating parameters in a system of eleven equations, where the eleven equations are the VAR process for consumption, employment and productivity and the eight model equations mentioned above. I assume that there are four variables whose values are observed in the data: productivity, consumption, employment and measured wages.⁴¹

Of course, the eight equations from the model contain other parameters in addition to ρ and γ , notably the parameters governing hiring costs and the matching function, as well as α_I , α_S , the discount factor β and the parameters ν and ϑ . For the purposes of the estimation, I fix the parameters governing hiring and matching as well as α_S , α_I and β at the values given in Table 1.⁴² I also set the values of ν and ϑ to match the target moments in Table 1. However, the values of ν and ϑ that this entails are highly sensitive to the values of the wage bargaining parameters γ and ρ . Therefore, when calculating the likelihood of each value of ρ and γ during the Bayesian estimation, I set ϑ and ν in each case to match the target moments in Table 1, given ρ and γ .

For the values of the observed variables, productivity, consumption, employment and wages, I use US data from 1979-2005,⁴³ taking aggregate consumption and output from the NIPA and employment and working hours from the CPS. I measure productivity using output per labor hour. I ignore government employment, and assume that aggregate employment corresponds to private employment in the model.⁴⁴ In order to remove trends in all variables, I use the band-pass filter of Christiano and Fitzgerald (2003) to isolate variation in the 2-120 quarter frequency band.⁴⁵

For wages, I use the series on wages of new hires out of unemployment constructed by Haefke, Sonntag and Van Rens (2013) from the CPS, adjusted for composition effects.⁴⁶ I use this series

⁴¹Assuming that these eleven equations are not mis-specified, the parameters γ and ρ are identified. This is because the number of observed variables, four, is equal to the number of shocks – where the four shocks are the three error terms in the VAR, and the iid measurement error in wages.

⁴²Since the calibrated values of α_S , α_I and β do depend on the values of ρ and γ , this requires an iterative process. Specifically, I set the values of α_I , α_S and β , and estimate the wage bargaining parameters. Then, using the estimated wage bargaining parameters, I recalibrate the values of α_I , α_S and β to match the target moments in Table 1. I then re-estimate the wage bargaining process and follow these steps repeatedly until convergence. In any case, I find that my calibrated values for α_S , α_I and β are not highly sensitive to the values of the wage bargaining parameters.

⁴³I consider these dates because the wage series I use is not available for other dates.

⁴⁴As an alternative, a government employment can be included as an extra variable in the VAR. This makes a negligible difference to the estimates, since it is the aggregate level of employment primarily that affects the bargained wage, rather than whether the levels of government and private employment separately.

⁴⁵120 quarters is of course much longer than what is usually considered the length of US business cycles. However, since the focus on the model is on persistent movements in aggregate variables, it is of importance that the response of wages to other variables is consistent with the data at frequencies below the business cycle as well as at high frequencies. This argues in favor of using data across a wide frequency band, such as 2-120 quarters.

⁴⁶I thank Thijs van Rens for making this data series publically available at <http://www.thijsvanrens.com/wage/>

because a large empirical literature has noted that measuring the wage using average hourly earnings of all workers may generate spurious evidence that the price of labor is rigid. This is for two reasons. First, wages of long-standing employees of a firm may adjust infrequently if they are set by implicit contracts. Therefore, aggregate wages may appear rigid. However, the marginal cost of labor to employers depends on how much they would have to pay newly hired workers, which may be more volatile. As such, I consider the average wage of new hires out of unemployment rather than the the average wage of all workers. The second concern with aggregate wage series is that they may generate spurious evidence that wages are acyclical or rigid due to composition biases. For instance, since workers who tend to lose their jobs in recessions are frequently those who earn relatively low wages, a rise in unemployment in a recession will tend to increase the average wage of all workers due to this compositional change. Failure to adjust for this generates a countercyclical bias in measured wages. Therefore, I use the series of Haefke, Sonntag and Van Rens (2013), which adjusts for cyclical variation in average levels of education and demographic variables among workers.⁴⁷ Haefke et. al. argue that this adjustment appears to eliminate most sources of compositional bias.

For the coefficients of the VAR, I use a normal prior with a standard deviation of 2 and a mean of 0, except for the coefficient of each variable on its own first lag, for which I use a normal prior with a mean of 0.9 and a standard deviation of 1. For the VAR shocks and the error term of wages, I use an inverse gamma-2 prior, with a mean of 0.01. For the parameter ρ , I use a lognormal prior. For the parameter γ , I assume that $\gamma = \gamma_0 \frac{\bar{Y}}{N}$ that is, it is equal to the proportion γ_0 of steady state productivity. For γ_0 , I use a beta prior with mean of 0.45, a standard deviation of 0.225 a minimum of 0 and a maximum of 0.9 , which is close to a uniform prior on the range $[0, 0.9]$. Priors and estimates of these parameters are shown in Table 2.⁴⁸

Table 2: Estimates of Wage Bargaining Parameters

Parameter	Prior Type	Prior Mean	Prior Standard Deviation	Posterior Mode	Posterior Stand. Dev.
$\log(\rho)$	Normal	$\log(0.5)$	3	$\log(0.02)$	1.09
γ_0	Beta	0.45	0.225	0.66	0.27

⁴⁷For further discussion of the relevance of wages of new hires rather than all workers, see Haefke, Sonntag and Van Rens (2013) and Pissarides (2009) and the references therein. For discussion of composition effects see Haefke, Sonntag and Van Rens (2013) and Daly, Hobijn and Wiles (2011) and the references therein. Arguably, the level of labor productivity should also be adjusted for compositional changes. To maintain comparability with Haefke, Sonntag and Van Rens (2013), I do not make this adjustment, since they do not. However, I find that applying the same compositional adjustments to productivity as I apply to wages makes negligible difference to the estimated labor market parameters.

⁴⁸I find that if I use the average wage of all workers, instead of the average wage of new hires, as the wage series for estimation purposes, then this moderately increases the level of estimated wage rigidity. Indeed, the posterior mode for ρ falls to 0.002. Similarly, I find that failure to adjust for composition effects slightly increases estimated wage rigidity. Higher levels of wage rigidity in the model produce larger slumps following financial shocks and strengthen the conclusions of the paper.

Although the priors are relatively loose, the posterior in ρ is relatively tight around a value of ρ close to zero. The posterior mode, $\rho = 0.02$, is very far from the standard Nash bargaining solution used in the search and matching literature, which corresponds to the special case $\rho = \infty$, as discussed in section 4.13. The much lower value of $\rho = 0.02$ implies that wages are not very sensitive at all to unemployment. Values of ρ above 0.18 can be rejected at the 5% level. This suggests that the data strongly favors wages that are substantially more rigid than the standard Nash bargaining solution. The value of γ_0 is less well identified, with the posterior standard deviation being close to the prior. For the model calibration, I use the posterior modes $\rho = 0.02$ and $\gamma_0 = 0.66$.

5.2 Calibration of Shocks

Table 3 shows the calibration of the shock processes. I calibrate the variance and autocorrelation of the shock e_c in order to match the variance and autocorrelation of the credit spread in US data for the 1970-2012 period, where I use the measure of the credit spread constructed by Gilchrist and Zakrajšek (2012), which measures the average excess return on corporate bonds relative to treasury bonds of similar maturity.

I interpret the entrepreneurial risk shock as representing a financial shock, and a severe increase in entrepreneurial risk as representing a financial crisis. This is for several reasons. First, the risk shock only affects the aggregate level of capital, output and employment insofar as it affects the costs of borrowing for entrepreneurs. That is, if we were to set monitoring costs equal to zero, thereby turning off financial frictions, the risk shock would have no effect on any aggregate quantities of interest. Second, the risk shock has an extremely strong effect on the credit spread as well as entrepreneurial leverage. Therefore, the risk shock induces strong comovements between credit spreads and financial conditions. Krishnamurthy and Muir (2015) have shown that movements in credit spreads are powerful predictors of the severity of financial crises. Third, Christiano, Motto and Rostagno (2014) have found in a estimated DSGE model featuring a similar entrepreneurial risk shock, that the risk shock accounts for 95% of the volatility in the credit spread at business cycle frequencies in the US. Moreover, they find that their measured risk shock rises and falls over time simultaneously with the standard deviation of the cross-section of stock returns of non-financial firms, suggesting it is indeed related to firm-level risk.

In order to compare the effects of the financial shock with those of other shocks, I also include the TFP and government employment shocks. I set the variance and autocorrelation of the public employment shock e_G so that the standard deviation of the autonomous component of public employment matches the value used by Quadrini and Trigari (2007), and the autocorrelation of government spending matches the estimate of Christiano, Motto and Rostagno (2014). Finally,

Table 3: Calibration of Shocks

Parameter	Value Used	Target Moment	Moment Value	Source
σ_ς	0.28	St. Dev. of Spread	0.0076	Gilchrist and Zakrajsek (2012)
ρ_ς	0.84	Autocorrelation of Spread	0.84	Gilchrist and Zakrajsek (2012)
σ_G	0.023	St. Dev. of Gov. Employment	0.023	Quadrini and Trigari (2007)
ρ_G	0.94	Autocorrelation of Gov. Spending	0.94	Christiano, Motto and Rostagno (2014)
σ_Z	0.072	St. Dev. of TFP Shock	0.072	King and Rebelo (1999)
ρ_Z	0.90	Autocorrelation of TFP Shock	0.90	Michaillat (2014)

I set the variance of innovations in the TFP shock to match the value recommended by King and Rebelo (1999) and set the autocorrelation to match the autocorrelation of the cyclical component of TFP estimated by Michaillat (2014).

6 Results of Model Simulations

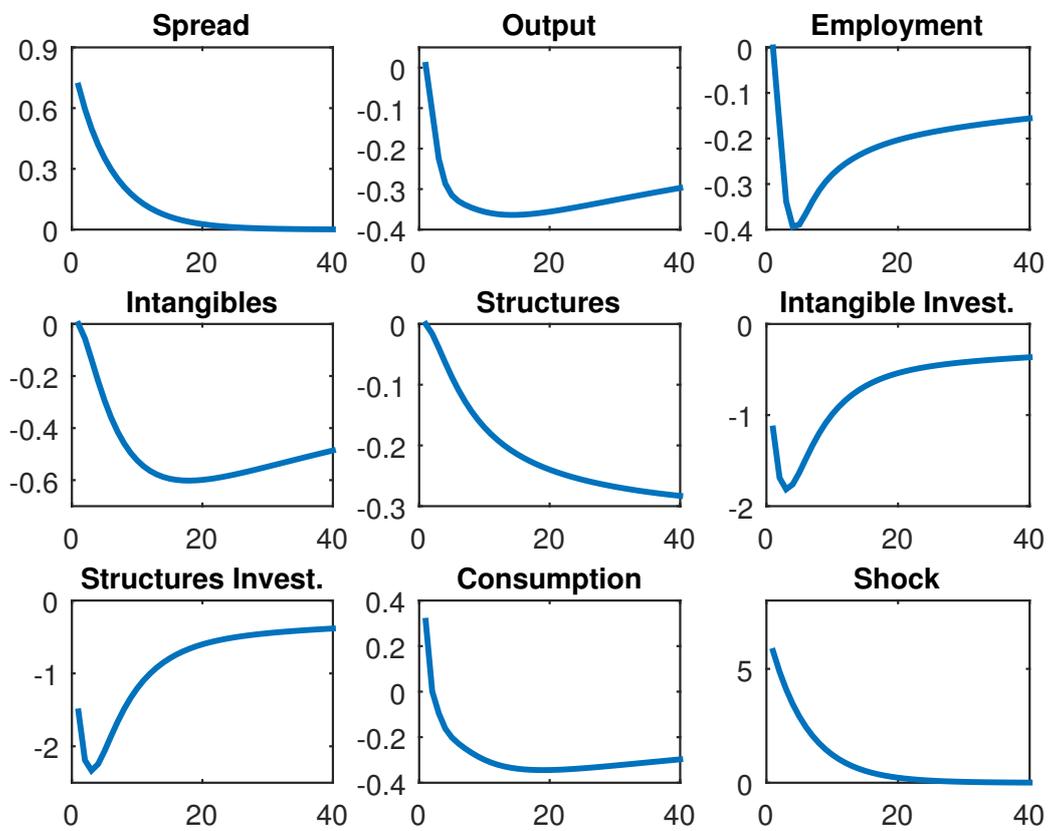
6.1 Effect of a Financial Shock

Figure 5 shows the impulse responses of model variables to a one standard deviation innovation in the financial shock.⁴⁹ All impulse responses are shown as percentage deviation from steady state values. The credit spread itself does not respond particularly persistently to the shock. This is unsurprising, since the shock process was calibrated to match the empirical behavior of spreads, which display little persistence. The bottom right panel shows the deviation of ς from its steady state value; this closely matches the behavior of the spread, indicating that the shock itself is no more persistent than the spread.

By contrast, the model generates a highly persistent response of output and employment. Output continues to decrease for the first four years after the shock hits and has recovered less than a quarter of its only way back to trend ten years after the shock hits. Moreover, the decrease in output is quantitatively non-trivial: 0.38% for a one standard deviation innovation in the shock. Employment likewise decreases by 0.39% in the first year after the shock hits, and has only recovered half-way to the steady state six years after the shock hits. Therefore, the

⁴⁹In the model simulations, I define ‘Output’ to be the sum of the output of entrepreneurs plus government spending.

Figure 5: Impulse Response to a Financial Shock: Baseline Model



model appears to provide a mechanism that could account for some of the large and persistent output decreases following financial crises. Note that the shape of the response of output in the model closely matches the shape of the response of intangibles. This is because much of the decrease in output is caused by the decrease in intangible capital and the decrease in labor demand that results from this. While investment in structures actually drops much more than investment in intangibles after the financial shock, the stock of intangibles drops further and faster, due to the higher depreciation rate of intangibles. Since α_I is relatively large, 0.39, this induces a drop in output. The persistent decrease in the stock of intangible capital also induces a persistent decrease in employment, through the contribution of intangible capital to labor demand. The decrease in intangible capital and the resultant decrease in employment together account for most of the persistent slump in output. However, employment and, to some extent, output drop more rapidly than the stock of intangible capital initially. Employment reaches its lowest level 4 quarters after the shock hits, whereas the stock of intangible capital reaches its lowest level 18 quarters after the shock hits. The rapid drop in employment occurs because the tightening of financial conditions makes it more costly for entrepreneurs to borrow and therefore to fund their hiring costs. This immediately leads entrepreneurs to cut back on hiring.⁵⁰

A counterfactual prediction of the model is that the level of consumption rises for one period after the shock and then falls. The initial rise in consumption after the shock is primarily an artifact of the assumption that time is discrete, with capital and labor chosen one period in advance. Therefore, output is for the most part set one period in advance.⁵¹ When the shock hits, investment decreases immediately, which forces consumption to increase for one period, since output is largely predetermined. However, after the first period, output falls and so consumption falls along with investment. Indeed, consumption is only 0.01% above its steady state value in period 2, and falls below its steady state value in period 3.⁵²

6.2 Sensitivity to Modeling Assumptions

This section considers the role played by various modeling assumptions in generating the quantitative results of the model. I analyse in turn, how the impulse responses to financial shocks change when three modeling features are excluded: intangible capital with a high depreciation rate; alternating-offer wage bargaining, and adjustment costs to the level of capital and investment.

⁵⁰This mechanism also occurs in other models combining financial and labor market frictions. See, for instance, Petrosky-Nadeau (2014).

⁵¹Output is ‘for the most part’ set one period in advance, because government wage payments and monitoring costs are not set one period in advance.

⁵²If the model were adapted in some way so that the shock could affect output contemporaneously, rather than with a one-period delay, this would presumably resolve much of the anomalous behavior of consumption in the first period. One possible way to adapt the model to this effect would be for workers to produce output in the same period in which they are hired, so that a drop in hiring decreases output in the same period.

In order to provide a sense of the quantitative importance of the high depreciation rate of intangible capital in the model, Figure 6 shows the response to a financial shock when the model is re-solved and re-calibrated to have only one kind of capital, with a depreciation rate of 6.7% and a steady-state capital-output ratio of 2.3, which Khan and Thomas (2011) find to be the averages in the US fixed asset tables over 1954-2002, after adjusting for growth. For ease of comparison, Figure 6 also shows the impulse responses in the baseline model. As is evident from the figure, it is still the case in this setting that a financial shock decreases investment, leading to a deaccumulation of capital and a persistent decrease in output. However, the decrease in output is only around two-thirds as large as it is in the baseline model, in spite of a larger decrease in investment than in the baseline model. The reason is that the lower depreciation of capital in this setting means that the capital stock is more stable than the stock of intangibles is in the baseline model. Consequently, it decreases less than 2/3 as much as the stock of intangibles decreases in the baseline model, leading to a correspondingly smaller decrease in output. The aggregate stock of capital also decreases only 3/4 as much in the model with one type of capital as in the baseline model, due to the lower depreciation rate of capital, where the aggregate stock of capital in the baseline model refers to the combined stock of structures and intangibles.

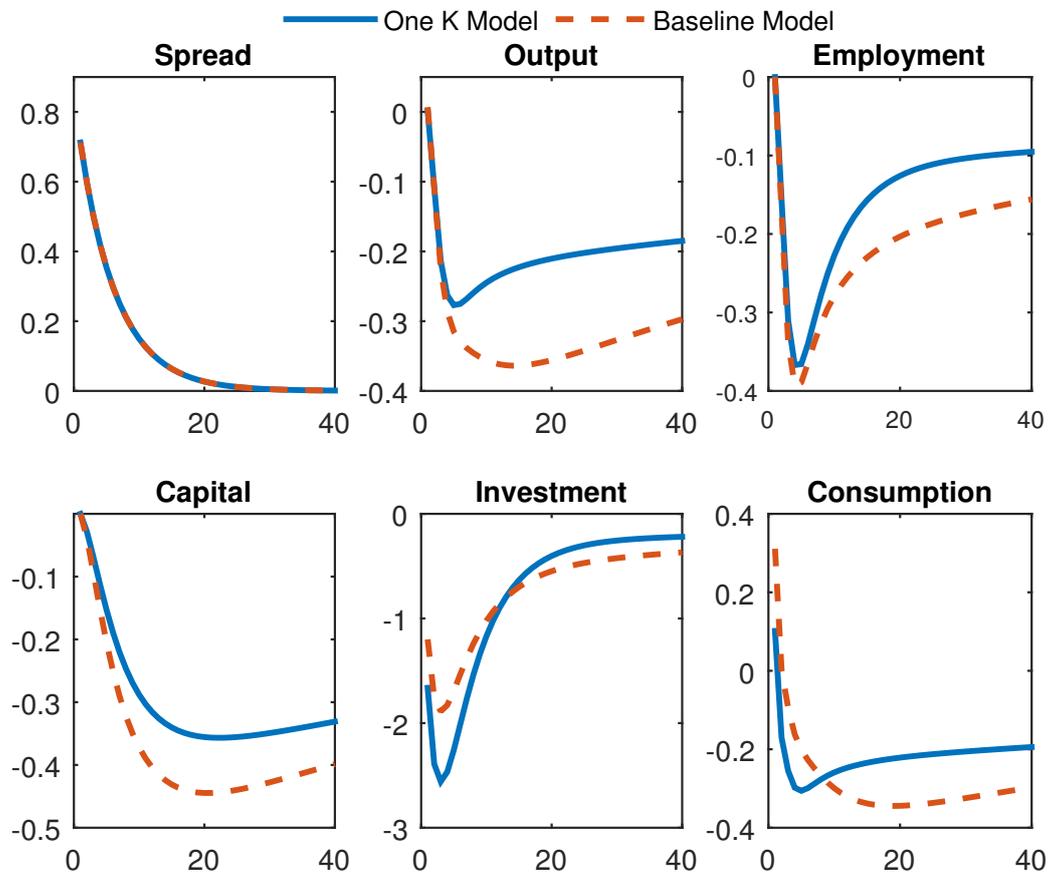
Note that while the drop in the stock of capital in the model with one type of capital is around 3/4 as large as in the baseline model, the resultant drop of output is only around 2/3 as large. The reason is that, in the baseline model, intangible capital contributes much more to production than do structures – with a share in production of 0.38, versus only 0.15 for structures.⁵³ Since intangible capital is disproportionately important in production in the baseline model, and falls more sharply than structures, this implies a relatively larger decrease in output in the baseline model. Therefore, the drop of output in the model with one type of capital is somewhat less than 3/4 as large as in the baseline model, even though the drop in the capital stock is fully 3/4 as large.⁵⁴

The model with one type of capital produces a decrease in employment from the shock on impact of virtually the same size as the baseline model. However, in the model with one type of capital, employment recovers more quickly. Indeed, in the model with one type of capital, employment returns half way to its steady state level 14 quarters after the shock hits, as compared to 25 quarters after in the baseline model. The reason for this pattern is that, in both models, the tightening of financial constraints directly hits entrepreneurs' ability to fund hiring costs and induces a rapid decrease in employment. However, this direct effect of tighter financial constraints on hiring is relatively transitory. The persistent decrease in employment in both models comes from capital deaccumulation. Since the baseline model experiences a decrease in

⁵³Section 5 above discusses why the high rate of depreciation of intangibles implies that that α_I should be high to induce entrepreneurs to invest in them.

⁵⁴It follows from this discussion that the baseline model is not isomorphic to a model with one type of capital and a higher average depreciation rate.

Figure 6: Impulse Response to a Financial Shock: Model with One Type of Capital

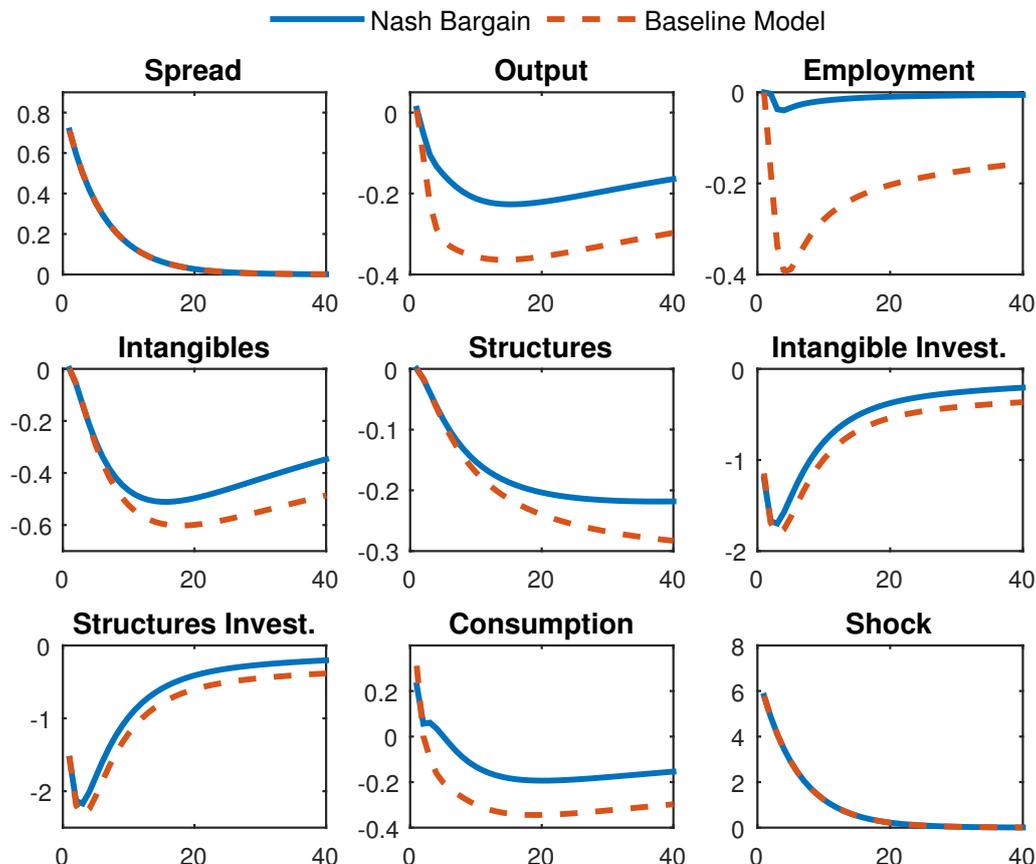


the stock of intangible capital of around 0.6%, compared to a decrease in aggregate capital of around 0.36% in the model with one type of capital, there is consequently a more persistent decrease in employment in the baseline model.

In order to assess the importance of wage rigidity in the model, I replace the alternating-offer bargaining protocol by the more standard Nash bargaining solution used in the search and matching literature, including Mortensen and Pissarides (1994). Moving to the standard Nash Bargaining solution is equivalent to using the alternating offer bargaining solution (44) and setting $\rho = \infty$. Therefore, I fix $\rho = \infty$ and recalibrate the rest of the model to match the same calibration targets as before.

Figure 7 shows the impulse responses to the financial shock in the model in which the Nash Bargain is used instead of the alternating offer bargain (i.e. when $\rho = \infty$). The baseline model results are also shown for ease of comparison. The most dramatic difference between the two models concerns the response of employment. In the model with standard Nash bargaining, the fall in employment is exceptionally small. This is because wages respond so strongly to changes in labor demand with Nash bargaining that the decreases in labor demand resulting from the deaccumulation of capital and tightening of financial decisions lead simply to a decrease in wages, rather than much of a decrease in employment. Since employment moves so little in the model with Nash bargaining, the response of output to the shock is also substantially smaller – output decreases after the shock around half as much as in the baseline model.

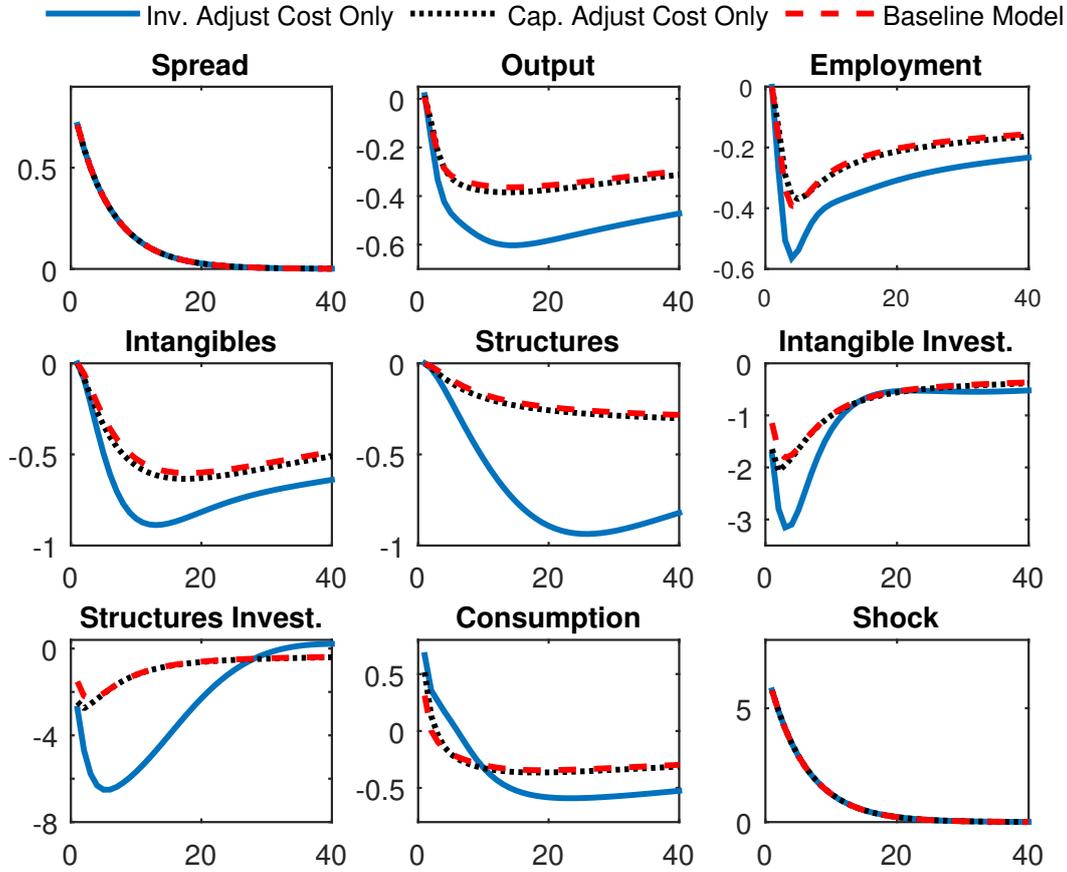
Figure 7: Impulse Response to a Financial Shock: Model with Nash Bargaining



Finally, in order to examine the role played by the adjustment costs to both the level of capital and to the level of investment in the model, Figure 8 shows how the impulse response to a financial shock changes as these are varied. The figure shows the impulse response in three cases. One case shown in the figure is the baseline model, with adjustment costs to the level of investment, $\kappa_1 = 1.2$ and adjustment costs to the level of capital $\kappa_2 = 0.7$. Another case shown is when the model is recalibrated to have only adjustment costs to the level of investment, that is $\kappa_1 = 1.2$ and $\kappa_2 = 0$. Another case shown is when the model is recalibrated to have only adjustment costs to the level of capital, $\kappa_1 = 0$, $\kappa_2 = 0.7$. In all these cases, I recalibrate the other parameters of the model to match the moments used in Tables 1 and 3.

It is evident from Figure 8 that varying adjustment costs to the level of capital affects the impulse responses much more than varying adjustment costs to the level of investment. When adjustment costs to the level of capital are removed and there are only adjustment costs to the level of investment, all variables move much more strongly in response to the shock. This is because, with fewer adjustment costs, investment can decrease much more strongly and so capital is deaccumulated much more quickly. This produces a much deeper slump. In the absence

Figure 8: Impulse Responses for Model with only Capital Adjustment Costs or only Investment Adjustment Costs

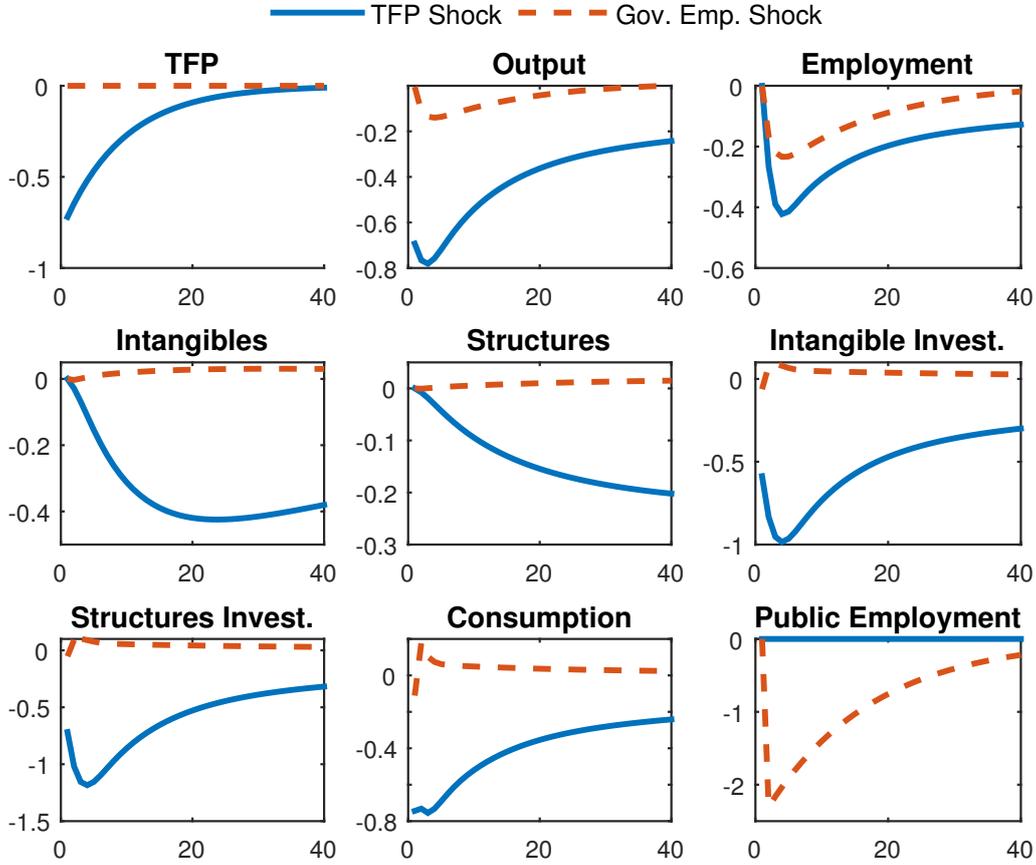


of capital adjustment costs, it is noticeable that the stock of structures decreases particularly sharply relative to the baseline model. This is because investment in structures falls more sharply, so that the ratio of structures to intangibles does not change too much from the steady state level. This prediction of the model without capital adjustment costs is counterfactual with regard to the US Great Recession because the stock of equipment and intangibles fell much more than the stock of structures in the recession, as discussed briefly in 6.4 below.

By contrast, removing the adjustment costs to the level of investment has little effect on most aggregate variables and produces impulse responses very similar to the baseline model. Nevertheless, it is notable that removing either kind of adjustment cost yields a sharper decline in investment and more countercyclical behavior of consumption. Essentially, reducing adjustment costs means that investment is free to decrease more quickly. Since output is relatively fixed in the very short run in the model, a more rapid decline in investment ensures that more resources are available for consumption and so consumption rises. This prediction of the models with fewer adjustment costs is at odds with the empirical fact that consumption clearly fell during

the US Great Recession and other financial crises.

Figure 9: Impulse Response to Other Shocks



6.3 Effects of Other Shocks

Figure 9 shows the response of output to contractionary public employment and TFP shocks in the model. These shocks have less persistent effects on output than the financial shock. Neither the TFP nor the public employment shock generates much of a hump-shaped response of output, unlike the financial shock. Moreover, output and employment have largely returned to steady state levels 40 quarters after these shocks hit, in contrast to after a financial shock. Notice that this is true in spite of the fact that the shock process for both of these shocks was itself calibrated to be more persistent than the financial shock, as shown in Table 3. The reason for the lower persistence is that these shocks affect investment less than the financial shock, and so the stock of intangibles decreases less. Consequently, the mechanism leading to a persistent decrease in output following a financial shock operates much less strongly for the other shocks.

That financial shocks in the model generate more persistent responses than non-financial shocks may account for why financial recessions appear to have more persistent output drops

than other recessions, as discussed in section 3 above.

From the perspective of most real business cycle models, it is surprising that government employment has such a large effect on aggregate employment and output in this setting. Notice that the increase in government employment has virtually no effect on private consumption or investment: the government spending multiplier is approximately equal to 1 in each period. The reason is that increased government employment increases labor market tightness, but this increase in labor market tightness has a very small effect on wages because of alternating offer bargaining. Consequently, job creation by the private sector is hardly hit.⁵⁵

6.4 Comparison to the Great Recession

In order to evaluate the empirical relevance of the model, I compare the model predictions to the movements of aggregate variables in the US over the period during and after the Great Recession, that is over 2007-2014. In order to assess how far the poor performance of the US economy in this period can be attributed to the 2008 financial crisis, I feed into the model the sequence of financial shocks that would lead the model to exactly match the behavior of the credit spread in the US over this period.⁵⁶ I assume that the economy was in the steady state in the first quarter of 2007. Given this sequence of shocks, I compare the implied behavior of other variables in the model to those in the data over this period. The results are shown in Figure 10. Data variables used in the figure are shown as percentage changes from their 2007 levels after eliminating a linear trend calculated over the period 1995-2006. Capital stocks at quarterly frequency were inferred using the perpetual inventory method, based on NIPA data on investment.

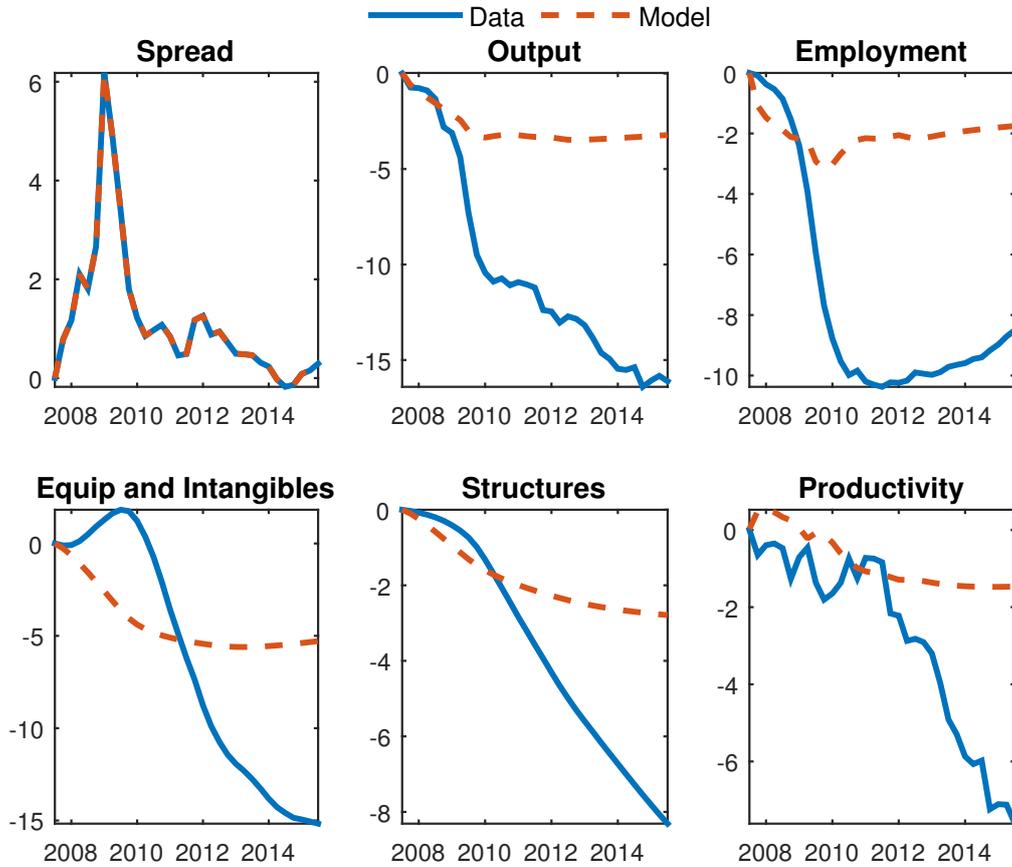
The model can replicate almost one third of the collapse in employment and output over the period 2008-2010. It is surprisingly capable of replicating the failure of these variables to return to trend by 2015. One area in which the model fails is accounting for the poor performance of productivity after 2011. For this reason, the model does not replicate the continuing decrease of output relative to its prior trend after 2011. It is possible that there has been a decrease in trend economic growth since 2008 in the US, which is absent from the model.

The model also accounts for around one third of the decrease in the stocks of structures and equipment and intangibles that is observed in the data. Here, it predicts an earlier decrease in these stocks than occurs in the data. That said, the NIPA measures used in constructing the figure may significantly understate the size and rapidity of the decrease in the stock of intangible

⁵⁵The mechanism for the public employment multiplier here is therefore similar to Michailat (2014).

⁵⁶Data on the Credit Spread measured by Gilchrist and Zakrajšek (2012) is unfortunately only available to the end of 2012. For the last two years, I measure the credit spread using the difference between the interest rate on BAA rated corporate bonds and the federal funds rate, which is highly correlated with the Gilchrist-Zakrajšek spread and has a similar variance. I subtract a constant from this, so that the two measures generate the same value of the credit spread in the last quarter of 2012.

Figure 10: Great Recession: Data vs. Model



capital in recent years. The measurement of equipment and intangibles I used for Figure 10 comes from the NIPA, due to limited data availability for recent years. This excludes marketing expenditures, which appear to be one of the most cyclical types of intangible investment. The limited available data on advertising expenditures in recent years appear to suggest that these expenditures fell strongly and rapidly in the Great Recession, suggesting that the decrease in equipment and intangibles was larger and faster than shown in Figure 10.⁵⁷

7 Conclusion

Movements in the stock of equipment and intangible capital may be a powerful force in the propagation of shocks in the economy and may have a major role to play in the persistent output drops following financial crises. Based on a panel of OECD and emerging economies, I discuss several patterns which suggest that drops in investment during financial crises might be related

⁵⁷In ongoing work, I construct an approximate quarterly series of the stock of intangible capital in the US in the post-war period based on data on advertising expenditures as well as the NIPA. I find that the stock of intangible capital is quite volatile over time and shows substantial decreases after spikes in the credit spread.

to the persistent output drops following these crises. I argue that financial recessions appear to have more long-lasting effects than non-financial recessions, consistent with a large empirical literature. Furthermore, I argue that financial recessions seem to exhibit larger decreases in the investment-output ratio than non-financial recessions, even in the year of and immediately after the recession, and that the crises which saw more persistent output drops also saw larger drops in the investment-output ratio.

I construct a model in which a financial shock decreases investment, particularly hitting the stock of equipment and intangibles, due to their especially high depreciation rates. I show that this can lead to a large and persistent slump in output and employment, if wages display a level of rigidity consistent with US aggregate wage data. I compare the behavior of aggregate variables in the US Great Recession to what would have occurred in the model, if movements in the credit spread in these years were caused by a sequence of financial shocks. I find that the model is capable of replicating around one third of the large and persistent decline in output and employment that occurred in the Great Recession. Moreover, it appears that the combined stock of equipment and intangible capital decreased substantially relative to trend in the recession, as is consistent with the mechanism of the model.

In the model, financial shocks lead to more persistent decreases in output and employment, relative to other shocks, because financial shocks particularly act to decrease investment. This is consistent with the empirical pattern that recessions marked by financial crises are associated with particularly large drops in investment and particularly persistent decreases in output. It is the drop in investment after a financial shock that is key to the persistent decrease in output in the model. As such, it is unsurprising that the financial crises in the past which were not followed by persistent slumps in output also typically did not witness large drops in investment during the crisis. That said, this empirical pattern still raises the question of why some financial crises did not feature large decreases in investment, and what it was that distinguished those crises. One possibility consistent with the model is that during those financial crises, the economy was primarily hit by shocks other than the entrepreneurial risk shock that is the focus of this paper. For instance, some sovereign default crises may affect the economy primarily through subsequent cuts to public employment. Consequently, a financial crisis occurring due to a sovereign default crisis might be followed by after-effects that resemble those of a public employment shock in the model: a decrease in output and employment that is less persistent, and a smaller decrease in investment compared to the entrepreneurial risk shock. Further investigation of this possibility is left to future work.

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Appendices

A Alternating-Offer Bargaining

A.1 Bargaining Protocol

This section provides a precise microfoundation for the wage bargaining equation in (44). Bargaining occurs over time, simultaneously with production. Assume that a period t can itself be divided into a continuum of measure 1 subperiods. Therefore, within a period, time is continuous. I index time within the period by $j \in [0, 1]$ to distinguish from t , which is discrete. All shocks are revealed before $j = 0$. Production can occur over $j \in [0, 1]$. Immediately after $j = 1$, investment, hiring, default and consumption all take place. The entrepreneur makes an initial wage offer to the worker at the instant $j = 0$. The offer will be a wage for the whole period. The worker can either accept or reject. If the worker accepts the offer, then production begins immediately and the offered wage is paid immediately at $j = 1$. If the worker rejects the offer, then the entrepreneur and worker start preparing new offers. This takes time. It is assumed that the entrepreneur becomes ready to make a new offer to the worker at Poisson rate $\lambda(1 - \vartheta)$. Similarly, the worker becomes ready to make an offer to the entrepreneur at Poisson rate $\lambda\vartheta$. Therefore, it is stochastic as to who will have a chance to make the next wage offer, the worker or the entrepreneur. The parameter λ governs the speed at which any agent can make an offer, and the parameter ϑ governs the relative speed at which workers and entrepreneurs can make offers.

Whenever the entrepreneur has the chance to make a new offer to the worker, then once again the worker can accept or reject. If the worker accepts then the worker commences work and production takes place for whatever is left of the period; total output produced will be lower than if an agreement had been reached at the beginning of the period because some time has passed. The agreed wage is paid at the end of the period. If the worker rejects again, then both entrepreneur and worker go back to preparing new offers and become ready to make a new offer at, respectively Poisson rates $\lambda(1 - \vartheta)$ and $\lambda\vartheta$.

Whenever the worker has a chance to make an offer, things proceed similarly. The entrepreneur can accept or reject. If she accepts then the worker starts work, output is produced in whatever is left of the period and the worker is paid at the end of the period. If the entrepreneur rejects, then both entrepreneur and worker go back to preparing new offers and can make another offer at Poisson rates $\lambda(1 - \vartheta)$ and $\lambda\vartheta$.

The entrepreneur and worker continue to make new offers in this way until an offer is accepted. However, the entrepreneur and worker would both prefer, all else equal, to reach an agreement as soon as possible because bargaining is costly in two ways. First, the longer time is spent

bargaining the less time there is to produce output, and the less output is produced. The worker's full marginal product of labor $\pi_t(\xi_t) = (1 - \alpha_I - \alpha_S) \frac{y_t(\xi_t)}{n_t}$ is produced if a deal is made at the beginning of the period but output is only $(1 - j)\pi_t$ if a deal is instead reached at point $j \in (0, 1]$.

Second, while the entrepreneur and worker are continuing to bargain – until they reach an agreement – there is a flow probability ρ that bargaining breaks down and the match separates. On the other hand, the worker benefits a little from bargaining in that he does not have to work while he is bargaining, and so does not face the disutility of working ν .

If the end of the period is reached, i.e. $j = 1$, and no offer has yet been accepted but at the same time bargaining has not broken down, then no production takes place this period and the worker is not paid. At this point, the worker and entrepreneur separate with probability δ_N , just as they would if they had reached agreement.

A.2 Continuation Values

Let $\hat{\mathcal{W}}_t(\xi, j)$ denote the continuation value of an employed worker in sub-period j who is matched with an entrepreneur of productivity ξ and *who has not yet reached agreement with his employer over wages this period*. Similarly, let $V_{N,t}(\xi, j)$ denote the marginal value of the worker to the entrepreneur in sub-period j if the entrepreneur is still matched with the worker but has not yet reached agreement with the worker over wages this period.

Recall that at the beginning of the period the entrepreneur makes an offer to the worker. As bargaining is totally wasteful, the entrepreneur maximizes her profits by offering the worker the wage $w_t(\xi)$ that makes the worker indifferent between accepting the offer and rejecting, and the worker will accept this offer.⁵⁸ The value of an employed worker $\mathcal{W}_t(\xi)$ at the beginning of the period is therefore the worker's value of accepting this initial offer.

Worker indifference between accepting and rejecting the offer implies that:

$$\mathcal{W}_t(\xi) = \hat{\mathcal{W}}_t(\xi, 0) \tag{49}$$

Here, the left hand side denotes the value of accepting the offer and the right hand side denotes the value of rejecting.

Now suppose, hypothetically, that in some sub-period $j \in (0, 1)$ is reached and the worker and entrepreneur are still matched but no agreement has yet been reached. Suppose that the entrepreneur gets an opportunity to make an offer to the worker at j . Then the entrepreneur will make an offer that makes the worker indifferent between accepting and rejecting and the

⁵⁸As is standard in the contract literature, it is assumed that the worker breaks indifference in favor of accepting the offer, since the resultant equilibrium can always be approached by a wage offer which the worker is strictly better off accepting.

worker will accept. Let $\hat{w}_t(\xi, j)$ be the offer the entrepreneur would make in sub-period j . Then $\hat{w}_t(\xi, j)$ satisfies:

$$\hat{w}_t(\xi, j) + \hat{\mathcal{W}}_t(\xi, 1) = \mathcal{W}_t(\xi, j) \quad (50)$$

where the left hand side denotes the worker's value if he accepts, and the right hand side denotes his value if he rejects. The value to the worker of acceptance here is the value of the wage paid for the remainder of the period, plus the continuation value that the worker would have at the end of the period even if no agreement had been reached, i.e. $\hat{\mathcal{W}}_t(\xi, 1)$.

Since the worker will accept, then the entrepreneur correspondingly receives a value of:

$$(1 - j)\pi_t - \hat{w}_t(\xi, j) + \hat{V}_{N,t}(\xi, 1)$$

That is, the entrepreneur gains a continuation value equal the value of the worker's remaining marginal product that will be produced in the period, i.e. $(1 - j)\pi_t$ (because the worker will immediately start work at j), minus the wage, plus the marginal value the entrepreneur would have at the end of the period from the worker even if no agreement had been reached.

Suppose that some $j \in (0, 1)$ is reached, the pair are still bargaining, and the worker gets an opportunity to make an offer to the entrepreneur. Then the worker will likewise make an offer that makes the entrepreneur indifferent between accepting and rejecting and the entrepreneur will accept. Let $w'_t(\xi, j)$ denote the offer the worker makes at x . Then $w'_t(\xi, j)$ satisfies:

$$(1 - j)\pi_t - w'_t(\xi, j) + \hat{V}_{N,t}(\xi, 1) = \hat{V}_{N,t}(\xi, j) \quad (51)$$

where the left hand side is the entrepreneur's continuation value if she accepts the offer and the right hand side is her value if she rejects. Since the entrepreneur will accept the offer, the worker will then get the value:

$$w'_t(\xi, j) + \hat{\mathcal{W}}_t(\xi, 1)$$

A.3 HJB Equations

The bargaining protocol therefore implies that $\hat{V}_{N,t}(\xi, j)$ and $\hat{\mathcal{W}}_t(\xi, j)$ satisfy the following HJB equations:

$$\begin{aligned} 0 = & -\gamma - \rho\hat{V}_{N,t}(\xi, j) + \lambda(1 - \vartheta)[(1 - j)\pi_t - \hat{w}_t(\xi, j) + \hat{V}_{N,t}(\xi, 1) - \hat{V}_{N,t}(\xi, j)] \\ & + \lambda\vartheta[(1 - j)\pi_t - w'_t(\xi, j) + \hat{V}_t(\xi, 1) - V_t(\xi, j)] + \frac{\partial \hat{V}_{N,t}(\xi, j)}{\partial j} \end{aligned} \quad (52)$$

$$\begin{aligned}
0 = \nu C_t + \rho[\mathcal{U}_t - \nu j C_t - \mathcal{W}_t(\xi, j)] + \lambda(1 - \vartheta)[\hat{w}_t(\xi, j) + \hat{\mathcal{W}}_t(\xi, 1) - \mathcal{W}_t(\xi, j)] \\
+ \lambda\vartheta[w'_t(\xi, j) + \hat{\mathcal{W}}_t(\xi, 1) - \mathcal{W}_t(\xi, j)] + \frac{\partial \mathcal{W}_t(\xi, j)}{\partial j}
\end{aligned} \tag{53}$$

Equation (52) is the HJB equation of the entrepreneur while bargaining.⁵⁹ During bargaining, the entrepreneur receives a flow value of $-\gamma$, that is, the cost of continuing to produce wage offers. With probability ρ the match terminates and the entrepreneur loses the marginal value of the match, $\hat{V}_{N,t}(\xi, j)$. At rate $\lambda(1 - \vartheta)$ the entrepreneur gets to make an offer to the worker and the worker accepts, in which case the wage will be $\hat{w}_t(\xi, j)$ and the entrepreneur will get value $[(1 - j)\pi_t - \hat{w}_t(\xi, j) + \hat{V}_{N,t}(\xi, 1) - \hat{V}_{N,t}(\xi, j)]$, that is, the value of the deal minus the value of continuing to bargain. At rate $\lambda\vartheta$ the worker makes an offer, the wage is $w'_t(\xi, j)$ and the entrepreneur gets $[(1 - j)\pi_t - w'_t(\xi, j) + \hat{V}_t(\xi, 1) - V_t(\xi, j)]$. Finally, even if bargaining continues without resolution, the entrepreneur's value changes at rate $\frac{\partial V_{N,t}(\xi, j)}{\partial j}$.

The worker's HJB equation (53) is very similar. The flow value of bargaining is given by νC_t , that is, the marginal value to the household of not working, evaluated in consumption units. If the match terminates — which it does at rate ρ — the worker gets to enjoy being unemployed for the remainder of the period — i.e. gets $\nu C_t(1 - x)$ from this and gets the value of being unemployed at the start of the recruitment phase $\mathcal{U}_t - \nu j C_t$ but loses the value of being employed $\mathcal{W}_t(\xi, j)$. If the entrepreneur gets to make an offer, the worker gets $[\hat{w}_t(\xi, j) + \hat{\mathcal{W}}_t(\xi, 1) - \mathcal{W}_t(\xi, j)]$, and if the worker gets to make an offer, he gets $[w'_t(\xi, j) + \hat{\mathcal{W}}_t(\xi, 1) - \mathcal{W}_t(\xi, j)]$. This gives rise to equation (53).

At the end of the period, if a deal has still not been reached, the worker and entrepreneur separate remain matched with probability $(1 - \delta_N)$. In that case, the entrepreneur will be able to 'sell' the worker at price h_t . Therefore

$$\hat{V}_{N,t}(\xi, 1) = (1 - \delta_N)h_t \tag{54}$$

Substituting (50) and (51) into equations (52) and (53) gives rise to two linear differential equations that can be solved for $\hat{V}_{N,t}(\xi, j)$ and $\hat{\mathcal{W}}_t(\xi, j)$ given boundary conditions (54) and (49).⁶⁰

⁵⁹Macroeconomists are usually accustomed to seeing the discount rate multiplied by the continuation value on the left hand side of HJB equations. Here, this does not appear because I assume no discounting *within the period* — discounting occurs between periods not within them. So the left hand side of each HJB equation is just zero. The discount rate is so low at the quarterly level that the quantitative impact of this assumption is small.

⁶⁰Details of the solution are available upon request.

Using $w_t(\xi) = \hat{w}_t(\xi, 0)$, and taking the limit as $\lambda \rightarrow \infty$, the solution gives

$$(1 - e^{-\rho}) \left[\frac{h_t + \pi_t(\xi) - w_t(\xi)}{1 - \vartheta} - \frac{\mathcal{W}_t(\xi) - \mathcal{U}_t}{\vartheta} \right] + e^{-\rho} \left[\frac{\gamma + \pi_t(\xi) - w_t(\xi)}{1 - \vartheta} - \frac{w_t(\xi) - \nu C_t}{\vartheta} \right] = 0$$

which was the wage bargaining solution considered in (44).

Taking the limit $\lambda \rightarrow \infty$ implies that agents can make wage offers infinitely quickly, which is obviously implausible. However, if agents can each make offers on average at least once per week, and there are 13 weeks in a quarter, this implies that $\lambda > 26$. At levels of λ this high or higher, the approximation error in assuming $\lambda \rightarrow \infty$ is incredibly small, provided $\rho < 4$ and $\vartheta \in (0.05, 0.95)$. Thus I let $\lambda \rightarrow \infty$ in order to produce a bargaining solution that is easily interpretable.